Mechanics of Material
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Pressure vessels and failure theories
Lecture – 90
General Principals

Welcome to the 32nd lecture in Mechanics of Materials. Till now we have been solving different boundary value problems, after having looked at what stress and strain force and displacement are on the equation that are connecting them, ok. Now we have been analyzing the success as though the consolation that we used holds throughout for the response of the material, ok. That is not the case, consolation holds only up to a particular limit. For example, the Hooke's law that we used holds only until the response of the material is elastic ok.

So, you have to tell the limit up to its say the consolation holds or in other words if the structure is if the material is failing you have to tell what is the stress beyond which the material will would not withstand the applied load, ok. So, this is where the failure theories comes in. First we had to understand what we mean by failure. Failure means not crushing of concrete, not crushing of wood necessarily. What it means is, is a inability of a structure to serve it is intended purpose. It can be that the displacement has exceed a particular limit, causing the structure not to perform its intended task as in the case of a Doppler antenna or your cell phone towers and things like that, where the geometry of a structure is more important than it ability to withstand the wind loads or the loads that are coming on the structure, ok.

Similarly, you know that all reinforce concrete structures there will be a crack in detention side which does not necessarily mean the structure has failed ok. We decided with the cracks so that it can still withstand the applied load ok. On the other hand the same reinforced concrete structure if you cracks in the water tank in the water tank is not allowable because water will seep in will cause some chemical degradation of the concrete and the steel which you can corrode.

So, in one scenario what is acceptable may not be unacceptable in some other scenario. So, you have to be clear on what you mean by failure. Now for our lecture here what we mean by failure is, the limit of applicability of a constitutive relation ok. In particular the limit of applicability of the Hooke's law; that is we are assuming that the metal response elastically, that is it regains its shape and it does not dissipate any mechanical energy. So, when will this assumption stop to hold is what we are interested in finding.

Now you can format failure theories using either a stress base approach or a strain base approach. There have been theories based on both stress and strain, but most popular theories are based on stress. There is a reason for this; the reason is the strain or what is the observable strain need not consistently cause failure in a body. I will give you an example: take a example of a freely expanding body due to temperature changes. You know that there is no thermal stresses or there is no stresses in the body because of a free expansion of a body due to rise in temperature ok, but there is a strain in the body.

So, theories are based on strain should in particular be based on the mechanical component of the strain and not on the total or the observed strain. For this reason this failure theory is based on strain as a limitation. On the other hand if the theory is based on stress there is stress in the body; means if it exceeds a particular limit it is going to fail ok. So, the stress in the body will be induced only by a mechanical strain. Even though the mechanical strain and mechanical stresses are related in a one to one manner, when the body is elastic or it undergoes a non dissipative response ok.

You can formed it both in terms of stress or strain, but it has to be the mechanical components of the strain that you have to get into the failure theory which is difficult to estimate in a general scenario. So, most failure theories are based on stresses. There is another reason why you based your failure theories on stresses, because there are bodies which can be residually stress. For example, in the last lecture we saw composite cylinder right, the same composite cylinder can be formed when you by a process called as a string fit process; wherein the inner steel pipe will have outer diameter which is slightly greater than the copper steel pipe inner diameter.

So what you do is you cool the inner steel pipe, so there is diameter reduces you put it into the copper pipe the outer pipe and then you allow it to come back to its room temperature. Then what happens is basically there will be a interfacial stress developed, because the copper the steel pipe is trying to expand which the copper pipe is trying to prevent ok. So, there will be in that case residual stresses or even though the there is no traction on the surface of the body. There will be logged in stresses are called as residual stresses in the body. Many manufacturing process will result in the body having residual stresses ok.

So basically these residual stresses have to be accounted for in some manner which you can do when a theory is based on failure theories or the limiting, boundary is different in terms of stress rather than the strain. For this reasons your failure theories will say that it is based on the stress ok.

Now what does this failure theories try to depict? There are 6 component of stresses and you know that the 6 component of stresses, the value of those 6 components of stresses depends upon the coordinate system that we choose to study the particular problem. That is a failure of a body depend upon the coordinate system that it choose to study the problem? No. If I pull a chock and if it breaks, the failure is not determined by the coordinate system I chose to represent it. It depends only on the stress state that the body is subjected to.

So, basically you cannot base a failure theories on the computed components of the stresses. It has to be independent of the coordinate system, some quantity is independent of the coordinate system has to be used to form to the failure theories ok. So, it should be based either on the principal stresses or the principal invariance of the stress. So, what is failure theories essentially tells us is, the region which is safe in the stress space from demarcated by the surface from the unsafe region.

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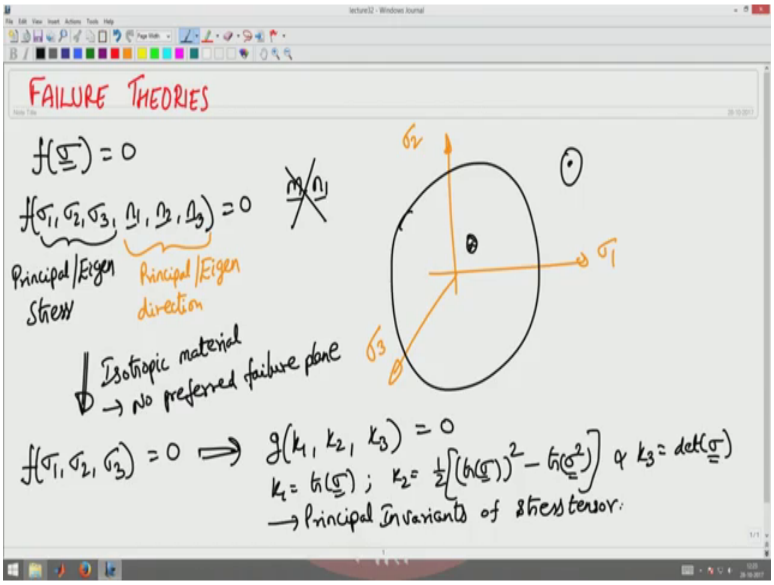

So, we have to consider, a 3 dimension space the principal stresses sigma 1 sigma 2 sigma 3 as the coordinates of that space. Then what is failure theories will give us is a surface in this 3 dimensional space ok, which will demarcate the safe region from the unsafe region. If it is inside the surface; the point the stress state is inside the surface it is safe. If it is outside the surface it is unsafe. If it is on the boundary of the surface, it is on the verge of failing or on the verge of losing its ability to model a given response ok.

So, essentially what we are after this in formulating this failure surface or the yield surface of the, or the limit of applicability of the constellation ok. So, from our discussions still now it is clear that I want to find a function f as a function of sigma equal to 0 or this a 6th dimensional space of the 6 components of stresses in general ok. Since is a surface is a scalar valued tensor function ok.

Now, for the scalar value tensor function to be objective: meaning, it should fail only when the stress state exceeds a particular limit and not depend upon the coordinate system use to study the problem this has to depend upon in general the principal values of stress and the principal directions n 1 n 2 and n 3, where sigma 1 sigma 2 sigma 3 are the principal or the Eigen stresses, and n 1 n 2 n 3 are the principal or the Eigen direction, ok. So, this has to be the case.

Now, if the body can fail in any plane there is no preferred plane of failure of a body then it will would not depend upon the Eigen directions, such a material is called as an

Isotropic material. This will reduce, so I assume Isotropic material that is, no preferred failure plane. Then the failure criteria will reduce to depending upon the principal values of the stresses sigma 1, sigma 2, sigma 3 equal to 0.

Now, what happens when there is a preferred failure plane like in a wood, wherein the fiber directions lesson fail easily compared to the direction perpendicular with the fiber orientation of in the wood, ok? Then it will depend upon n 1 n 2 n 3; then it will depend upon n 1 n 2 n 3 which are the principal directions, ok? In particular there will be factors coming in like, where m is the direction of the fiber m dotted with the n 1 kind of contraries will come in the failure theory, ok? We would not look into these in this course. Ok? We will look at only the Isotopic materials which means depends only on the principal values of the stress.

Since it is difficult to compute the principal values of stress, you know that you have to solve cubic equation to find the principal values of stress in general what you do is you based it on the principal invariants instead. You based it on k 1 k 2 k 3; so principal in various of these stresses where k 1 is nothing but, trace of sigma k 2 is nothing but 1 half trace sigma squared minus trace of sigma square and k 3 is nothing but, determinant of sigma. These are the principal invariants of stress, ok; so the principal invariance of the stress tensor, ok. So basically you based on k 1 k 2 or k 3.