Mechanics of Material
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Buckling of columns
Lecture - 98
Euler critical load for simply supported column

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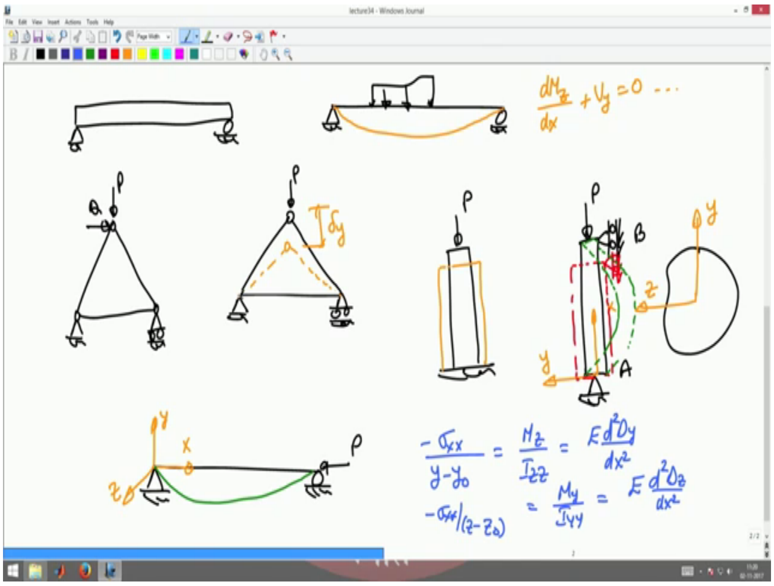

Now, if the let us convert this into one line diagram I have column I have roller there applied axial force p, this is x that is y and this is z ok. Now, the question asking is a deformation of the form like this possible if the deformation of this form the question asking. So, the deformation of the form like this possible ok. Now, to answer that what should happen there should be a smooth displacement field from which you get this strain, then it has to satisfy the constellation to find the stress and then you have to satisfy the equilibrium equations ok.

If it deforms like that it is like a beam bending problem even though we derived it for a case where it was subjected to pure bending moment, we saw that the equation for bending, which was minus sigma xx by y minus y naught equal to M z by I zz equal to E d square delta y by d x square was for bending along the for bending deflection occurring along the y axis right.

Similarly you are a equation minus sigma xx by z minus z naught equal to my by I yy equal to E times d square delta z by d x square for bending along the z direction. So, basically now if it were to bend like this, this equation has too old from the equilibrium point of view and from the strain displacement strain stress equilibrium equation point of view.

So, now the question is what is this moment M z or M y it is going to come in this problem. So, let us go ahead let us cut this beam somewhere here.

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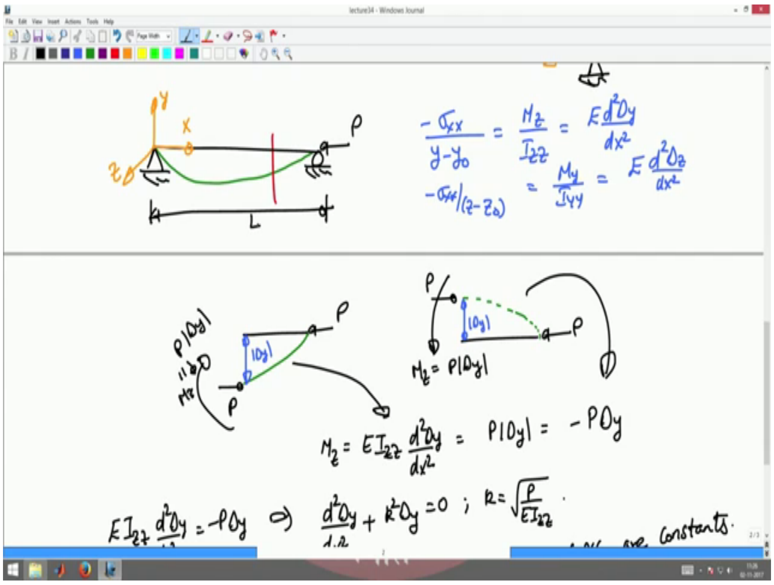

Some are there and write the equilibrium equation for that cut portion and axial force p, since there is no vertical load applied, there would not be any shear force coming in the system.

So, basically I will have now the force P acting like this, if this displacement this displacement were to be absolute value of delta y width of their sign that displacement were to be absolute value of delta y, then I will have a moment P into delta y coming in this system which is a which is anti clockwise moment. So, I should apply a clockwise moment tr to contract this P into delta y moment which will be P into absolute value of delta y P into absolute value of delta y.

Will be this moment M z moment this is say M z moment ok. Now, if you try to deflect up then also say it did not deflect down, but it deflected up add a P it went up like this instead of coming down since we saw a scale go on either direction then also I will have a P here and there is this let the distance be absolute value of delta y then I will have a moment now this produces a clockwise moment. So, the moment that is produced M z the M z moment would be like this and this will be P into absolute value of delta y ok. Now going back to this bending equation in here I now found M z for this system.

So, I use the equation to find delta y ok. So, I use that equation to find delta y. So, what is the equation I have? I have M z is E times I zz d square delta y by dx square which is nothing, but P times absolute value of delta y ok.

If I have to resolve the absolute sign I am writing this for this case writing it for this case now delta y is negative I want the absolute value of delta y. So, I have to replace this with minus delta y so; that means, delta y is negative this equation becomes gives a positive value ok. Now you find that the same equation holds for this case also; because this moment M z is a negative moment, but delta y is positive.

So, when delta y is positive, this is a negative moment. So, this gives a right sign for the bending moment. So, this is the right expression for bending moment whether it bends down like this or whether it bends up like that. So, the equation remain the same. So, now I have to solve this equation, now what I have is I have e I zz d square delta y by dx square equal to minus P times delta y ok. This I can rewrite it as d square delta y by dx square plus k square delta y equal to 0 where k is square root of P by E I zz ok.

Now, this is a second or differential equation for which the solution is known, delta y would be of the form delta y will be of the form C 1 times cos kx plus C 2 times sin kx ok. C 1 and C 2 are constants to be found from boundary conditions ok. Now, you know that delta y it is a simply supported beam that I am looking at.

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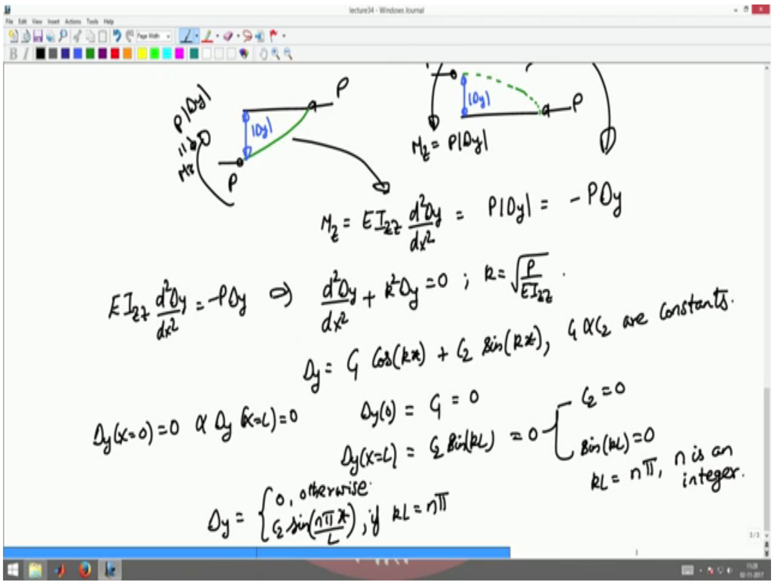

So, delta y at x equal to 0 and let us assume the length of the beam is L. So, you know that for a simply supported beam delta y at x equal to 0 has to be 0 and delta y at x equal to L has to be 0 ok. This gives you the condition that when delta y of 0 is nothing, but C 1. So, C 1 has to be 0 ok. Delta y at x equal to L is C 2 sin kL this has to be equal to 0 ok.

Now, you have two possibilities one is C 2 being 0 or sin kL being 0 sin kL will be 0 if kL is n times pi per n is an integer ok. From here you get that two solutions your C 2 is 0 the two solutions are delta y has to be identically 0 or it can be C 2 times sin k is n pi x by L if kL is equal to n pi this is otherwise 0 is when it is otherwise ok.

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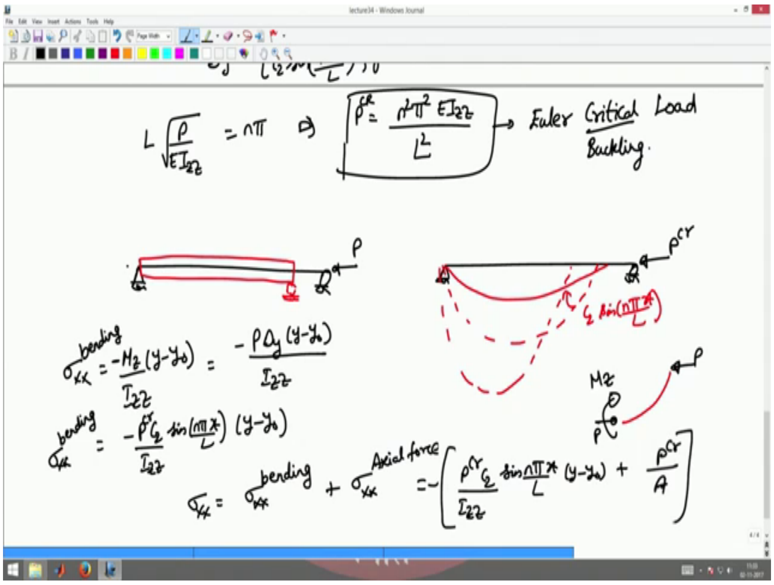

Now, let us go ahead and look at what this condition kL equal to n pi means, K we said that k was square root of P by E I zz into L must be equal to n pi implies P is n square pi square E I zz by L square that is if the load we have to take this particular value then an alternate solution arises which is given by sin n pi x by L, when the deformation is given by its part of a sin wave ok.

So, this load is called as a critical load PCR and this expressions is called as the Euler critical load or another name given for critical is Buckling load ok. Now what this means is until P reaches P critical, this beam the simply supported beam subject to an axial load will deform into this shape.

It will expand laterally and contract horizontally ok. So, this will be the deform shape ok. If P is equal to P critical if P is P critical then an alternate solution arises which is this which is that solution wherein this is given by C 2 times sin n pi x by L still I am determined what C 2 is.

What this tells us is there is no bound on C 2 so it can do continue to do like this you can continually deform like that in particular a mid span deflections which is a maximum deflection is a undefined. So, we continue to deform. Now and it continuously deform let us see what happens to this stress sigma xx we did not use this part of the equation yet, we did not use this part of the equation yet right.

So, let us use that sigma xx is M z by I zz minus into y minus y naught this is due to bending ok. Now M z is minus P times delta y into y minus y naught by I zz which is nothing, but minus P times C 2 sin n pi x by L into y minus y naught by I zz this P critical sigma zz due to bending this. What is the net sigma access would I have two components there will be a bending component plus an actual force component. As an actual force also coming in here right because we draw a free body diagram of this if I draw a free body diagram of this structure I will have a net P coming in there.

I draw the free bar diagram I add a P net axial compression and a movement M z there. So, due to this movement we found what is the due to this movement we found what is a stress, I have to add to there the stress caused due to this axial force P there will be P by a area of the cross section ok.

That is going to give me sigma x axis P critical C 2 sin n pi x by L divided by I zz into y minus y naught plus P critical by area of cross section.

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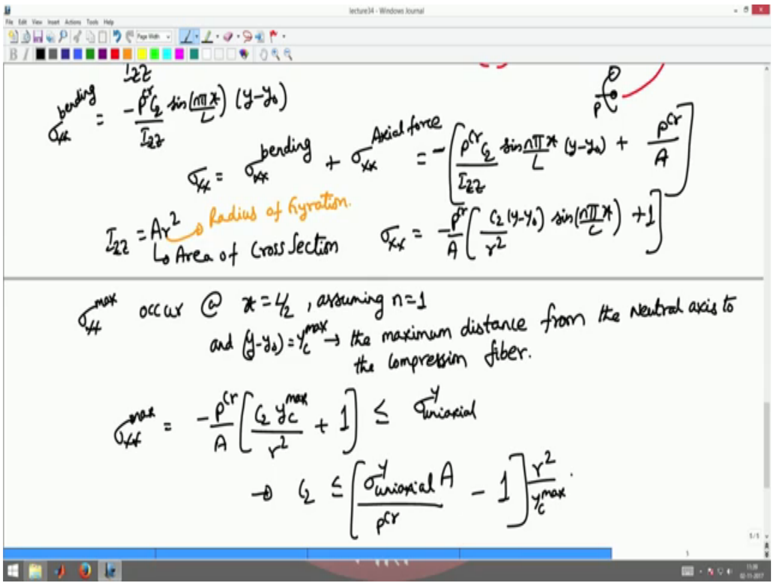

Now, I can write I zz as A times r square per A is area of cross section and the r square is radius of gyration ok. So, I zz is area of cross section in to radius of gyration square. So, if I substitute that sigma xx becomes minus P critical by area of cross section, C 2 by r square into y minus y naught into sin n pi x by L plus 1 ok.

Now let us find what is the maximum stress now, the stress being done by here sigma xx max will occur at x equal to L by 2 wherein sin n pi x by L would be maximum if n is an odd integer ok.

Assuming n is 1 or assuming n is equal to 1 and then what do you get is at x equal to L by 2 assuming n equal to 1 and y minus y naught is equal to y max in compression the maximum distance from the neutral axis to the compression fiber which will be on the top usually.

So, I will get sigma xx max as minus P critical by A into C 2 y max C by r square plus 1 ok. Now a c tends to infinity you find that sigma axis max tends to infinity that is not possible that be a stress wherein this it is like kind of a uni axial state of stress finding as a sigma axis varying like this.

So, once this stress reaches say you are doing it on a metallic shield rod which obeys 1 minus its criteria this should be less than equal to sigma y uniaxial this sigma axis max should be lesser than sigma y uniaxial because this like a university state of stress and if you are finding the yield stress and uniaxial state of stress to be sigma y it should be lesser than sigma y ok. So, basically now that tells you what is the maximum value of this implies C 2 must be lesser than or equal to sigma y uniaxial divided by P critical into A minus 1 into r square by yc max ok.

Commonly this attributed that the material failure is different from stability failure in the sense that is independent of the material yield stress value; but the material yield stress value governs what is the maximum deflection that you can have there is not independent of the material failure, but what happens here is because geometry they sets a continues to deform because they are writing the equilibrium equation at deformed shape it continuous to deform or it has multiplied solutions possible arising at the particular stress value and that ns causes this stress to exceed a particular limiting value given by a metal failure limit ok.

It is not that several induced failure or not material failures ok. Now let us next see what value of n I should assume next is what is the value of n to be taken here, I have taken n as 1.

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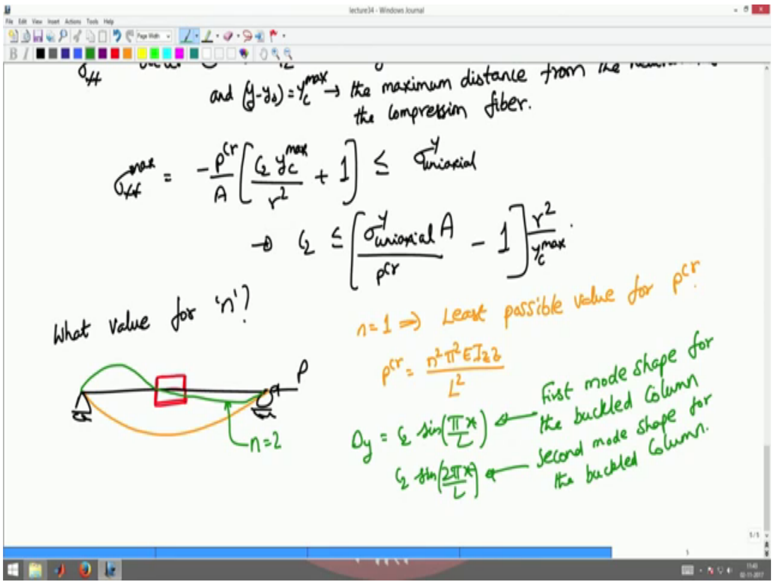

Let us say why did I take n as 1 what value for n now this depends upon what is the mode of deformation that is possible for a structure. If I add just a simply supported structure simply supported column, then axial force P applied at a mid span can show any deflection at it once you have, and it can deform in particular into a part of a sinusoidal curve ok.

So, here n can be 1 or when n is 1 that will results in the least value of PCR this yields the least possible value for PCR the critical axial load ok. If you recollect we found PCR to be n square pi square E times I zz divided by L square.

So, this will be least when n equal to 1. Now, which means their body will fail when the n is equal to 1 when this critical load is occurring. However, if I where to strengthen this structure at a mid span by putting some extra rigid supports or I stiffen the member by providing extra place at the mid span this deformation is not possible and what should happen now is if you deform into something like this which is when happens when n equal to 2 ok. When equal to two this happens. So, in that case for this case n would be 2 ok. If I where to put additional stiffeners at L by 4, L by 2 and 3 L by 4 then higher modes of buckling will be occur.

This shape that you get for a deformed shape this delta y being C 2 sin pi x by L is the first it is called as a first mode shape for the buckled column ok. C 2 sin 2 pi x by L similarly it is called as the second mode shape for the column buckled column ok.

So, these are the deflections called as a mode shape and a critical load is called as a critical load ok. Now, I want to do some more analysis on this Euler buckling analysis.

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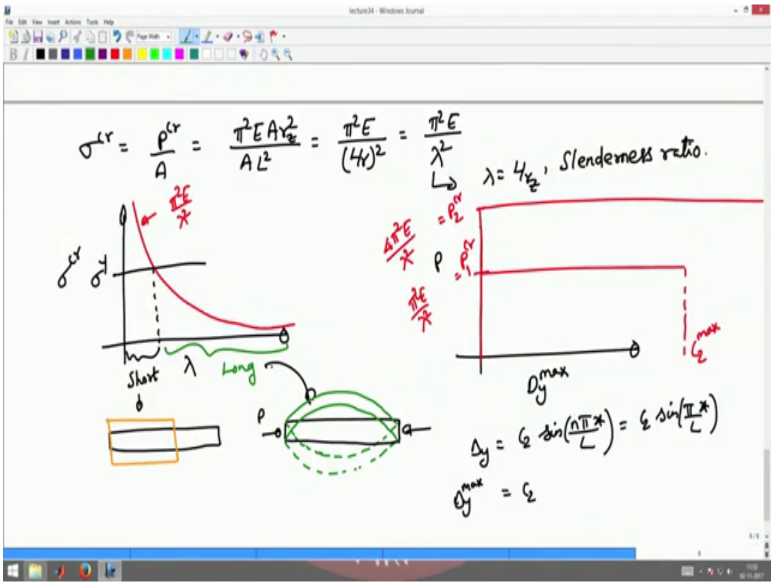

I want to rewrite this PCR in terms of a stress like quantity, sigma critical would be PCR by A which would be pi square E I write it as A r square divided by A into L square basically I have written in the movement of inertia is the area of cross section time, the radius of gyration squared.

So, what I did before? So, this will give me pi square E by L by r the whole square ok. Now, this L by r is called as the slenderness ratio lambda this is lambda is L by r it is called as the slenderness ratio. I was telling you at a beginning of the lecture that if the slenderness the beam is slender if it its start stout, then it will fail by bucking right that is this is the measure of the slenderness ok. This is the measure of the slenderness, which is length by the radius of gyration of the cross section ok.

To be more specific this is a radius of the gyration of the cross section the z direction. So, let us say rz ok. Now, let us plot this sigma critical, as a function of slenderness ratio lambda ok. Now, this will look like a curve I have shown here ok. As lambda tends to infinity the stress will go to 0, as lambda tends to 0 the stress will go to infinity ok.

So, basically this is how the relationship between sigma critical and the slenderness ratio is, this is given by pi square E by lambda square these are the ratios. Now, you know that, if the material we have to compress the critical load cannot exceed the yield strength of the material.

So, if I mark the yields in the strength of the middle by this line sigma y, then there is no ways that if I apply on a stout column or on a column where the slenderness ratio is very small, it is going to get compressed and it is going to failed by yielding or by the failure theory that governs the compressive failure rather than by slenderness by deforming laterally.

What I mean by this is up to this regime, the column is called as short column a short column wherein this will would not failed by buckling or it will would not failed by bending, it will would not failed by delta y being non-zero ok.

In the short column it will would not failed by generating a delta y deformation, it will have only the axial compression possible it will would not be bending axial coming in which will cause this failure axial compression will cause their failure.

Once you cross this point the mode of failure is gone by the buckling mode of failure or the stability induced failure, wherein the uncontrolled lateral deformation will govern this failure mode. So, this is what you have to understand ok. In a short column the failure is by if I have a bar, it will compress axially and it will failed in this mode. In this regime I was called as a long column just like a ruler that I showed you.

In this regime what will happen is, a same column and subjected to a compressive force P will bend like this we will bend like that and failed ok. So, there is how the long column fails, this is how the long column fails this is how the short column will failed ok.

So, if the failure modes will be different for a long column and a short column ok. Now in all this if I where to look at the load versus delta y max what will this curve look like? We found that delta y is given by C 2 sin n pi x by L now in particular we are looking at first mode chip. So, this is C 2 sin pi x pi L ok. So, basically now what happens is delta y max would be C 2 right.

At L by 2 or at L by 4 or 3 L by 4 and the delta max value would be C 2 ok. Now, what happens if I where to plot delta y max versus P is we will get a something like this. It will be 0 up to PCR and then I will show some infinite displacement which is C 2 which is a undefined parameter until this reaches the C 2 max that we found before from the yield stress value ok. Here will be a C 2 max that we found before there will be a very large value of deformations ok. Now if I go to second mode.

Then also it will be C 2 max, but that value will be different from this value it will show a much larger deformation. This is P critical 2, which is 4 pi square E by lambda square ok. This P critical one which is pi square E by lambda square, now it will show a much larger deformation. So, this is how the deformed shape will look like the mid span deflection.

But now experimental with the column here with the scale here you find at the moment I apply some load it deforms laterally. So, it does not agree with our analysis here. So, we will see what we have to do to accommodate the fact that the column begins to deform the movement I applied a slight load. So, that is something we have to do next ok.

Now we saw that the column can deform in either down or up on the analysis remaining the same right from the beginning. I told you it can deform like this or it can deform a long column can buckle upward; what this is depends upon the probability, depends upon what is a disturbance there is causing it to go from the equilibrium state to a unstable state. So, this is a random so, called random probability that you can go down or it can go up.

But whether it will go along the whether it will defect along the y or z depends upon the slenderness ratio ok. We found that for a simply supported beam the critical load is pi square E by lambda square where lambda was L by rz where it is a radius of gyration for I for the about the z axis ok. On the other hand I can similarly defined a radius of gyration about the y axis L by ry this will be lambda y ok.

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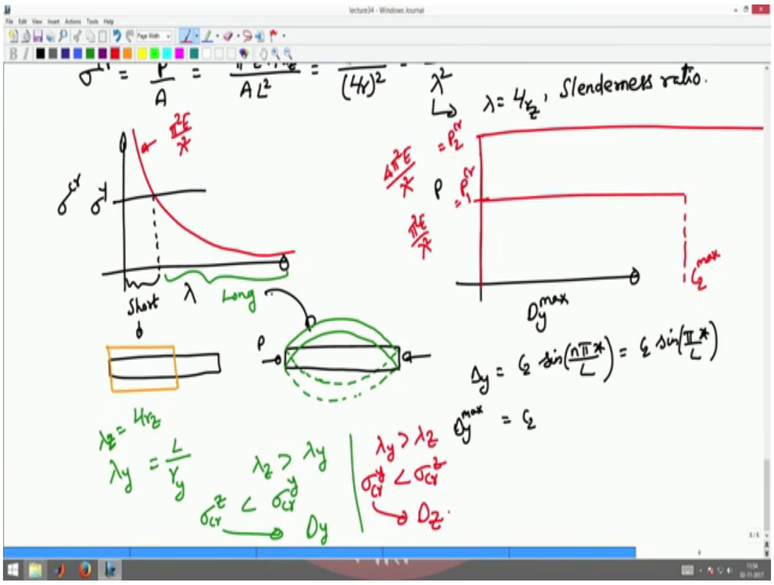

The above expression is lambda z where this L by rz ok. Now, which one is more we will determine whichever is more we will lead to a lower value of stress right.

So, if this is more if lambda z is greater than lambda y ok, then sigma critical for the z axis would be lesser than sigma critical for the y axis ok. If this happens then the since sigma critical along z axis is less it has to deform along the delta y direction.

It should show a delta y deflection ok. If it where sigma if it where the other way around if it where such that lambda y was greater than lambda z, then sigma critical in the y direction would be lesser than sigma critical along the z direction this implies there will be a delta z deformation; Because the axis about which the beam bends is perpendicular to the axial about which its shows the deflection.

So, in all these cases we have still assume that the bending equation holds for even this case was an assumption, but it is found experimentally that it is a good enough assumption for these problems. So, the direction of bending and the axis about which you will bend you have to be clear it depends upon which slenderness ratio is more ok.

If lambda z slenderness ratio is more it will deflect along the y direction, if lambda y slenderness ratio is more it will deflect along the z direction. So, that you can visualize for this scale you know that the movement of inertia about this axis is more.

Because it is this cube times this thickness divided by 12. On the other hand if I add cross the axis oriented like this it will be this cube times this thickness. So, basically now that is why it bends like this. So, apply axial load it bends along this direction because there is a movement generated along this direction in here.

So, that is a reason why it bends like that ok. With this we conclude today's lecture we will look at more different boundary conditions and how to model realistic columns in the coming two lectures.

Thank you.