Mechanics of Material
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Buckling of columns
Lecture – 99
Euler critical load for column with any boundary condition

Welcome to the 35th lecture in mechanics of materials. The last lecture we started looking at stability induce failure in particular we analyze an ideal column ok. What I mean by ideal column as, the column which is prismatic, straight the load acting to this center of gravity of the cross section without any eccentricity in the load applied and things like that ok. We will provide more detail explanations in this lecture on what are ideal column is ok.

We said that we have write the equilibrium equations in the deform shape consequently there arises more than one solution that is possible, when you write the equilibrium equation the deformed shape ok. We took an example of a simply supported column, we analyzed it and we found that two solutions are possible for certain critical loads ok. For certain loads which we call as critical loads, in particular for a minimum critical load given by pi square E by lambda, a lambda is L by r minimum square there is a direction, which the minimum radius of direction occurs squared ok.

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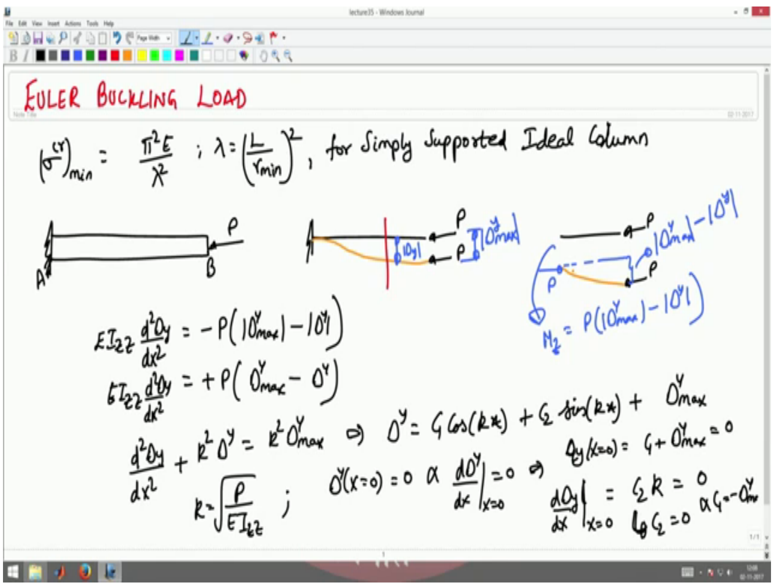

Then in that case your critical load is given by a pi critical stresses given by pi square E by lambda square, as a Euler critical stress Buckling stress ok.

And then this was derived for a simply supported column basically ok. now let us see how to get this Euler critical load for a different boundary conditions of the beam ok. First let us look at a cantilever beam say I have a cantilever column subject to an axial force P ok. Now, what happens is, the procedure is the same as a force acting P like that now this would have deformed the some shape like this and the load would act until to act perpendicular to the in the horizontal direction and if I where to say this displacement is absolute value of delta y ok.

And if I do a section analysis if I cut a section here, if I cut a section there then I will have P here P here and let us say this tip displacement is delta y max let us say the tip displacement was delta max y max ok.

So, now this distance should be this distance should be delta y max minus absolute value of delta y right minus absolute value of delta y ok, this is also an absolute value of delta y max now then the moment is this produces clockwise moment. So, I will have a anticlockwise moment coming in here ok. So, I will have a anticlockwise moment M z which is P times delta y max absolute value minus delta y absolute value ok. Now I go back to my governing equation bending equation which tells me that E times I zz into d square delta y by dx square must be equal to M z which is this is a negative moment here.

So, I will have minus P times absolute value of delta y max minus absolute value of delta y ok. Now, I know it is a downward displacement. So, delta y is negative ok. So, this will be minus or minus minus plus P times delta y max minus delta y because both delta y max and delta y are in a negative direction, both I have to put a negative sign I pull that negative sign out it will become positive ok.

So, I have the equation E times I zz d square delta y by dx square is that ok. As before this equation boils down to d square delta y by dx square plus k square times delta y is equal to k square times delta y max delta y max ok. So, basically for this the solution is delta y is again given by C 1 cos kx plus C 2 sin kx plus k square delta y max no plus delta y max plus delta y max ok.

Now, I substitute the boundary conditions the boundary conditions by the end a being fixed and nb being free are delta y at x equal to 0 must be equal to 0 and d delta y by dx at x equal to 0 must be equal to 0 ok. This will tell us that C 1 are delta y at x equal to 0 is C 1 plus delta y max this has to be equal to 0 and then delta y at x equal to d delta y by dx at x equal to 0 at x equal to 0 will give us C 2 k to be 0 ok, which means k is not 0 because k is a load which is applied which is nonzero value would depends upon the load there is applied by the way k here again means square root of P by E times I zz.

So, k is not 0. So, C 2 has to be 0 from here you get the condition that C 2 has to be 0 and C 1 is minus delta y max ok.

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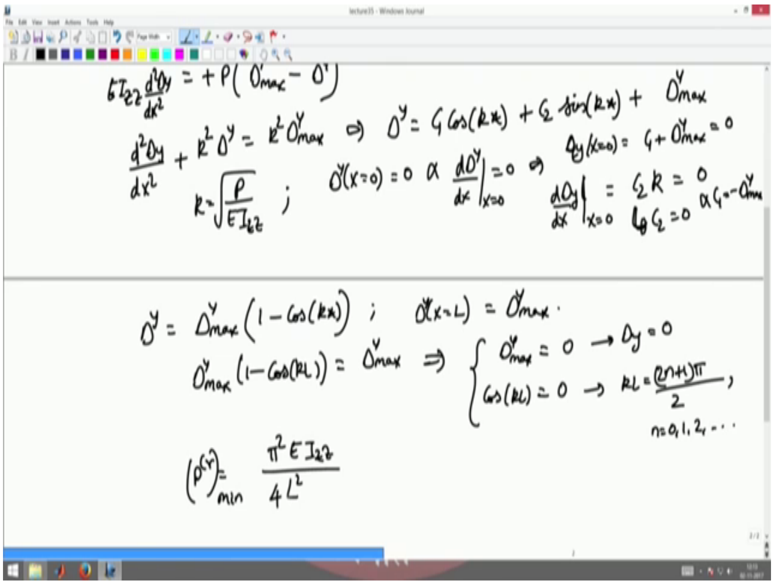

So, combining these things results combining these results you have delta y to be given by delta y max into 1 minus cos kx right ok. I substitute C 2 is 0 and C 1 is minus delta y max, but I want to use the fact that I use the condition that delta y at delta y at x equal to L is equal to delta y max that is a condition I have used because delta y max is delta y at x equal to L right.

So, if I applied that boundary condition, now I will get that delta y max into 1 minus cos kx must be equal to k L must be equal to delta y max this would imply that two cases delta y max has to be 0 or cos k L has to be 0 ok. This leads to a trivial solution by delta y this gives as delta y to be 0 and this requires that k L must be 2 n plus 1 pi by 2 l pi by 2 where n is n is 0 1 2 and so on.

Now, from here you get the condition that P critical has to be P critical minimum has to be pi square E I zz by L square four l square ok. Substituting n equal to 0 this is the first mode j for this wandered problem ok. You find that the critical load for a cantilever beam is one forth the critical load of a simply supported b that is the observation ok.

Now, let us proceed and do it for a statically indeterminate structure let us do it for a probe cantilever; at assume that I have probe cantilever subject to an actual load P ok. Now this would be form into some shape like this, some shape like that ok.

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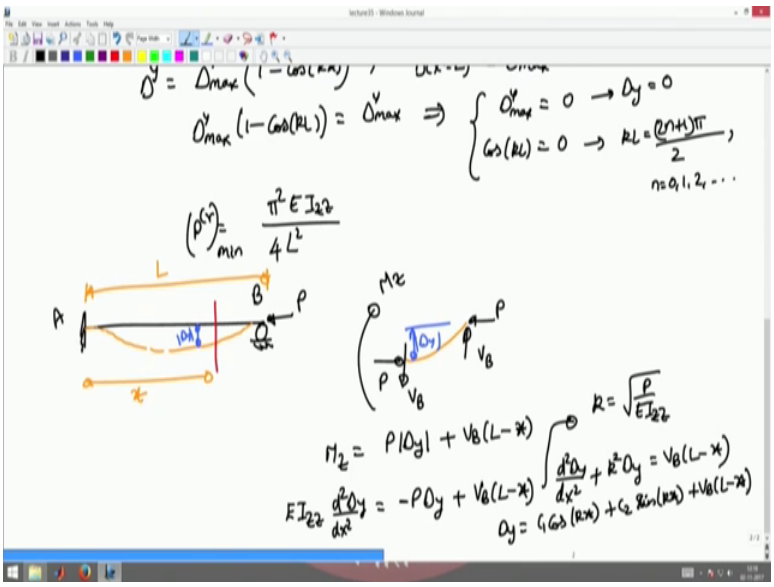

Wherein I will now what will happen is even though there are no vertical forces applied you will find it there is a reaction force develop because this sense supports a moment to be develop ok.

So, basically that is why I am illustrating into the respect to a indeterminate structure ok. So, I have a P applied now I cut the beam here as before ok. I cut the beam there and draw the free body diagram I have that, I have a P, I have a P here and I have to have a vertical direction force V B let us say this is A and B.

I should have a vertical direction force B there, there will be a vertical direction force V B here this is required to enforce a condition that a displacement at B is 0 either would not be a difference between a cantilever beam and a probe cantilever beam this is since is x and length of the beam is L length of the beam is L, this will be L minus x. If delta y as usual this reflection is absolute value of delta y that would be P times delta y P times absolute value of delta y because this is since is delta y absolute value of delta y ok.

Now, the M z moment would be in the clockwise direction because both P into delta y produces a clockwise moment and anticlockwise moment and V B into l minus x which is that distance produces a anticlockwise moment. So, M z would be P into absolute value of delta y plus V B into L minus x ok. This is the M z moment now are in got the M z moment the procedure remains the same I have to go (Refer Time: 12:52) and solve the differential equation E times I zz d square delta y by dx square is equal to P times delta y is a negative displacement.

So, I have minus delta y plus V B into L minus x ok. This I can convert back using the standard procedure defining k as square root of P by E I zz as d square delta y by dx square plus k square times delta y equal to V B into l minus x ok. The solution for this differential equation is delta y is C 1 cos kx plus C 2 sin kx plus V B into L minus x ok.

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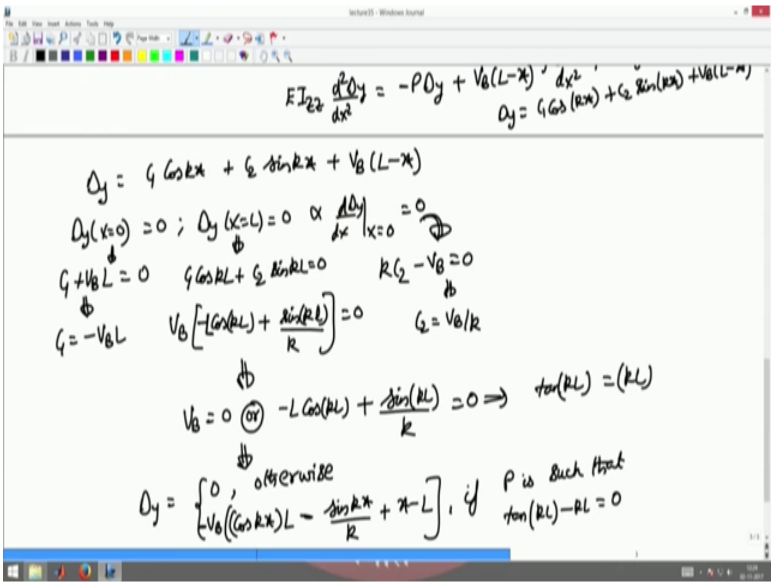

Now, let us apply the boundary conditions now delta y is C 1 cos kx plus C 2 sin kx plus V B into L minus x, a delta y at x equal to 0 has to be 0 delta y at x equal to L has to be 0 and d delta y by dx at x equal to 0 has to be 0. Now you add three conditions that is and we are three constants C 1, C 2 and V B. So, to add where from these three conditions delta y at x equal to 0 equal to 0 tells us that C 1 plus V B into L must be equal to 0 get this from there and delta y at x equal to L equal to 0 implies C 1 cos k L plus C 2 sin k L has to be equal to 0 and this condition on the derivative tells us that C 1 k times C 2 minus V B has to be equal to 0 ok.

Now, from here I get C 1 to be minus V B times L and from here I get C 2 to be V B by k substituting that in here I get V B into minus cos k L plus sin k L by k will have L here by k equal to 0 ok. This implies that either V B has to be equal to 0 identically or minus L times cos k L plus sin k L by k has to be equal to 0 this implies that delta y is identically 0 because of V B is 0 C 1 and C 2 are 0 V B is also 0.

So, delta y is 0 which soon results in a trivial solution, this is a otherwise or from here I get it as tan k L must be equal to k L ok. Otherwise and delta y is given by V B into cos kx into L minus plus sin kx by k plus x minus L. No there will be a minus sign here plus x minus here ok. If P critical if P is such that if P is such that tan of k into L minus k L is equal to 0 ok. Solving this equation you can find the P critical value, I am not going to go into that now ok. I leave it as an exercise for you to do to find the P critical value ok.