

Mechanics of Material
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Pressure vessels and failure theories
Lecture – 91
Different failure modes

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FAILURE THEORIES

$f(\underline{\sigma}) = 0$

$f(\sigma_1, \sigma_2, \sigma_3, n_1, n_2, n_3) = 0$ ~~n_1/n_2~~

Principal/Eigen Stress Principal/Eigen direction

↓ Isotropic material
 → no preferred failure plane

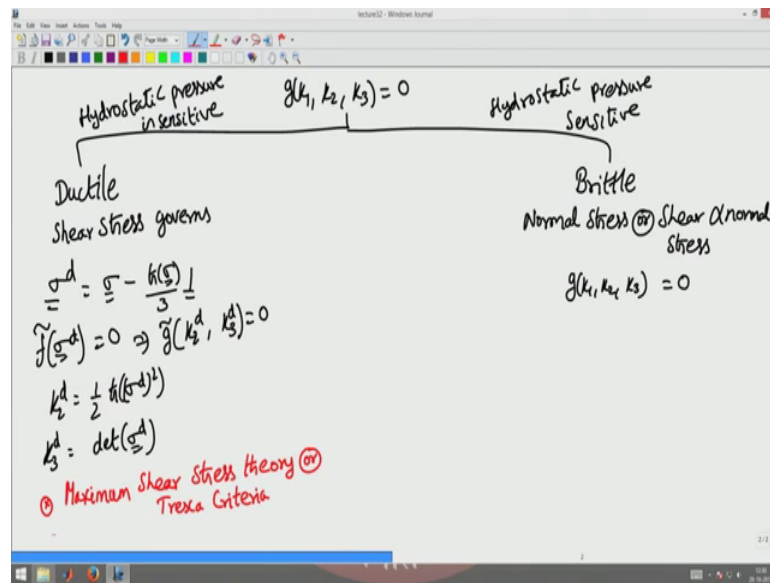
$f(\sigma_1, \sigma_2, \sigma_3) = 0 \Rightarrow g(k_1, k_2, k_3) = 0$

$k_1 = \text{tr}(\underline{\sigma})$; $k_2 = \frac{1}{2}[(\text{tr}(\underline{\sigma}))^2 - \text{tr}(\underline{\sigma}^2)]$ & $k_3 = \det(\underline{\sigma})$

→ Principal Invariants of stress tensor.

Now, comes up demarcation, wherein there are certain materials which are hydrostatic pressure sensitive and certain materials are hydrostatic pressure insensitive. Let us understand what we mean by this.

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Now, we got g as a function of k_1, k_2, k_3 equal to 0 now there is a demarcation as hydrostatic pressure insensitive and hydrostatic pressure sensitive materials ok.

What do you mean by hydrostatic pressure sensitive and insensitive material is, if I take a spear of a particular material and immerse it in a specific coefficient will it fail. When a immerse spear of a material in a specific coefficient in a fluid is subject to an hydrostatic pressure. What is the hydrostatic pressure? Hydrostatic pressure is a state of stress whereas, no shear stress in any plane.

We will such a state of stress caused a failure in a material it is a question we asking. It turn out that there are many materials like steel metals in general, which will would not fail when it subject to an hydrostatic pressure. Hydrostatic pressure can be compressive or tensile in any of these stress state is a material should not fail that is in stress state which are purely normal stresses, there is no shear stresses this metal wont fail such metals are called as hydrostatic pressure insensitive materials.

On the other hand materials like glass, concrete to some extent in tension fails due to hydrostatic pressure ok. So, these materials are called as hydrostatic pressure sensitive materials ok. In general you associate hydrostatic pressure in sensitive materials to what is called as a ductile mode of failure and hydrostatic pressure sensitive materials. So, what is called as a brittle mold of failure ok?

The difference is in ductile mode of failure the shear stress governs; shear stress governs the failure mode ok. In brittle material mostly normal stress or to a certain extent shear and normal stress can govern failure ok.

Now, a given material need not always behave in a ductile manner or a brittle manner this again temperature dependent. So, a given material in certain range of temperatures may be ductile and certain lower range of temperatures may be gave in a duct brittle manner ok. So, it is not that steel is always ductile and glass is always brittle ok. So, you have to understand that.

Next since the mode of failure is ductile or it is a shear stress that governs the failure not the hydrostatic pressure, these pressure insensitive materials are govern by the deviatoric stress $\sigma_{\text{deviatoric}}$, which we are define in a previous lectures as $\sigma_{\text{deviatoric}} = \sigma - \frac{1}{3} \text{trace}(\sigma) \mathbf{I}$. So, this is called as a deviatoric stress because trace of σ corresponds to the hydrostatic pressure and that hydrostatic pressure will would not cause failure in these bodies, which are ductile which shows a ductile mode of failure ok.

So, now instead of this depending upon g a function of k_1, k_2, k_3 or f also function of general function of σ it will depend upon a special function of $\sigma_{\text{deviatoric}}$ equal to 0 for ductile mode of failure ok.

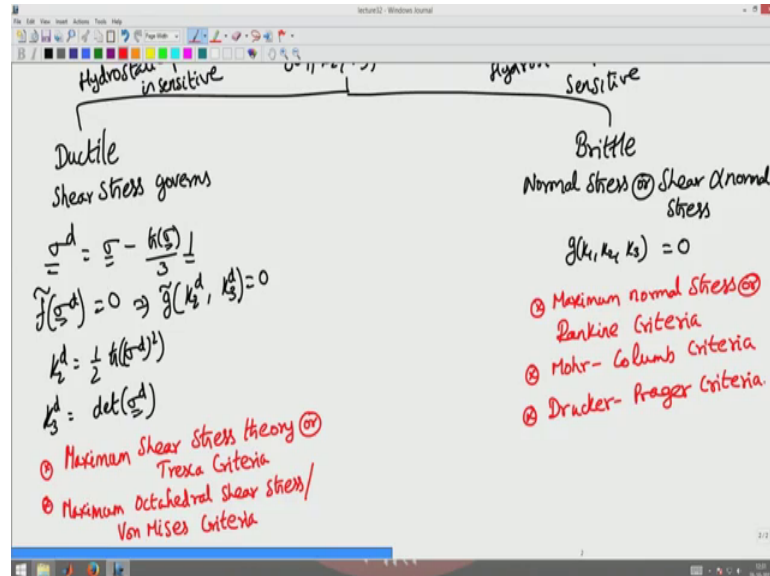
Since trace of $\sigma_{\text{deviatoric}}$ is always 0 this will depend from similar argument that we made before, this will depend upon g of k_2, k_3 and k_3 being 0 this will depend upon k_2 and k_3 are 0 k_2 is defined as one of trace of $\sigma_{\text{deviatoric}}^2$ and k_3 is defined as determinant of $\sigma_{\text{deviatoric}}$ ok. You can (Refer Time: 05:21) this in terms of σ also, but that I will leave it as a exercise we will do it as a when required where k_2 and k_3 are d are this.

When for normal stress and shear stress govern the failure are both or normal stress along governs the failure still we have to retain the original form which is k_1, k_2, k_3 being equal to 0 this is a general expression for failure theories of this class.

Now, there are different failure theories within the same hydrostatic pressure sensitive and hydrostatic pressure insensitive failure theories, we will study couple of them in each of these cases ok. The hydrostatic pressure insensitive case we will study 2 failure

theories one is called as the Maximum Shear Stress theory or what is called as Tresca criteria and other theory.

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That will look up is Maximum Octahedral Shear Stress more popularly called as Von Mises criteria.

Similarly, in pressure sensitive materials will look up three theories one is Maximum normal stress or Rankin criteria second is Mohr-Columb criteria and last is Drucker Prager criteria ok. So, you look up these three theories in brittle mode of failure and this two theories in ductile mode of failure.