Mechanics of Material
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Buckling of columns
Lecture - 97
General Concepts

Welcome to 34th lecture in mechanics of materials, we started off looking at some basic concepts in mechanics and then we looked at four equation that connected those four concepts, and then we went ahead to solve some boundary problems using those concepts and equations, from there we assimilated what the displacement and the stresses in the body are, and then we use the failure criteria which was a material failure so to speak wherein we said that if the stress exceeds a particular limit, then the failure of the body would occur. In that in the last three lectures we were looking at various failure criteria's for pressure hydrostatic pressure sensitive and hydrostatic pressure insensitive materials.

In particular we looked at Trescas criteria and Von Mises criteria for hydrostatic pressure in sensitive materials and then we looked at Rankine or maximum normal stress criteria, Mohr-Columb criteria and Drucker-Prager criteria in quick for pressure sensitive materials ok. And then we said that if this stress exceeds a particle limit defined by the failure surface of these failure theories then it will fail or this on the verge of failure, if it is on the failure surface.

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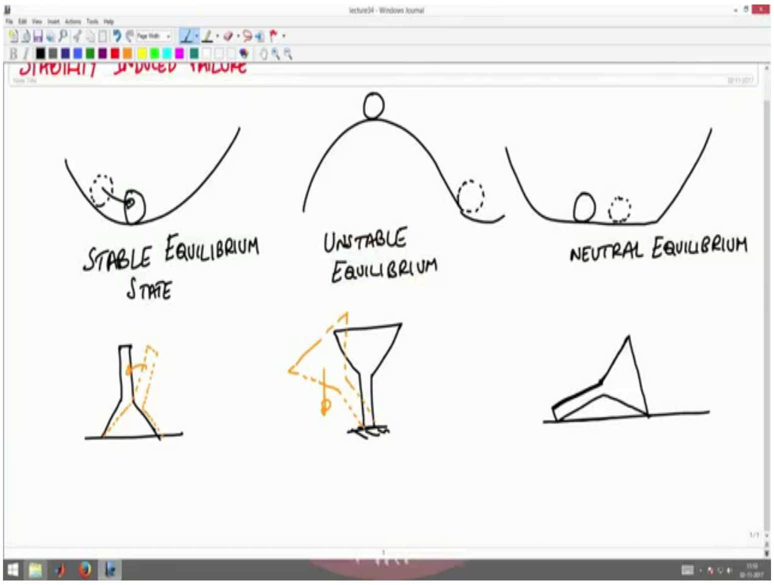

There is one mode of failure. The another mode of failure wherein a given body would fail just because of stability reasons meaning if you have a ball which is sitting on a trough like this, ball sitting on a trough like this ok. If I move this ball a little bit on either side of the trough it return back to its equilibrium position ok. My other hand if I have a ball place on a surface like this, if I move this ball on either side it move goes to a totally different state of equilibrium state ok.

So, basically what happens here is a small disturbance alters the equilibrium state of the body ok. Similarly, I can have a situation wherein, I have the ball somewhere here small disturbance will take it to somewhere here and it will stay put their. I want be further deformation whereas, the second case is a small disturbance would take it to some position here we will take it to some position here or much more deeper depending upon how the shape of the surfaces. Here if I disturb it will come back to its original position. Here if I disturbed this disturb with somewhere here it will return back to its original position ok.

So, you want the body to be in this state, which is called as the stable equilibrium state you do not want the body to be in this state which is called as unstable equilibrium or neutral equilibrium is, but you still do not want because it produces a large definitions this is called as neutral equilibrium state to understand this in terms of a structure, think of a funnel place on its base. There is an equilibrium configuration of the funnel fluted funnel based on its base ok.

Now, what happens if I give a small disturbance, if I tap this funnel to occupy some position like this some position like that, then it will restore back to its original state it will come back to its original state because there is a corresponding moment which will help it come back to its original state ok. On the other hand if I have a funnel based on a strip like this a funnel based on a strip like this, then if I turn it the cg of the funnel faults outside the base of the funnel. So, it tip it somewhat like this ok, this is the cg fault outside is based and hence it will top lower ok.

On the other hand for neutral equilibrium I can have a funnel resting on its a funnel resting on its sides funnel resting on its side it is an example of a neutral equilibrium configuration wherein if I perturb it its say put in the perturb position ok.

So, basically now you want the body to be in a stable equilibrium configuration, because that is a state which you prefer the body to be in. So, that it does not show large deformations or it does not undergo see large stresses in the process of going to unstable equilibrium states ok.

Now, next seen you have to understand this, till now we have been writing our equilibrium equations in the undeformed shape ok.

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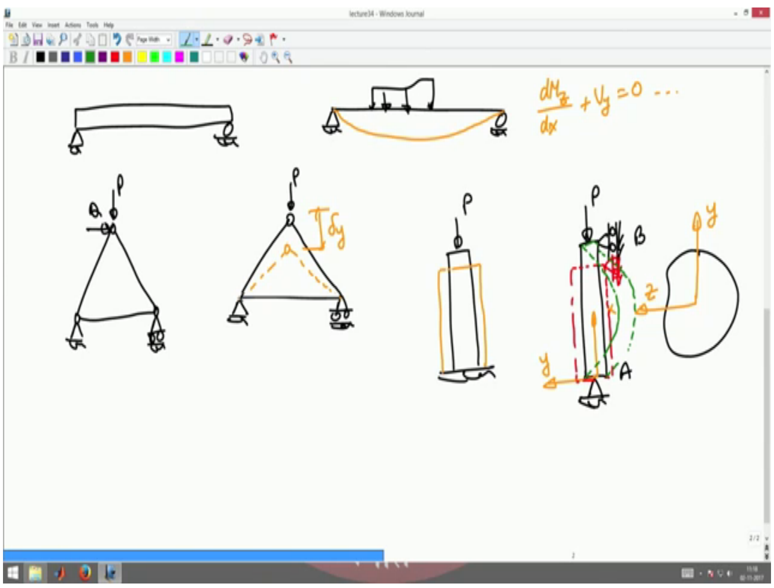

For example if I take a beam element, if I take a beam which is simply supported and we idolize it as a line element like this, parent was the acts of the beam and you add some loading coming in here, you know that the beam will deform into some shape like this, if it deforms into some shape like that, then you did not account for the fact that you did not write the equilibrium equations, the deformed configuration that this you did not say whether this force remains vertical or perpendicular to the deform surface ok.

If remains vertical and water we did was somewhat appropriate, but if it remains perpendicular to the deform surface and the lever arms will change those effects we did not consider, we just took that the load to be acting on the undeformed shape and we wrote the equilibrium equations you get dM x Mz by dx plus Vy equal to 0 and so on the equilibrium equation that we got.

But based on undeformed configuration equilibrium; this is evident when you look at a truss analysis, which you are done in your previous course in engineering mechanics. So, when I have triangular truss like this, which is simply supported at these ends and I add some loading coming in your P and Q. You did not ask the question what is the deformed shape before writing the equilibrium equations, you wrote the equilibrium equations in this undeformed shape and you assume that the deformed shape will be close to the undeformed shape that will would not matter significantly ok.

That is a good enough argument if the deformations are small, but what happens is these tell you that there is only one deformation that is possible for a given truss. Since you are writing the equilibrium equation the undeformed shape if the truss you have to deform then you will find that there will be multiple equilibrium stage at arises which are possible. To illustrate that let us look at an example wherein I have this two element truss with a point load p automatically down.

Now, if I assume that this is not a small deformation problem at a large deformation problem, then I have to account for the fact that this stress will deform into some shape like this, wherein this deformation delta y is significant ok. Then what happens it will see that, there are more than one solution that becomes possible. I am going to illustrate this in this lecture by not using this complicated truss which is simple, but it complicated for our purposes, but I will demonstrated with respect to a single member in compression ok.

For example when you think of a member in compression say a column subjected to a compressive force P, you will think that it is going to deform it is like a uniaxial set of stress and then. So, you will thing that it will deform into some shape like this right. This will happen if the column is what is called as a short column or a stout column, but if the column is slender like a scale then you will find that this is not what happens, but something different happens.

Let us see what happens. I have the scale which is slender here in these dimensions ok. I am applying a compressive stress you can see that it deforms quite significantly in the lateral direction, it is not getting compressed it is not going down like this. If you apply a compressive force here you can see that it is bending on either direction it bends on a either direction, but it does not bend in this way, it does not bend in this way, but it bends only in this direction or in this direction depending upon some criteria it bends like this or it bends like this it does not do what we saw as it would do if it you are a stout or a short column ok.

So, basically now you find that there is a discord between you are unless this expectation and the vary body deforms. So, we have to reconcile this. The reconciliation is by what you are going to do in the structure ok. The reconciliation arises because you wrote the equilibrium equation for this column in the undeformed shape whereas; you have to write in the deformed shape ok.

So, now what we will do is we will take this column instead of a cantilever fixed and a free in column, we will look at a simply supported column. The pin here and a roller here you understand that a roller should be in this direction because only then we will allow for axial compression, otherwise the member will would not deform what are load you apply this support reaction at b will take all the axillar that is applied ok. If roller inch or if your roller in the other direction the support direction, we will take all the reaction force and not allow the column to compress ok.

Now, let us assume a coordinate system, let us still assume this is x, this is y and outside the plane is z, out of plane axis is z ok. So, let us assume the cross section have some cross certain shape with its arbitrary, which is y and z acting like that ok.

Now, you would expect that this column to deform into some shape like this, wherein the roller moves down here and it becomes a roller that to it the applied load. But let us investigate another there is an alternate deformation possible which is this that is the column to bend like what the scale did in our case ok. You can bend along y direction or you can bend along z direction what is the criteria we will find out in a short while now which direction it bends.