# Mechanics of Material

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## Pressure vessels and failure theories
## Lecture – 92
## Tresca Condition

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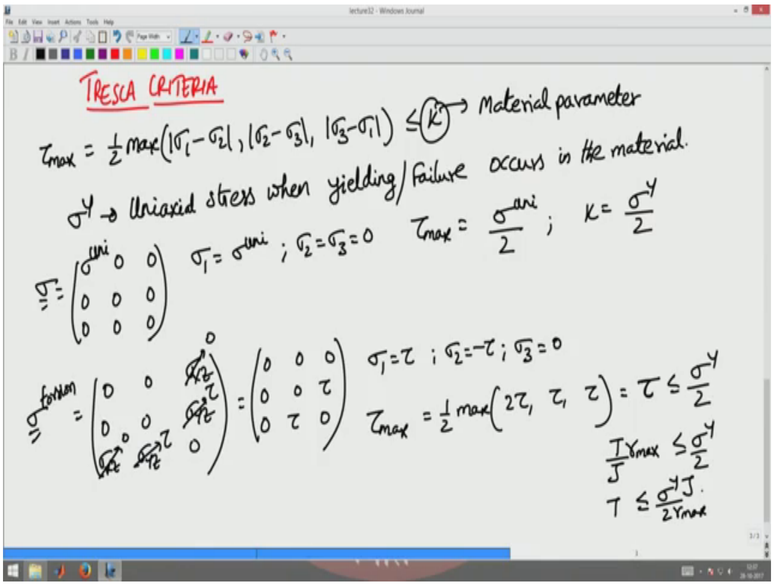


Let us start with Tresca criteria ok. What this criteria tells us is, the maximum shear stress in the body which is given by 1 half maximum of absolute value of sigma 1 minus sigma 2, absolute value of sigma 2 minus sigma 3, absolute value of sigma 3 minus sigma 1 should be lesser than or equal to kappa where this is a material parameter just like Young’s modulus or Poisson’s ratio ok.

So, basically now how do you find this material parameter? You have to do some experiment to find this material parameter. So, typically we have been using Uniaxial experiments to find the Young’s modulus and Poisson’s ratio. So, you will use the same experiment to find this parameter.

So, let us say the body began to yield or began to fail at a stress when this Uniaxial stress reached sigma y called as the yield stress of the material. So, let us sigma y represent Uniaxial stress when yielding or failure occurs failure here means yielding essentially occurs in the material ok.

Now what is this kappa how is this kappa related to sigma y is a question ok. For Uniaxial state of stress, I know that this stress tensor is given by sigma Uniaxial 0 0 0 0 0 0 0 0 0 ok. So, for this the principal value is sigma 1 is sigma Uniaxial and sigma 2 and sigma 3 are 0 ok. So, the maximum shear stress tau max is given by sigma Uniaxial by 2 ok.

So, the kappa is when this Uniaxial stress reaches sigma y. So, kappa would be sigma y by 2. When the Uniaxial stress reaches sigma y the material has failed. So, that is my limiting value of that kappa value. So, kappa has to be sigma y by 2 ok.

Now, I am found kappa, now I can use this to estimate what will be the failure load in shear. For example, I do a torsion experiment; I know that sigma for torsion experiment is given by 0 0 sigma xz, 0 0 sigma yz, sigma xz sigma yz 0 right ok. In particular if I am interested in finding a point, where it is parallel to where the shear stress acts parallel to the y axis for example, point along the x axis then I will have this goes to 0 and along only that and let me say this is tau and let me say this is tau the shear stress acting (Refer Time: 04:13) shear stress is acting at that location ok.

So, I have the sate of stresses is pure shear. So, you know that the Mohr circle for this is centered about the origin and it has a diameter it has a radius of tau ok. So, for this case when the Mohr circle you can read out that sigma 1 would be tau, sigma 2 would be minus tau and sigma 3 would be 0 are the principle stresses for the stress state and for this stress state tau max would be 1 half maximum of 2 tau tau and tau right ok. So, this will be maximum of that will be 2 tau, 2 tau divided by 2 will be tau this has to be lesser than or equal to sigma y by 2 right ok.

So, now you have found the condition on the shear stress in terms of the Uniaxial yield stress that you found from a Uniaxial experiments. So, this tau in terms of torque is given T by J into r or r is a radial location of the point I am interested in the maximum shear stress. So, it will be r max, the maximum value of the radial location this has to be lesser than sigma y by 2 or my torque must be less than or equal to sigma y by 2 r max into J that gives you the limiting torque that circular shaft can see when it obeys a Tresca criteria.

Now, I can find kappa not necessarily from Uniaxial experiment you know find kappa even from a shear experiment right. I can do the ulta bulta you know I can use the shear experiment to find the kappa value and Uniaxial experiments to find what is a limiting Uniaxial stress then let us see what happens.

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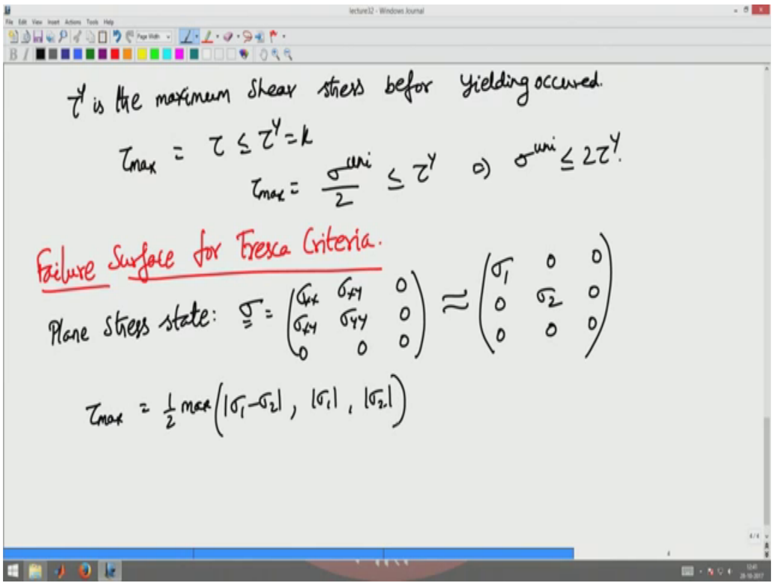


Now, say I am giving you that tau y is the maximum shear stress before yielding occurred then I have tau max for pure shear is given by tau this should be lesser than or equal to tau y. So, your kappa this is equal to your kappa in that case my Uniaxial if I go back and predict my Uniaxial state of stress, then I know that tau max for Uniaxial state of stress is this tau max for Uniaxial state of stress is this, this should be limited to tau y now.

So, basically if I use this to predict the Uniaxial state of stress there is possible, tau max for Uniaxial state of stress is sigma uni by 2 must be lesser than or equal to tau y ok.

In other words its implies sigma Uniaxial stress must be lesser than or equal to 2 times tau y. So, I can use any one experiment to estimate kappa, once I estimated kappa I can use that estimated value of kappa to predict the stresses in any other state of stress ok.

Next you are interested in plotting the failure envelop for the Tresca criteria ok. Let us assume that the state of stress is plane I am interested in plotting the failure surface for Tresca criteria what I am interested is not the general case, but plane state of stress plane stress state. So, sigma is given by sigma xx, sigma xy 0, sigma xy sigma yy 0 0 0 0, this I can equivalently represent as sigma 1 0 0, 0 sigma 2 0 0 0 0 for this tau max is given by 1 half maximum value of absolute value of sigma 1 minus sigma 2, absolute value of sigma 1, absolute value of sigma 2 ok.

Now, what will this b? This will be equal to sigma 1.

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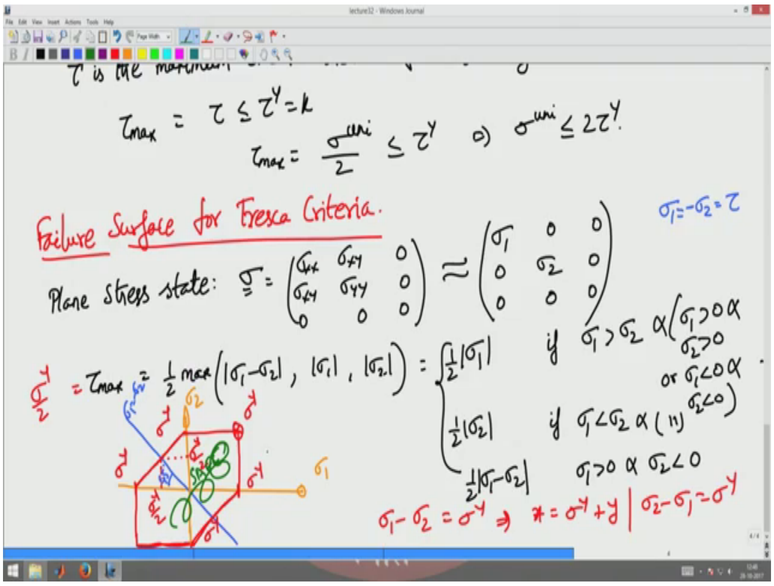

Absolute value of sigma 1 if sigma 1 is less than sigma 2 and sigma 1 is greater than 0 and sigma 2 is greater than 0 or sigma 1 is less than 0 and sigma 2 is less than 0 and sigma 2 is less than 0 right and sigma 1 is positive and sigma 2 is positive, sigma 1 minus sigma 2 will be lesser than sigma 1 or sigma 2 because I am subtracting to positive numbers.

So, whichever is greater will be the value of maximum stress there 1 half of that maximum stress, sigma 1 is greater than sigma 2 here the sigma 1 is greater than sigma 2 the maximum value of these tau max would be the maximum value of sigma 1 or sigma 2 ok. Similarly if sigma 1 is less than sigma 2 and the same conditions are satisfied it will be 1 half of sigma 2 absolute value of sigma 2.

On the other hand if sigma 1 if positive, the sigma 1 is greater than 0 and sigma 2 is less than 0, then sigma 1 minus sigma 2 will be the maximum value because they are of opposite signs. So, they will add up rather than they are getting subtracted. So, now, it will be 1 half maximum of sigma 1 minus sigma 2.

So, with this results, now we are now position to plot the yield surface let us assume that this is sigma 2 axis, this is sigma 1 axis and I know that kappa from Uniaxial stress condition is sigma y by 2.

So, basically if I do a Uniaxial tests the yields stress sigma y I can do a Uniaxial stresses in 1 direction or 2 direction then the yield stress will remain same as sigma y. So, from that argument I get this point a sigma y and similarly this point would be sigma y along the 2 direction where the other component of stress is 0.

When both sigma 1 and sigma 2 are positive and of a same magnitude, then also I will get sigma y as my state of stress the failure state of stress ok. So, I get it I get this point in equal biaxial state of stress will still fail as sigma y because sigma 1 is less than sigma 2 until sigma to reaches sigma y and I both of them go in the same raid also both reach sigma y simultaneously and that will be a failure scenario here that will be sigma y there.

I did not distinguish between tension and compression because only the direction of this shear is going to change between tension and compression that is why there is absolute value coming in here ok. So, in compression also we will have the same sigma y as the failure stress and sigma y as the failure stress there, and hence this will be the failure envelope in tension and tension tension compression compression zones ok.

What happens in tension compression zone? Basically let us look at a special case where in I was interested in pure shear pure shear is a case where in sigma 1 is equal to sigma 2 minus sigma 2 equal to tau right where in I add tension and compression coming in the different directions. So, since sigma the magnitude of tension and compression are same that will represent a 45 degree line here to represent this 45 degree line here where the sigma 1 is minus sigma 2 ok.

In that line you know that the maximum stress that it can reaches sigma y by 2 the maximum stress that it can reach here is sigma y by 2 maximum stress that it can reach here is this is sigma y by 2, and this is sigma y by 2 ok.

And I know that the equation that governs when sigma 1 is positive sigma 2 is positive is sigma 1 minus sigma 2 equal to sigma y ok; In other words this equations of the form x equal to sigma y plus y. So, the equation of this line is what I represented here it is this line represents sigma 1 minus sigma 2 equal to sigma y. Similarly if sigma in the other in the other region where sigma 1 is positive sigma whose negative, the absolute value would be sigma 2 minus sigma 1 equal to sigma or y and hence it will be this line. This line is what that equation represents.

So, the Tresca criteria the failure surfaces in the form of an hexagon ok. So, the failure surface for Tresca criteria is in the form of hexagon. Now, we are estimate the failure surface that is the stress state is within this region it is safe if it is on the surface it has begun to yield, it turns out that no material can cross this surface and still it is not possible for stress state to cross this surface since there is a yielding surface it is not possible materials to cross the surface ok. So, this unsafe region, but technically there cannot exist stress which are in this region in this region there can be no stress states in this or in this region there can be no stress states possible.

Next what we want to see is we want to see in what plane the body would have failed you have found the failure stresses, but how will the body fail is what we want to see next.

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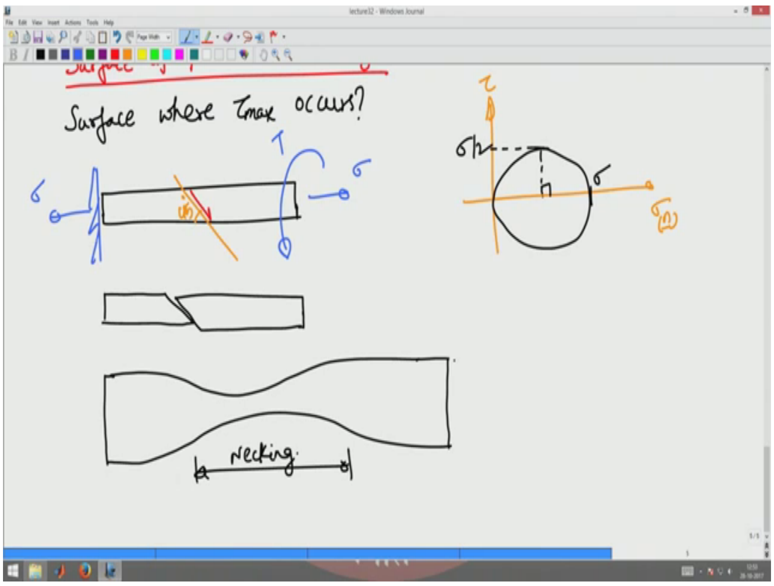


So, let us take the example of a Uniaxial tension surface of failure in the body that is we said that when tau max occurs it will fail right. Now, what we are interested is finding that surface where tau max occur. So, that will determine this our failure.

For example, if I take a Uniaxial bar a bar subject to Uniaxial state of stress a bar subjected to Uniaxial state of stress sigma sigma you know that from the Mohr circle there is a shear stress, there is a normal stress, this is where you connect with what do you have done previously for this the Mohr circle is you cannot forget what do you have done previously would be like that and you know that 0 and sigma are the maximum or minimum normal stresses in the Mohr circle, and tau max is sigma by 2 that we found also occurs at a 90 degree orientation from this plane and 90 degree orientation of Mohr circle corresponds to a 45 degree rotation. So, basically if I segment this a 45 degree cut, I know that the shear stress here is maximum.

And hence this body will failed by shearing along this region it will shear of along this region shear of along that region. So, the failure surface would look like some region is going to slide like that.

Now, what happens is, once it begins to slide it shows large deformations. So, what will called as necking occurs necking occurs and hence the final failure surface of a body in tension for example, for a material like steel look like this would look like this ok. Where there will be considerable deformations and it the cross section area would reduce significantly and this is called as necking it is called as necking ok. Now, this how a brittle material fail.

Now, let us look at another case when in I subjected this bar to a torsional moment subjected this bar to a torsional moment fix this let me look up a case.

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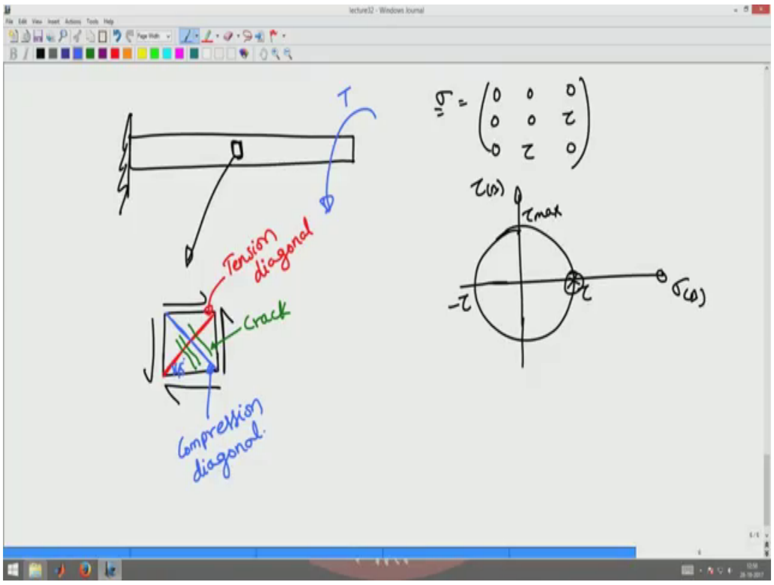


When I subject a bar, fix this in subjected to a torsional moment T ok. Now I subject this to a torsional moment the maximum shear stress occurs where maximum shear stresses in the plane of the problem ok. So, the state of stresses pure shear 0 0 0 0 0 tau, 0 tau 0 for example, and the Mohr circle for this is sigma of n taus of n shear stress it will be a circle there where this is tau this minus tau this tau max.

In the given plane the maximum shear stress occurs. So, what will happen is here the surface and I twist it we saw the surface which are in torsion, which are in shear. So, they will just sheared off like this in torsion what will happen is the surface will shear of like that that is if the shear stress where to you govern the failure theory.

Now, let us say this material was like a chalk piece which is not ducktail, but it is a brittle material then what happens see maximum normal stress of the rankine failure criteria govern the failure then and let us see the material is we can tension then what happens? This stress is what is going to cause it to fail then the failure surface for this has to be at 45 degrees to the direction of this shear right.

So, if I take a element here, I take this element in from here blow it up I will have shear stresses acting like this and this diagonal this diagonal is going to be in tension and this diagonal is going to be in compression ok. Since this diagonal is in tension and the material we can tension cracks will develop along this direction this will be the crack, this is tension diagonal this is the compression diagonal ok. So, crack will develop perpendicular to the tension direction because tension causes is a surfaces to be cracked if I add tension along this direction to crack now open up like this that is it will be parallel to the compression direction compression diagonal direction.

So, this you know from Mohr circle that this tension and compression occurs at 45 degree inclination to the of the acts of the beam ok. So, now, let us take a chalk piece and check whether this thing happens ok. So, I have this chalk piece. So, I am twisting it, you can see that the failure envelop is roughly 40 degrees failure envelop here is roughly 45 degrees ok. So, that shows that what are we are studying make some practical sense also.

So, what we have seen in this lecture is we have see formulated the failure theory we have said that the failure theories can be stress base or strain base bust stress base failure theories I have preferred, the reason we saw and then we saw a there are two kinds of materials which are hydrostatic pressure sensitive hydrostatic pressure in sensitive materials. And hydrostatic pressure sensitive materials behave like a ductile failure show a ducktail kind of a failure or the shear stress it will govern the failure mode.

Whereas in brittle material the normal stresses or a combination of normal and shear stresses will govern the failure mode. And we saw one example of hydrostatic pressure insensitive failure criteria or the yield criteria, which is a maximum shear stresses or Tresca criteria and we saw what a failure surfaces now how to determine the surface along which the body will fail ok. We will repeat same things for the other 4 failure theories that we were we want to study as part of the course ok.

Thank you.