Mechanics of Material

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Stresses and displacement due to torsion or inflation
Lecture – 85
Cylindrical polar coordinate system

Welcome to lecture 30 of Mechanics of Materials. In the last lecture we saw how to analyze member subjected to twisting moment or torsion, in today's lecture we will see a different boundary problem wherein you have a annular similar which is inflated from inside held at a constant length ok.

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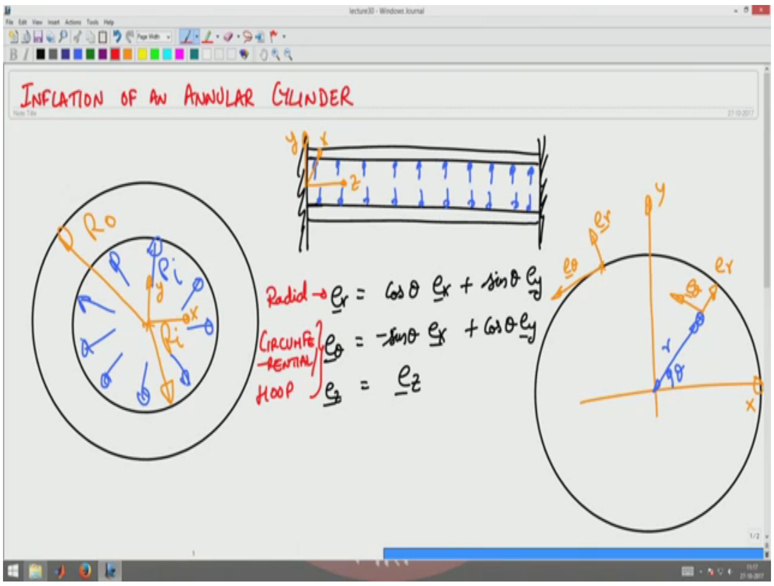

So, what I have is, I have this annular cylinder I am holding it fixed and then I am inflating it from inside and I am inflating it from inside like this ok. And I am interested in analyzing this structure for the stresses and displacement ok. Since the geometry of the structure is circular the cross section is circular, it is prudent to use cylindrical polar coordinate system to solve this problem ok.

So, we will understand first the components of stresses in cylindrical polar coordinate system now let us assume that this is z, that is x, and that is y of the cylinder and the cross section is shown here, which is essentially in the x y plane this x this is y ok. Now if I want to study a problem in cylindrical polar coordinate system, I have to see; what is a component of stresses mean cylindrical polar coordinate system.

You know that in cynical polar coordinate system, you have to transform from x y z, e x, e y, e z basis to radial circumferential and axial basis where e r is given by cos of theta e x plus sin theta e y. I will mark what theta e r e theta is shortly minus sin theta e x plus cos theta e y and e z in cylindrical coordinate would be same as e z in the Cartesian coordinate system ok. Now this is y and that is x, that is x any point in here as coordinates given by the radial distance r from that to here and this is d theta the angular position of the point.

Now, e r then is defined as the radial vector pointing along the radial direction, that is this is c r and e theta is tangential to the circle that passes at that point e theta ok. That is if I draw a circle with radius small r e theta will be tangential to the circle at that location for example, if I am looking at a point here e r is the outward or normal to the circle which is e r, and e theta would be tangential to that circle e theta.

So, now resolving e r into the basis e x and e y, you can see that you get this relationship that I have written here. It is just like taking component of e x, e y along e r, e theta e z direction ok. So, that is e r, e theta the point you notice the radial vector and circumferential vector in cylindrical polar coordinate system changes from point to point ok. This is called as the radial basis and this is called as circumferential or hoop basis ok. Let us c theta is called as a circumferential or the hoop basis and e r is called as a radial basis.

Now, to understand what different components of stresses the six component of stress cylindrical polar coordinate mean, I have to section now to find out what the six components of stresses in cylindrical polar system mean.

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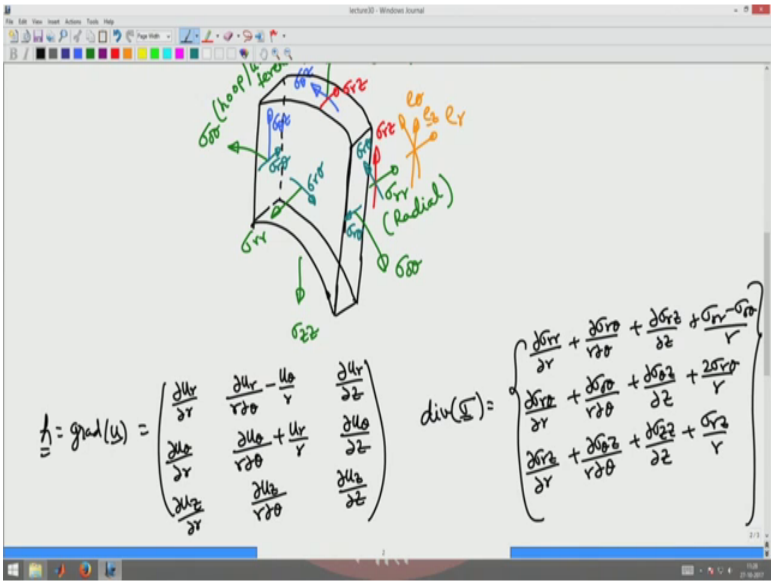

Just like we drew a stress cube or draw a segment of a cylinder and show you those six components of stresses ok. So, let us do that next ok. So, this is a segment of a cylinder, but e r, e theta e z r this is e r, this is e theta and this is e z.

Now, let us look at the positive oriented e z face that is I am looking at this I am looking at this area this cut surface whose normal coincides with the positive oriented e z normal ok. Now what is this normal stress? This normal stress would be sigma z z is also called as the axial stress.

Now, next let us look at the tangential a circumferential basis let us look at the circumferential basis. Now this is the cut surface for which e theta is positive, the circumferential basis is positive and hence this would be the sigma theta, theta are your hoop or circumferential stress or the circumferential stress.

Now, let us look at the positive surface, whose normal coincides with the radial basis, that positive surface is this surface in here that is the normal e r acting on this face coincides with the positive oriented radial basis ok. And hence this stress is sigma r r are the radial stress ok. Now corresponding in the negative basis, you will have this as sigma z z will have this as sigma theta theta and this would be a sigma r r on the respect to other side of the cut surface ok. Now for completeness will complete the shear stresses also ok; now what will what will this shear stress b? The normal to the cut surfaces is z and the shear stress is acting along the theta direction.

So, that will be sigma theta z are to be precise sigma z theta I am not going to distinguish between theta z and z theta ok. Same thing here this will be sigma theta z and then this stress would be sigma r z, and the stress acting on this face would be sigma r z ok. Now we have seen 5 components, the remaining component is sigma r theta is the component we have not seen till now, that would be in the radial basis, it will act like this that will be sigma r theta or in this face it will act like this, sigma r theta on this positive face starting in the theta normal surface in the r direction. So, this is a sigma r theta and this will also be sigma r theta that is the other component of the shear stress in them.

Now, have you understood the components of the meaning of the components of stresses in cylindrical polar coordinates and the direction in which they act. Now let us go ahead and see what is the gradient of displacement and divergence of sigma in cylindrical polar coordinates. I am not going to derive this equations, I am going to take it as given to you.

So, h which is gradient of displacement field, where h is gradient of the displacement field is given by dou u r by dou r u r by r dou theta minus u theta by r dou u r by dou z dou u theta by dou r, dho u theta by r dou theta plus u r by r dou u theta by dou z, r dou theta by dou z.

Similarly, the divergence of sigma in cylindrical polar coordinates is dou sigma r r by dou r plus dou sigma r theta by r dou theta plus dou sigma r z by dou z plus sigma r r minus sigma theta theta by r dou sigma r theta by dou r, plus dou sigma theta theta by r dou theta, plus dou sigma theta z by dou z, plus 2 times sigma r theta by r, r z by dou r dou sigma theta z by r dou theta, dou sigma z z by dou z, plus sigma r z by r ok.

Those are the expression for diversion of sigma and gradient of the displacement field.