Mechanics of Material

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Pressure vessels and failure theories
Lecture - 88
Example problems: Thick walled cylindrical vessel

Ok welcome to the 31st lecture in mechanics of materials, you are looking in the last class on inflation of ion less cylinder basically we used it as an example of how to formulate a boundary problems somatically using theory of elasticity that is we assume a displacement field from a displacement field we computed what is the strain; from the strain we computed the stress using the three dimensional consolidation, then we found the unknown component the displacement field from the equilibrium equation.

That is (Refer Time: 00:48) sigma equal to 0, assuming there is negligible body forces and the bodies in static equilibrium, we the equilibrium equations boil down to requiring there is no sigma to be equal to 0 using that we found that.

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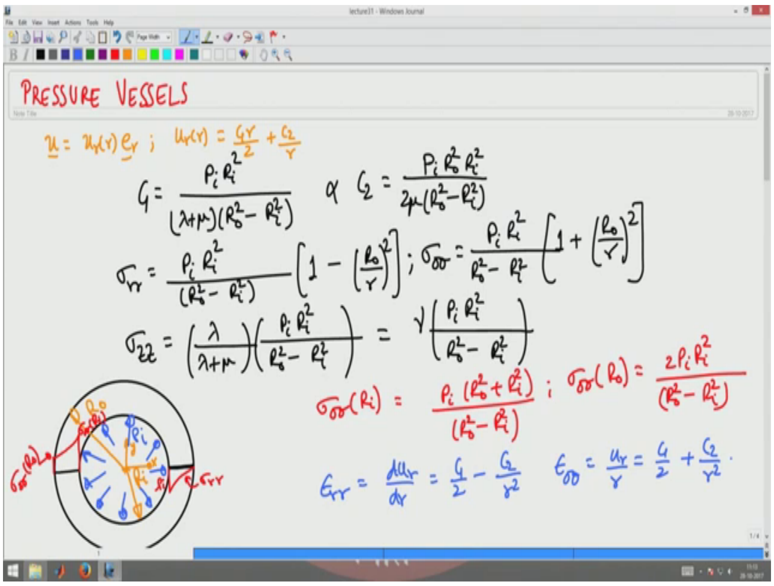

The displacement field u of r has to be given by C 1 r by 2 plus C 2 by r and then we found the constant C 1 and C 2 such that the boundary conditions for the given problem are satisfied. In particular in this problem, we are pressure appear at the inner surface and no pressure applied at the outer surface.

So, that was sigma rr at R i the inner surface must be equal to minus pi the applied pressure at the inner surface and sigma rr at R naught must be equal to 0 the applied pressure at the outer surface. From that condition we found what C 1 and C 2 r and then we estimated the stresses again going back to computing the strain from the stain problem get in the constellation to compute the stresses as given here and we rationalized that the predominant stress that is produced.

In this boundary value problem as the hoop stress or sigma theta theta stress, which is approximately 10 to 20 times more than the radial stress radial stress is sigma rr and it has a variation I have shown here it varies from minus pi are the inner surface to 0 or the outer surface the boundary condition it was specified and where as sigma theta theta is a tensile stress which varies from a maximum value of the inner surface to a minimum value of the outer surface, which is a roughly 2 times pi R i square by R naught square minus R i square and then we can go back.

And compute what the strains are also from the expression that sigma epsilon rr is though you are bid or with the C 1 by 2 minus C 2 by r square and from the fact that epsilon theta theta its nothing, but ur by r that is C 1 by 2 plus C 2 by r square ok. From these expressions you can compute the strains also, because you know what u of r is ok. Now we will proceed further and see what happens when the boundary condition changes and what are the various variations in this problem that we can study ok.

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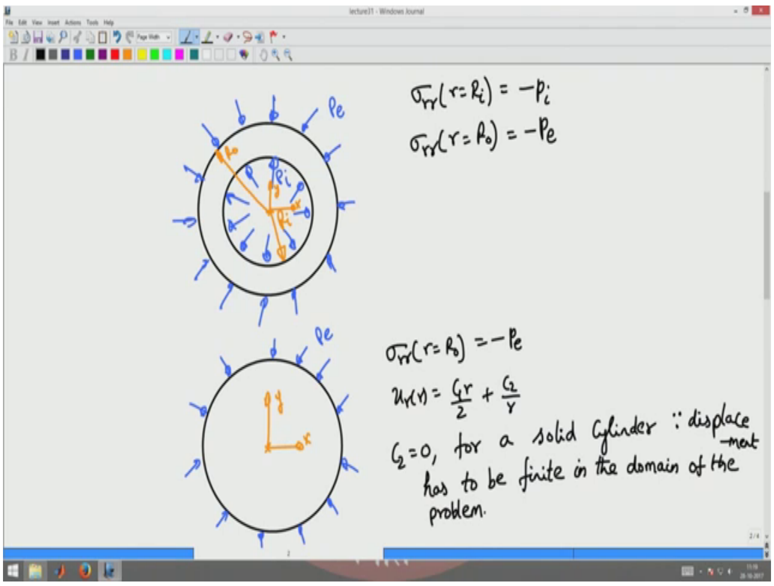

The first thing is. So, I pressurise is it both from inside and outside I am not going to solve this entire problem, but just indicate how to go about solving this problem especially from outside by a pressure P e say; now this pressure is again opposite to the how to draw normal to the surface. So, this is the negative pressure. So, the boundary conditions now would be sigma rr at r equal to R i s equal to minus pi same as what we saw in the last class last lecture and sigma rr at r equal to R naught instead of being 0 was going to be minus p external pre externally applied pressure.

Negative because it is opposite to the direction of the how to draw a normal, the stresses which are parallel to the how to draw normal are always positive stresses which are opposite the forces which are writing opposite to the how to draw normal or negative stresses ok. So, you got these two expressions now.

So, your evaluation of the constant C 1 and C 2 will be different for this case will involved pi and pe and you can go about doing the same thing plugging into the constellation find a strain plugging into the constellation and find this stresses ok. Now let us say instead of cylinder, which is ion less cylinder I have a solid cylinder pressurised from outside then what will happen.

Say I have a solid cylinder pressurised from outside this or to writing pressure would be normal to the surface ok. So, this pe is acting in a solid cylinder with the x and y boundary like that, now what are the boundary conditions you have? You have only presumably one boundary condition right you have sigma rr at r equal to R naught to be given by minus pe what is the other boundary condition that you will use?

The other boundary condition comes from the fact that the displacement field should be bounded in the domain of the problem you we got u of r of r to be C 1 r by 2 plus C 2 by r as R tends to 0, you find that C 2 by r goes to infinity which means the centre of the cylinder are saw infinite displacement if this were to be the displacement field which is not allowable ok. And then you set c t C 2 to be 0 C 2 is 0 for a solid cylinder.

Since displacement has to be has to be finite in the domain of the problem ok. So, that is the second boundary condition that you are now you have only one constant which you can find from the condition that sigma rr at R naught is minus pe.

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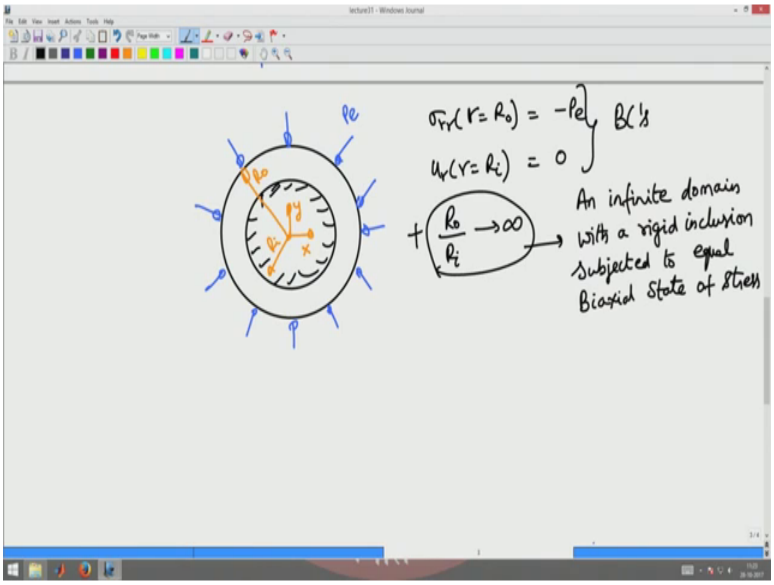

Now I can now, another boundary condition wherein the inner surface can be yield fixed say I have a rigid body a rigid steel rod put into a rubber pipe and I am pressurising this pipe from outside. Now I am pressurising this pipe from an outside pressure say from outside pressure pe what is the boundary condition now?

Still let me have my axis as x and y here, still my boundary conditions would be sigma rr at r equal to R naught the outer surface is minus pe where this is R naught and this is R i. Now what is the second boundary condition it will have? The second boundary condition is since the inner surface is resting against a rigid body, this surface defined by r equal to this inner surface will not get radially displaced since it is resting against a rigid body.

So, the boundary condition would be u of r at r equal to R i has to be equal to 0. So, this is the second boundary condition it will use solve this problem ok. Now I can after setting this to be 0 and using this boundary condition, I can look at the problem wherein I can look at R naught by R i tending to infinity meaning I have a infinite domain the centre of which is a rigid body ok. So, in that case if I apply an equal biaxial pressure that is the idealization that you get from solving this problem in the limit R naught by R i tending to infinity.

So, this will be this boundary conditions plus this condition gives us the solution for infinite domain with a rigid inclusion subjected to equal biaxial state of stress ok. Similarly I can have a case wherein the external pressure alone.

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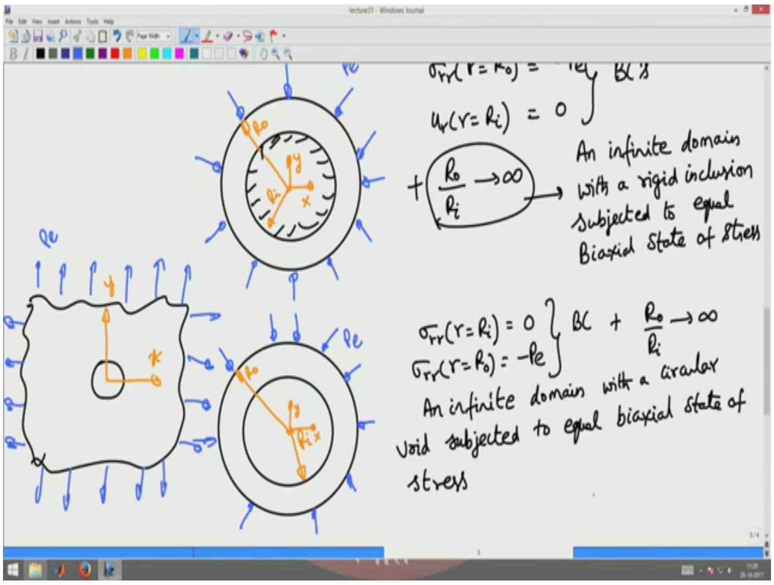

Similarly, I can have a case of an external pressure alone applied in this domain pe an external pressure alone applied in this domain wherein now the boundary conditions is obvious should be sigma rr at r equal to R i has to be equal to 0 and sigma rr at r equal to R naught would be minus pe, and for this boundary condition plus the limit R naught by R i tending to infinity, you will have an infinite domain with a circular hole in in the middle subjected to an equal biaxial state of stress ok.

This solution will correspond to an infinite domain with circular void subjected to equal biaxial state of stress this infinite domain means basically you have some domain like this which is extending towards infinity both in x and y direction and you have a void a small void, in other words located at the centre of the coordinate system ok.

So, this as a part shape of the domain means it is a infinitely to extending to a large area and you are subjecting it to equal biaxial pressure pe in all directions. Since a domain is infinite the pe will become normal to the surface its always normal to the surface. So, it will become equal biaxial state of stress. So, basically it is not now in all this we had a basic displacement field as u is a function of u of r er.

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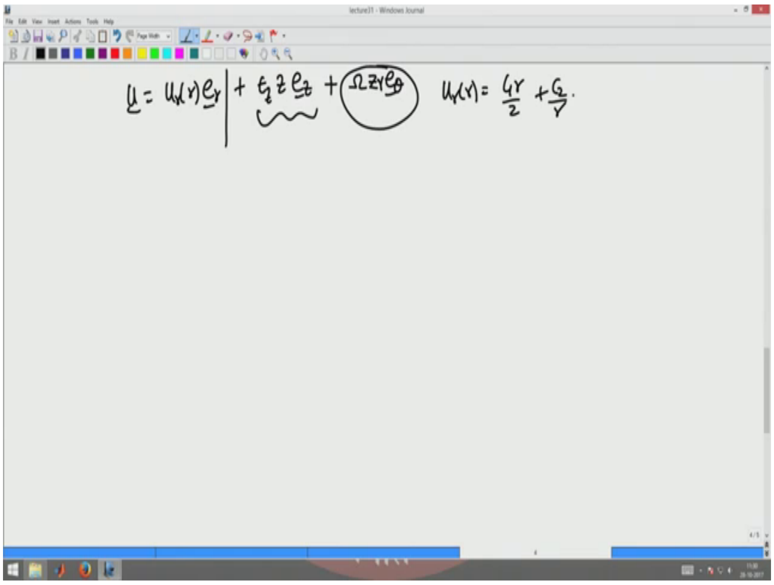

But this also can change I can superpose axial deformations which will be plus epsilon z times z ez, which is a axial expansion or the contraction.

The axial displacement is given by epsilon z times z ok. I can superpose the torsional displacement which in this case will be given by omega z e theta omega z e theta which I can superpose in this also which will be the angular displacement. And still for this displacement fields are also u of r by r would be C 1 r by 2 plus C 2 by r. Because these displacement are orthogonal to the radial displacement and super impose allowed which means this is the small deformation linear constellation that we are studying.

Hence you can superpose solution. So, you can suggest solutions which have only this component or which has only this component or one which has only the torsional displacement component ok, which has the torsional displacement component or I can study only this torsional rotational component ok.

In the each of these cases you have to rederive from the first principals find the strain is a constellation to find this stress, use the equilibrium equations to find the unknown components for a displacement field and then you go back plug that in into the strain field and find the stress again and that will be your solution for a problem.