Design of Masonry Structures
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Module - 03
Lecture - 11
Strength and Behaviour of Masonry Part – I

We start with our third module, where we examine different actions and a combination of these actions. The primary purpose of the third module is to examine Behavior of Masonry in a detailed manner and be able to arrive at a closed-form solution/ closed-form analytical expression for masonry under different actions and combinations of actions. That becomes important because we are then able to establish the strength under different actions of different sizes of masonry components; maybe you are interested in looking at slender walls, squat walls, beams, columns and so on.

Starting with compression- concentric compression, eccentric compression; bending- out of plane bending and then finally in-plane bending or shear acting on a masonry wall and then of course we are interested in understanding how they interact with each other- Axial load with moment, axial force with shear force and have our failure surfaces under these actions ready and understood as far as the behavior of masonry is concerned. This becomes the analytical basis for design, right. So, the next module which will deal with design is going to be really feeding into the behavior of masonry under different actions.

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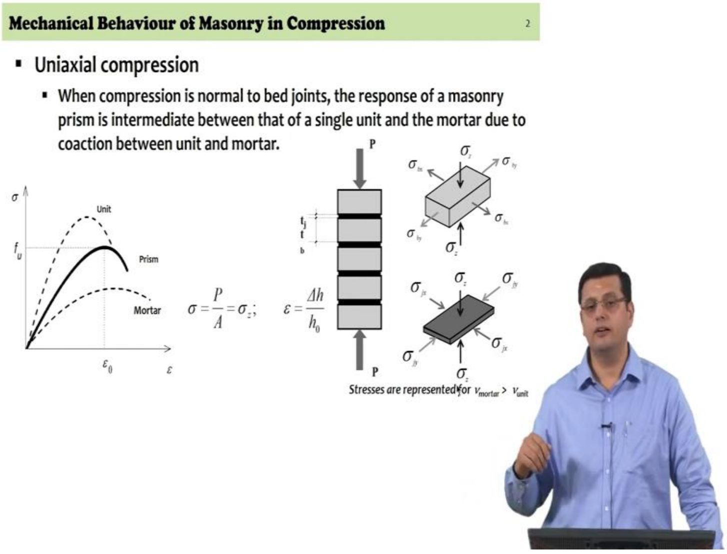

So, I begin with the mechanical behavior- the mechanics of masonry in compression, this is something we have already seen quite carefully, particularly the co-action between the unit and the mortar. However, the intention is if you were, if you had with you the strengths of the constituents and the properties of the constituents - the unit and the mortar, will you be able to estimate the compressive strength of masonry?

It is not often that you will be able to carry out a an experiment in the laboratory and say this is the failure strength of the masonry that I am going to be working with, you need to have an understanding analytically how this is performing and the unit strength and the mortar strength is something that you can establish quite easily even in practical situations. So, with that knowledge is it possible to be able to arrive at, for a complex behavior as we have seen, because of the coaction what is the failure strength of masonry itself in compression, ok.

So, we are examining in today's class, we are going to be examining uniaxial compression only- concentric. We have still not, we would not be going into strain gradients due to eccentricities of the compression and we are looking at the strength in compression of the masonry. It is not the strength of a wall; we are not bringing in the slenderness of the masonry wall where second order effects can come into the picture. So, we are still examining the masonry as a material under compression. So, we have studied this carefully and we discussed that the strength of the masonry is going to lie somewhere between the strength of the unit and the strength of the mortar and this is because of the co-action that occurs between the two; between the unit and the mortar.

The unit is going to be the stronger of the two, but the more brittle of the two, less deformable of the two; the mortar is more deformable, weaker in strength, but what you get as the combination of the two because of the bond that is established between the two constituents is something in between and we have also examined how the state of stress, in the multiaxial state of state of stress in the unit characterized by the uniaxial compression and the bilateral tension forms and how the triaxial state of compression in the mortar, because of the confinement offered by the unit itself.

So, let us keep that in mind and use this as a basis for the formulations. So, I am going to be examining the formulation under an assumption of linear elastic behavior of the unit and the mortar which is of course not true as you know. We are dealing with materials which have different strengths and can enter nonlinearity/inelasticity quite early and therefore we will examine a second theory which accounts for the inelastic behavior of both the mortar and the unit in arriving at the failure strength of the masonry itself.

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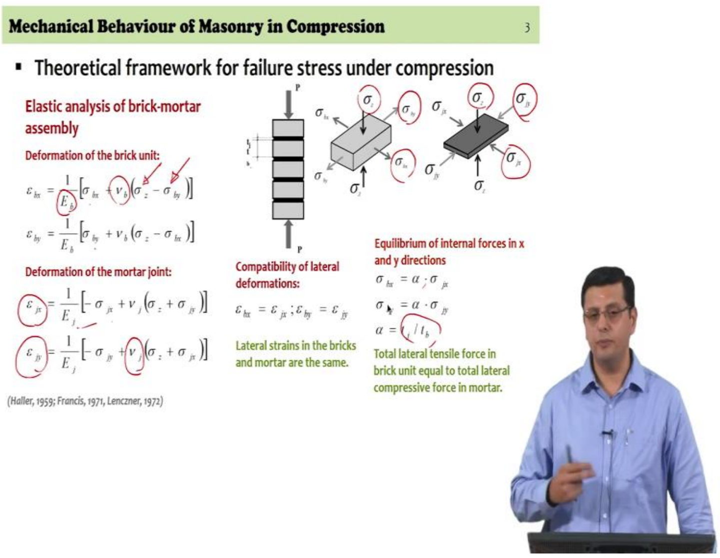

So, to begin with the linear elastic theory- this is a well-established theoretical framework; it comes from the work of Haller-Francis several decades ago. Assumption the fundamental assumption here is the entire assembly is working in an elastic manner, right and that is where the problem arises for us, ok. So, we were talking about the

Poisson's effect being fundamental in creating the states of multiaxial stress in the unit and the mortar.

So, if I were to examine the brick unit, just to get the know the notations familiarized. We are looking at the z-axis, the stress acting in the vertical direction-uniaxial compressive. The axial compression as σ_z, the lateral bilateral tension in the brick unit in the two directions σ_{bx} and σ_{by}. Whereas the multiaxial state of stress of compression in the unit in the mortar σ_z as the axial compression and the other two directions which are also in compression σ_{jx} and σ_{jy}. Here the subscript b is used to refer to the brick unit and j is referring to the joint itself. So, we have σ_{bx} and σ_{by}, σ_{jx} and σ_{jy}.

So, in terms of the dimensions of the unit and the mortar joint, we are looking at the thickness of the brick unit as t_b and t_j as the thickness of the mortar joint, ok. Now based on an understanding that under the state of triaxial stress, Poisson's effect comes into the picture, we can write down the deformation in the brick unit and the deformation in the mortar with respect to the states of stress in the different directions. So, if you look at the deformation in the brick unit, we can write down the strain in the brick unit in the x direction.

$$\varepsilon_{bx} = \frac{1}{E_b}\left[\sigma_{bx} + \nu_b(\sigma_z - \sigma_{by})\right]$$

So, ε_{bx} in terms of the triaxial state of stress, E_b here refers to the modulus of elasticity of the brick σ_{bx} is the direct stress in the direction bx, ν_b here is the Poisson's ratio of the brick unit and the other two directions and the stress in the other two directions σ_z compression and σ_{by}.

So, you are looking at the Poisson's effect with respect to the triaxial state of stress. Similarly, ε_{by} is written down.

$$\varepsilon_{by} = \frac{1}{E_b}\left[\sigma_{by} + \nu_b(\sigma_z - \sigma_{bx})\right]$$

We have the states of stress σ_{by} which is going to be the direct contributor to ε_{by} and the Poisson's effect in the other two directions causing changes in the strain in the direction by. Similarly, we write down the deformation of the mortar joint itself; mortar joint is in triaxial compression.

$$\varepsilon_{jx} = \frac{1}{E_j}\left[-\sigma_{jx} + \nu_j(\sigma_z + \sigma_{jy})\right]$$

$$\varepsilon_{jy} = \frac{1}{E_j}\left[-\sigma_{jy} + \nu_j(\sigma_z - \sigma_{jx})\right]$$

So, you have ε_{jx}, so the compressive strain in the joint in the x direction and the compressive strain in the joint in the y direction where E_j is the modulus of elasticity of the mortar joint; again the strain caused by the stress which is in direct correlation to the direction. So, σ_{jx} and σ_{jy} for ε_{jx} and ε_{jy} and the other two directions caused because of the Poisson's effect and ν_b here refers to the Poisson's ratio of the mortar or the joint material itself.

However, we know that a bond exists between the two materials and therefore there is compatibility of lateral deformations that will occur; there is going to be a compatibility of the strains, a compatibility of the lateral deformations because of the existence of the bond, which means that the strain in the x direction in the brick unit is going to be the same as a strain in the x direction in the mortar material as well. So, ε_{bx} should be equal to ε_{jx} and ε_{by} should be equal to ε_{jy}. That is basically, the lateral strains at the interface should be same as long as equilibrium is maintained- till you do not have failure. This is acceptable, right.

We also have to consider equilibrium of forces, so if you are going to be looking at an equilibrium of internal forces in both the x and the y directions, the lateral directions; we can examine the thickness of the mortar joint, the thickness of the brick unit over which these stresses are acting and write down the equilibrium in these two directions.

$$\sigma_{bx} = \alpha \cdot \sigma_{jx}$$

$$\sigma_{by} = \alpha \cdot \sigma_{jy}$$

$$\alpha = \frac{t_f}{t_b}$$

So, this is the basic set of assumptions that we are working with and the force equilibrium and the compatibility of deformations is what is helping us write down this set of expressions.

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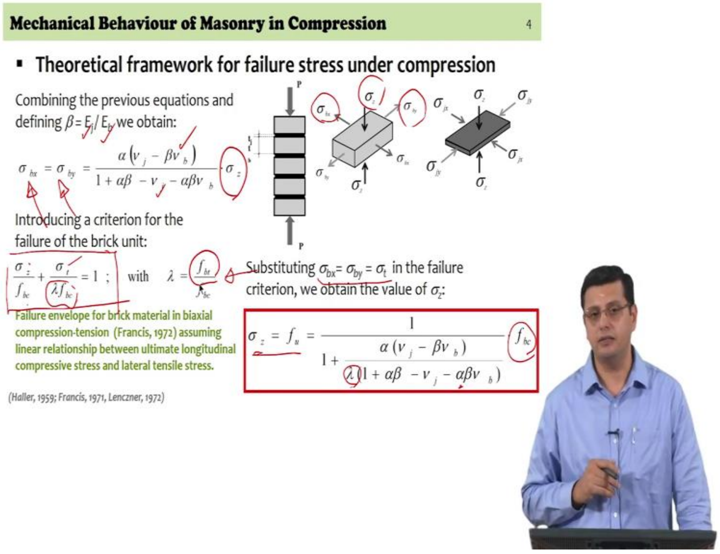

If we were to combine the previous set of expressions, the final output we have, what we are interested in arriving at, is can we get an expression for the stress at which the masonry will fail under σ_z ; σ_z is the vertical compression. We want an expression for σ_z which is a function of the constituents- The modulus of elasticity of the brick, the modulus of elasticity of the mortar, the geometry of the unit and the mortar, particularly the joint thickness and the Poisson's ratio of these materials. So, knowing the constituent materials and their constituent strengths- the tensile strength of the brick unit, the compressive strength of the mortar, will we be able to arrive at σ_z or the failure value of σ_z itself ?

So, in the previous expression that we have seen, α is representing the joint thickness ratio. Similarly, if we were to use β as the ratio between the modulus of elasticity of the mortar material to the brick unit, we are writing down σ_{bx} is equal to σ_{by} in terms of σ_z. So, it is a mere rearrangement of the previous set of expressions which are based on the deformation, now those were expressed in terms of the strains; we are writing them in terms of σ_{bx} and σ_{by}. Eliminating the different stresses in the unit and the mortar with aspects that we are able to measure and know- the geometry and the individual values of Poisson's ratio - of the mortar, of the brick and the the moduli of elasticity of the two materials.

$$\sigma_{bx} = \sigma_{by} = \frac{\alpha\left(\nu_j - \beta\nu_b\right)}{1+\alpha\beta-\nu_j-\alpha\beta\nu_b}\cdot\sigma_z$$

So, the expression that you see here basically eliminates the unknown states of stress. So, we have an expression of the local stress here, which is the tensile stress that the brick unit is subjected to σ_{bx} and σ_{by}. It is important to be able to write it in terms of that stress because we know that the brick unit will fail in tension right. So, we have an idea of the mechanism since the brick unit is expected to fail in tension, we now have an expression that relates the stress in the brick unit, the tensile stress in the brick unit to this overall stress - vertical compressive stress that you are actually applying and under which the prism is going to fail. This is the generic expression.

We now need to be able to plug in strength values and the reason why it is in this form is you already have the stress in tension of the brick unit right, σ_{bx} and σ_{by} are actually tensile stresses. But we know that the brick unit is going to split; it is going to split at it is tensile strength. So, if I have an estimate of the tensile strength of the brick unit, I can plug it in here to be able to get an expression in σ_z so that is the way it is designed.

So, you need a criterion for the failure of the brick unit, if you remember the graph that we had looked at- the figure that we had looked at in terms of the behavior, the stress path in the unit, the stress path in the mortar will come to that in a moment we were assuming and it is a fairly reasonable assumption that the failure plane in the brick unit under compression and tension is a straight line right. So, that forms a criterion for us you have a straight-line interaction between the compressive strength of the unit and a tensile strength of the unit- that is a straight line failure plane. So, if f_{bc} is the compressive strength of the unit; the uniaxial compressive strength of the unit and f_{bt}. So, f_{bt} is the uniaxial tensile strength of the unit. If you were to do a direct tension test and get the tensile strength of the unit, you do a uniaxial compression test flat wise compression test on the brick unit you get f_{bc}.

So, with f_{bc} and f_{bt} available a straight line interaction between the compressive stress in the brick unit,

$$\frac{\sigma_z}{f_{bc}} + \frac{\sigma_t}{\lambda f_{bc}} = 1;$$
$$\lambda = \frac{f_{bt}}{f_{bc}}$$

And it is only to make things a little simpler that we are looking at a ratio such as lambda here, we have discussed that the tensile strength of brick would be about ten percent of the compressive strength.

So, you could you could work with if you do not have the exact value coming out of direct tension test, you could still use this relationship between the tensile strength of the unit and the compressive strength of the unit itself. So, you have this straight line failure surface of the brick in tension-compression which is what we will feed into to be able to establish the criterion.

So, if we now substitute in the expression, we have been using σ_{bx} and σ_{by} which are nothing but tensile stresses in the unit. So, we have an expression here in σ_t, so σ_{bx} and σ_{by} are being substituted by σ_t in the failure criterion and rewriting in terms of σ_z, we get an expression for σ_z since we are now bringing in the failure strength in compression of the unit the σ_z is going to be referring to the failure in compression of the masonry assembly itself.

$$\sigma_z = f_u = \frac{1}{1 + \frac{\alpha\left(\nu_j - \beta\nu_b\right)}{\lambda\left(1 + \alpha\beta - \nu_j - \alpha\beta\nu_b\right)}} \cdot f_{bc}$$

So, we have an expression now which is the failure strength of masonry under uniaxial concentric compression, where the parameters that you require are the Poisson's ratio of the unit, the Poisson's ratio of the mortar, the modulus of elasticity of the unit, the modulus of elasticity of mortar. You have the term alpha which is nothing but geometrical parameter-joint thickness versus the brick thickness and finally lambda which is nothing but the ratio of the strengths f_{bt} by f_{bc}. So, this is an expression that is fairly good, but linear analysis is the basis; linear elastic analysis is the basis of this expression itself.

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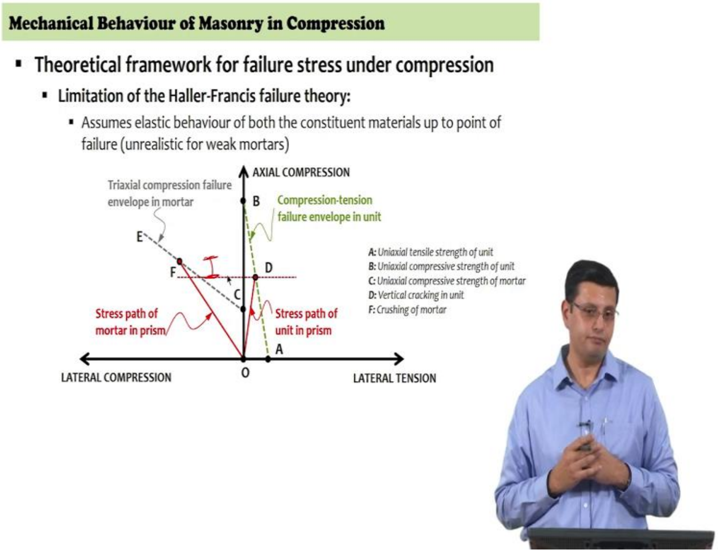

So, the fundamental limitation of this theory which is often referred to as the two people who propounded this initially Haller-Francis theory, is the fact that we are assuming linear elastic behavior and we know that the mortar and the unit to an extent much more the mortar, is not going to behave in a linear elastic manner. And the problem of considering this sort of a basis for the behavior of the masonry composite is that you remember this picture where you have axial compression which is the load that is being applied the stress caused by the load.

The compression load that is being applied against the lateral compression lateral tension. The point is the stress path in the unit is assumed to be linear and then, it hits the failure criterion that we have used which is the straight line in compression and tension envelope for the unit. We are again assuming that the stress path in mortar is the stress path of the mortar in the prism, is also linear elastic, we would not be able to explain how the unit and the mortar failed together in reality.

In reality, you expect a simultaneous occurrence of the tensile cracking in the unit and loss of confinement and the crushing failure of the mortar. If you go by this theory, depending on what the mortar strength and the unit strengths are you will get a disparity between the points of failure. So, according to this theory brick will fail and then after some time you increase the load and then the mortar will fail which does not happen there is a simultaneous failure of the two.

So, I will stop here, we will come back and examine the same situation, but with mortar strengths different from the unit strengths as shown here and then examine how we go into a situation where inelastic behavior of the mortar can give us a much better prediction of the mechanical behavior under compression.

Thank you.