Mechanics of Material
Dr. U. Saravanan
Department of Civil Engineering
Indian Institute of Technology, Madras

Pressure vessels and failure theories
Lecture - 89
Thin walled pressure vessels

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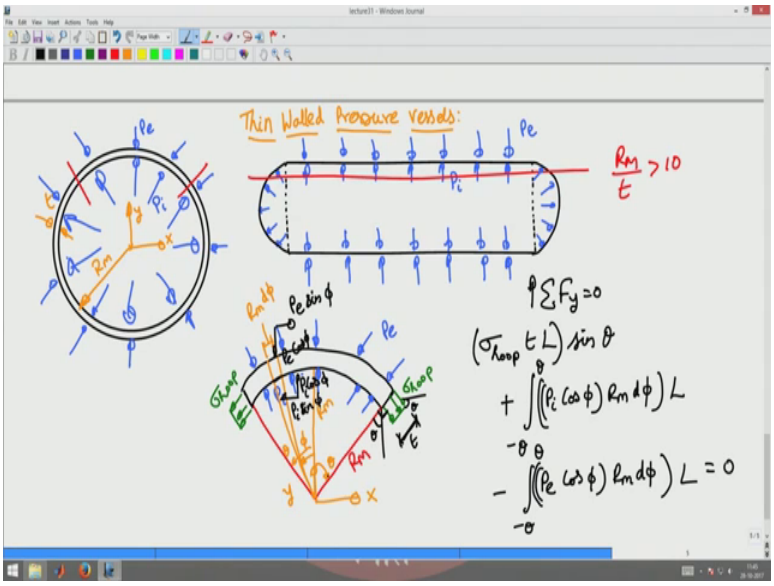

Now, we will move on from looking at regress an accessory based solutions to vessels like this, structures like this to and approximate equilibrium based approach where in what we are doing now is we are looking at thin walled pressure vessels ok. What we have is essentially the same domain, it is a cylindrical structure with the end closed, this can be viewed as a cross session of a gas cylinder or cross session of an aeroplane kind of a thing where in the ends are closed in both the cases.

So, you are interested in finding what are the stresses that are induced in the structure with the condition that R m by the condition that R m by t is greater than 10 preferably around 15 or t20 ok; So, R m by t is it is a very thin walled structure compared to the radius of the structure.

Now, how do you analyse these structures, what are the stress components that will be present we saw from our thick walled cylinder solution that the predominant stress there is present is the hoop stress. So, I have to estimate first the hoop stress. So, to estimate the hoop stress I have to induce I have to make a cut statutory hoop stress comes out of in the free body diagram ok.

So, what I do is I cut here like this and like this which will be a cut made in the cylinder like this. This is a cut I am making in the cylinder. So, once I cut that what will I get? I will have the hoop stresses which is a circumferential stresses I exposed ok. So, the free body diagram will be if I section this the free body diagram of this will look something.

Like this where I exposed the hoop stresses there is a external pressure acting like that and internal pressure acting like this now I want to write the vertical and horizontal force equilibrium, where in my x and y axis are is x and y axis ok. I want to write the vertical force equilibrium for this structure sigma F y equal to 0 with upward acting forces positive.

First let us look at the hoop stresses there is a stress or multiply by the area first now I am going to write the equilibrium equations for this thin walled pressure vessel, first I am going to write the vertical force equilibrium which will give me the hoop stresses. So, I have to find what is the net force acting in the vertical region because; of the hoop stresses. The net force would be sigma hoop into t this area, this width along which the hoop stress acts matching the hoop stress is constant over the thickness of the cylinder, it is a reasonable assumption given that the vessel is thin walled ok. And then it acts over a length L of the cylinder.

So, that is into l that gives me the force I have to find the component of this force along the vertical direction I know that if this is theta, this angle is going to be theta and the ends and the hoop stress is tangential to the cylinder cut surface.

So, this angle is going to be 90 degrees and the ends the hoop stress will make an angle theta with the horizontal and the ends the vertical component would be sin theta ok. Now to this I have to add what is the effect of the inner pressure pi, the P i inner pressure acts at an angle phi, let us look at an angle of orientation phi from the centre of the cylinder circle.

So, basically I will have integral I have to write the component of the stress along by vertical direction. So, that will be P i cos phi and the horizontal components going to be

P i sin phi, because it makes an angle phi with the vertical direction it being normal to the surface.

So, I have P i cos phi which is the component of the applied pressure along the y direction into I have to multiply by this distance in here which is R m times d phi. R m times d phi into the length of the cylinder again L and this I would integrate from minus theta to plus theta because I measuring theta and anti clock wise directions in one side and clockwise directions in the other side ok.

So, this will be minus theta to theta and this stress acts in the positive x y direction and hence it is positive. Similarly for the external pressure I have minus because this axis in negative y direction.

You can see that the component of the stresses would be P e sin phi and P e cos phi vertical component ok. So, there will be P e cos phi into R m d phi into l in a same argument using the same argument this will be minus theta to theta this has to be equal to 0 ok.

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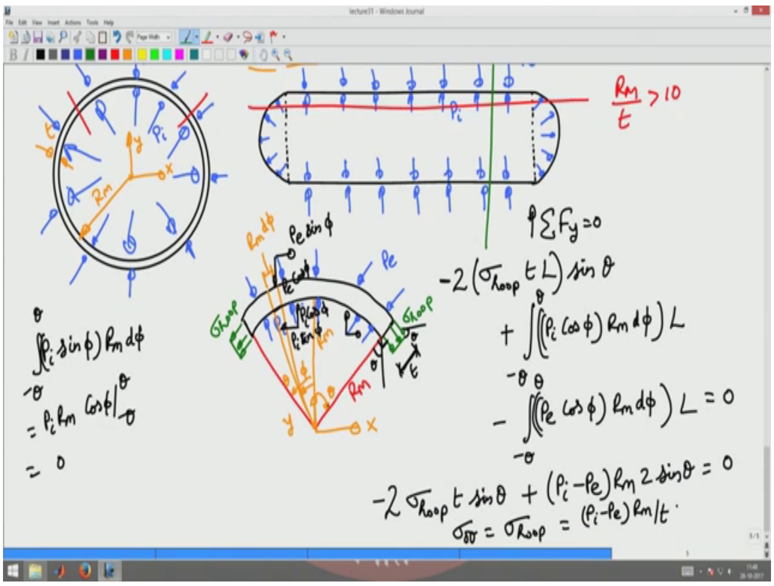

From here we will get that sigma hoop into t L gets cancelled out integrating cos phi you will get it as I left one more component, there will be I took the component of hoop stress in one side that is hoop stress on the other side too. So, that will that should be multiply by 2 there yeah and the hoop stresses are in the downward direction.

So, there should be a negative sign in there. So, basically I have minus two times hoop stress into t into sin theta plus P i minus P e I cancel the l out R m cos phi if I integrated I will get it as sin phi sin phi integrated from minus theta to plus theta would be two times sin theta ok. This would be equal to 0 and from here you get sigma hoop to be P i minus P e into R m by t ok. So, that is the hoop stress now let us see what happens to the horizontal component I will just show what happens to P i sin phi R m d phi integrated from minus theta to plus theta this will be P i R m cos phi from minus theta to plus theta.

Which is 0 because cos of minus theta is cos theta that is intuitive because on either side of the centre line either side of the centre line of this structure, the direction of the horizontal forces is opposite to each other. So, that will get cancelled out. So, hoop stress is given by P i minus P e into R m by t. So, that is the hoop stress next I am interested in computing what will be the axial stress. In other words this hoop stress is nothing, but sigma theta theta I am interested in computing the actual stress which will be sigma z z for this structure to compute the axial stress what should I do? I have to induce a different cut I have to induce a cut.

Which is vertical I have to induce a cut like this. So, that only then the axial stresses will be exposed now let us analyse.

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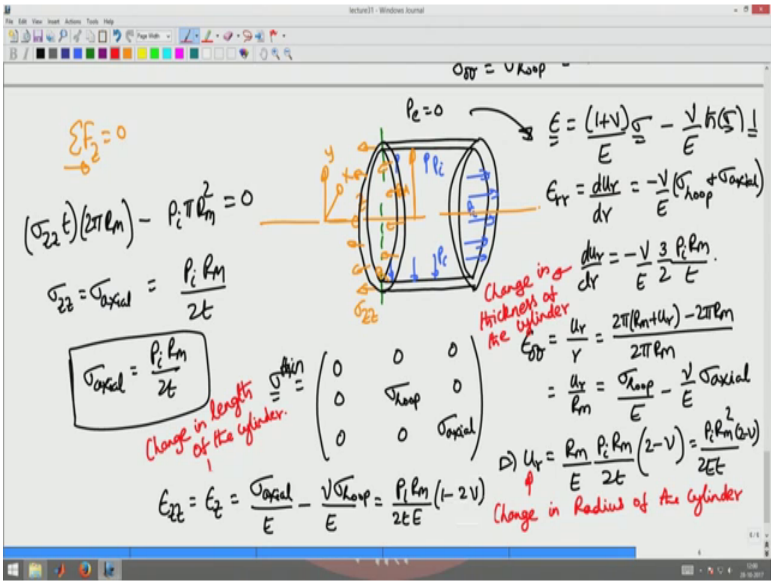

What happens when I cut the cylinder like that I cut the cylinder like I showed before, I will get that is the cut surface. So, basically I will have these stresses exposed which is sigma zz and then I have pressure acting like this on this wall P i and pressure acting like this on this lid there, which is also of pressure P i magnitude ok. Now and you know that this from the centre line of a cylinder this radius is R m.

So, now what happens I am going to write the equilibrium equation along F Z equal to 0 taking this as the positive direction if you recollect our orientation of the axis was the axis of the cylinder was z the vertical axis was y and this axis was x. So, I write a first equilibrium along the z direction, which will give me sigma zz and the thickness of this cylinder into this sigma zz is going to act over the circumference of the cylinder there is the circumference of the cylinder are known which it will act. So, that is the circumferences of the cylinder is given by 2 P i R m there ok. That is it acts over this circumference acts over this entire circumference sigma zz is going to act like that over this entire circumference ok.

So, that is 2 P i r into t, that is the area that is the cross sessional area of this cylinder on which sigma zz acts ok. Now, this has to be balanced by the pressure exerted by this P i; this P i on the lid there ok.

So, what happens is now this minus P i into it acts over the circular area P i R m square the internal pressure acts over a circular area P i R m square, where either it is this area at we are talking about it acts over in that area and the ends this P i R m square this as to be equal to 0 ok..

From here you get sigma zz or sigma axial stress to be P i R m by 2 t the external pressure can or need not cross an axial stress that depends upon the arrangement of the vessel and so on. So, basically that is why we did not consider external pressure P e to produce any axial stress because it is in general it will would not cause an actual stress ok. So, sigma axial now is half of the half of the hoop stress we have to recollect that this axial stresses will arise only when the cylindrical vessel is closed at both the ends, this is not like what we had for the thick walled vessel.

The effect of Poisson's effect, where in the hoop tension is causing a contact in the axial direction which you are prevented by making it fixed along the axial direction ok. This is a stress that arises only when the cylinder is closed ok. Now, we have found the stresses, next what we are interested is we are finding the displacement of this pressure vessel.

So, displacement means I have to estimate the strain you know that strain is given by the constellation for a strain in terms of stress is given by 1 by mu by E sigma s mu by E trace of sigma identity and now then you have to find it what this sigma s and what is epsilon s in terms of the displacement field ok. Now sigma we have found there is a hoop a circumferential stress.

And as a axial are stress axial are sigma zz stress what about the radial stress? The shear stresses are 0. So, I know that this is 0, this is 0, this is 0 I know this is hoop the circumferential stress I know this is 0 0 0 sigma axial stress is what is here sigma zz what about the radial stress? Radial stress will vary from P i at the inner surface to 0 at the outer surface. It being thin walled you assume that there is no radial stresses induced in the vessel that is an assumption you are making otherwise you have to distinguish between the inner surface.

And the outer surface and do a appropriate computation. So, we will assume that the radial stress is a 0 for thin walled vessels ok. Now for this we will find that epsilon rr the radial component of the strain, which we found in the previous lecture that this is related to displacement at d mu r by dr from the constellation now you will find that it is minus mu by E sigma hoop plus sigma axial ok.

So, this is minus mu by E into 3 by 2 P i R m by t ok. Epsilon theta theta it is from the previous lecture we found it as ur by r that is a change in the circumference divided by the original circumference of the cylinder.

Changes in circumference would be given by 2 pi R m plus ur the radial displacement of the centre line of the vessel minus 2 pi R m the original circumference of the cylinder divided by two P i R m ok. So, this will be equal to ur by R m from the constellation it will be sigma hoop by e minus mu by e sigma axial.

So, from here you get u of r to be R m times by e into P i R m by 2 t into two minus mu or this will be P i R m square by 2 et into 2 minus mu ok. Similarly epsilon zz which is which was epsilon z the axial contraction that we had from axial contractions are expansion that you will see is given by from the constellation is given by sigma axial by E minus mu sigma hoop by E which is nothing, but P i R m by 2 t E into 1 minus mu ok. In all this I assumed the external pressure to be 0 in this cases. So, up to constellation the relationship remains same, but afterwards to find the expression for ur and dur by dr I have assumed P e to be equal to 0 I assumed P e to be equal to 0 in these derivations ok. Now, it is clear that what u of r is u of r is the radial displacement of the cylinder. So, I can associate with the circumference change.

But what is this dur by dr or what is this radial component of the strain. The radial component of the strain gives you what the change in thickness of the cylinder is; this gives change in thickness of the cylinder that gives you the change in thickness, this gives you the change in radius this gives you the change in radius of this cylinder and this gives you the change in length of this cylinder ok.

So, basically what we have done is, we have analysed thin walled pressure vessels there is one more concept that we have to a understand what happens in a composite thin walled cylinder. So, let us spend some time in understanding the composite thin walled cylinders let us say the inner cylinder is made of steel and the outer cylinder is made of copper.

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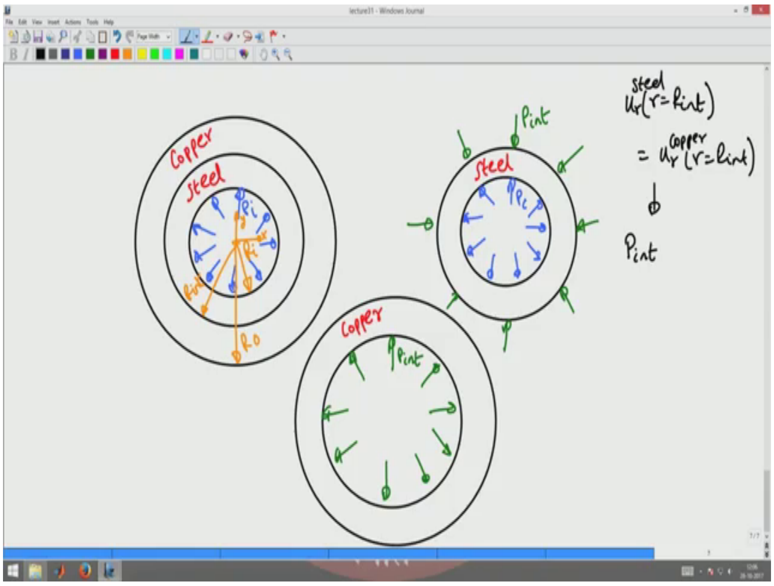

And there is a inside pressure it is applied to this composite steel, and let us assume that this is R interface let us assume that this is R e R R 0. So, basically now we want to analyse what happens to this composite structure the ends you know that young’s the young’s modulus and Poisson stress of steel and copper are different.

So, basically how do you analyse this structure is, you draw the free body diagram of steel and copper exposing that there be a radial pressure acting at the interface of steel and copper. So, you isolate the steel pipe from the copper pipe this is steel and this is copper.

And you expose the interfacial pressure you know that there is P i arcing here this P i acting there and we expose the interfacial stress radial stress P interface acting like that ok. Similarly there will be a p interface acting in the copper like this. There will be interfacial pressure acting like that in the copper vessel.

So, now, you solve this problem as thick walled cylinder or thin walled cylinder depending upon what the R m by t ratio is and you have to enforce you can solve this problem find a displacement field and solve you can solve this problem and find a displacement field you can solve this problem or find the displacement field what is required is the displacement at the interface what do you required is, ur at r equal to R interface of the steel must be same as ur of copper at r equal to r interface ok.

A radial displacement at the interface has to be the same that is because it is a perfect burning between steel and copper and there is no possibility of steel penetrating copper or steel opening up from the copper bonding. So, you have to have this condition that u of r of steel and copper has to be the same..

You use this condition to find the p interface the interface pressure because otherwise you have sufficient boundary conditions. So, find the displacement field in a thick walled cylinder or in a thin walled cylinder.

So, basically this is how you have to proceed when you have a composite cylinder either thick walled or thin walled ok. With this we will conclude today’s lecture from next lecture on we look at, how to formulate the limits of applicability of the constellation or the failure theories in general ok.

Thank you.