Mechanics of Material

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Stresses and displacement due to torsion or inflation
Lecture – 83
Thin walled closed sections

Now, let us look at how to analyze thin walled closed sections ok. Till now we have been looking at thin walled open sections are closed sections from which will would not warp now let us analyze thin walled closed sections which will warp. For example, if I have a box kind of a geometry.

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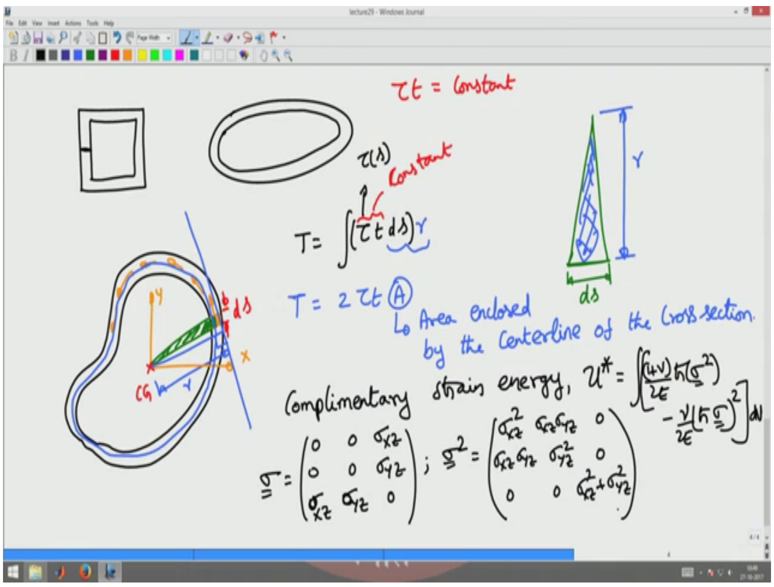

Cross section say I have a box shaped cross section or a elliptical cross section or an elliptical cross section like this with uniform thickness or varying thickness how do you analyze these sections? Because all the cylinder is a thin walled section which will wont warp because of the geometry these sections will warp again because the shear flow is not continuous in the cross section ok. So, let us analyze how to analyze the structure.

So, now for this we will think of it as we will make an assumption that the shear stress at a particular section is uniform across the thickness of the cross section ok. What happens in the thick wall section is, the shear stress across the thickness of the cross section will vary, but this being thin walled we assume that the shear stress does not vary across the thickness of the cross section ok. That is if I cut here and expose see the shear stress variation along this line, we will assume that it is constant. So, let us assume arbitrary geometry.

Wherein the thickness also need not be constant some arbitrary geometry let us assume the C G of these geometry is here this is C G of the cross section and the center of rotation of the cross section this is y this is x ok. Now the shear stress in this cross sections it being thin walled even though it has a varying thickness is tangential to the geometry of the cross section it will be like this the shear flow will be like this the shear flow will be like that ok. Now what I am assuming is I am assuming that in this thickness region, the shear stress is constant along this thickness region.

So, now let us consider an input decimal element of dimensions d s here ok, where in this uniform shear stress acts in that d s element then what is the torque for this? The torque would be given by integral tau, which is a shear stress acting in that input decimal element times the area of that input decimal element which is t times d s. This torque this shear stress is a function of s this is a function of s because the thickness changes the shear stress can change ok. This can be a function of s in general. So, basically this is tau times t times d s into that is a net force acting in that cross section times I have to multiply by the lever arm. The lever arm is the perpendicular distance between the C G and it say that is a tangent to the curve at that point, then I want this perpendicular distance this is 90 degrees I want this perpendicular distance which is say r. So, that will be times that r.

Now, now let us understand what this d s r mean what this d s into r mean when I integrate over the entire cross section ok. Now this d s times r represents the area of this triangle up to the center line of the cross section ok. Because d s is a base I have a triangle like this whose base is d s and whose perpendicular distance from this point to this is r, that is the lever arm ok. So, now, what is this d s into r would be twice this area of this rectangle triangle d s times r will be twice area of the triangle there.

So, when I integrate it over the entire cross section, I will get it as 2 times tau t into area of the enclosed area of the cross section. What I mean by enclosed area of the cross section is, it is the area given by here say this is the center line of the cross section the blue line I am interested in the area enclosed by the center line of the cross section that is this is area enclosed by the center line of the cross section that is the area enclosed by the center line of the cross section. So, might I have found a expression for torque ok. Here you have to note that even though tau s is a function of s tau t here; I have pulled out tau t outside the integration because tau t I am assuming is a constant, t can be a function of s tau can be a function of s, but tau p is not a function of s.

So, I have the expression tau t equal to a constant that I assumed in this derivation ok. So, now, we have derive the expression for the torque as 2 times tau t into the area enclosed by the cross section. Next I want to relate this to the angle of twist per unit length right. So, further we will take a energy approach this time for this we have to construct the complementary strain energy, which we introduced in lecture 15 u star, this was integral over the volume of the body 1 plus mu by 2 E trace sigma square minus mu by 2 E trace sigma the whole square into d v the volume of the body ok. Now what is sigma for this case? Sigma for this case in x and y coordinate system they said that is a shear stress in the plane of the cross section alone.

So, it will have 0 0 sigma x z, 0 0 sigma y z 0 0 0 sigma x z and sigma y z and 0 there ok. There is a set of stress because I can resolve the effective shear stress tau into components sigma x z and sigma y z now what is sigma square? Now sigma square is sigma x z squared sigma x z sigma y z 0, sigma x z sigma y z, sigma y z squared 0 0 0 sigma x z square plus sigma y z square.

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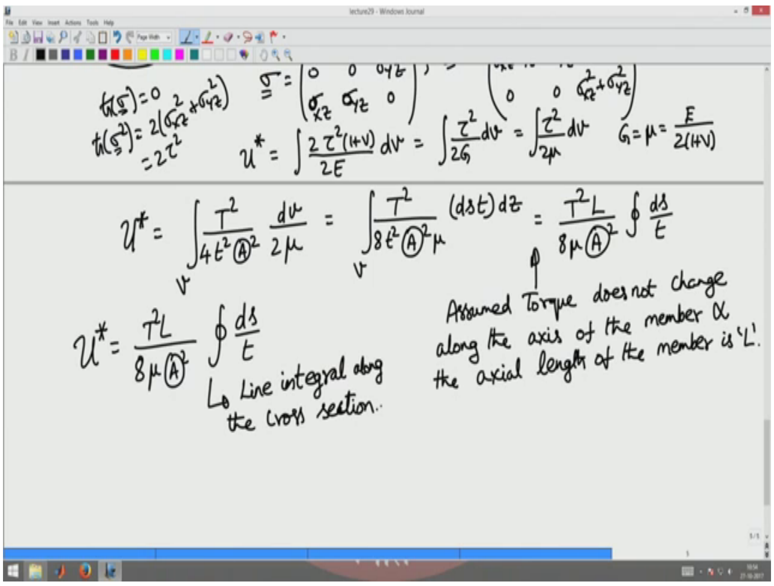

Now, next I am interested in finding the complementary strain energy, which is u star. I know that trace of sigma is 0 and trace of sigma square is 2 times sigma x z square plus sigma y z square that is nothing, but 2 times tau square. So, what do I have the complementary strain energy is 2 times tau square 1 by nu into 1 plus nu by 2 E d v ok. From a definition of shear modulus are from the fact that a (Refer Time: 10:22) constant is equal to the mu of the (Refer Time: 10:25) constant is equal to the shear modulus, we can rewrite this equation as integral tau square by 2 G d v or integral tau square by 2 mu d v right because G equal to mu is equal to E by 2 times 1 plus nu ok. I have use that relation to rewrite it as this ok.

Now, what I am going to do next is, I am going to substitute for the torque from from the expression here to the shear stress in here. Next I am going to substitute for the torque from this expression into the shear stress expression here. So, what do I get? I get u star to be integral T square by 4 t square enclosed area squared d v by 2 mu ok. Now what is d v for this case? The volume of the body I can write that ask T square by 8, T square enclosed volume square into mu d v I can write it as d s into t is the area of the cross section times d z whose integration along the length.

So, now t alone can depend upon the axis the actual location of the member. But let us assume that this v here I am assuming that to write this I am assuming that the torsion moment does not change along the axis of the member ok. So, 8 A square integral d s by t, here I have assumed torque does not change along the axis of the member and the actual length of the member is L.

So, now what is what have we got? We got the complementary strain energy as T square L by 8 mu area of cross section squared d s by t. This is the contour integral or the line integral this is the line integral of the cross section along cross section ok. Now next I have to construct the total potential which is u star minus the load potential ok. So, next again this load potential and total potential are defined in lecture 15, so I request you to go back to that lecture if you are not familiar with these definitions ok.

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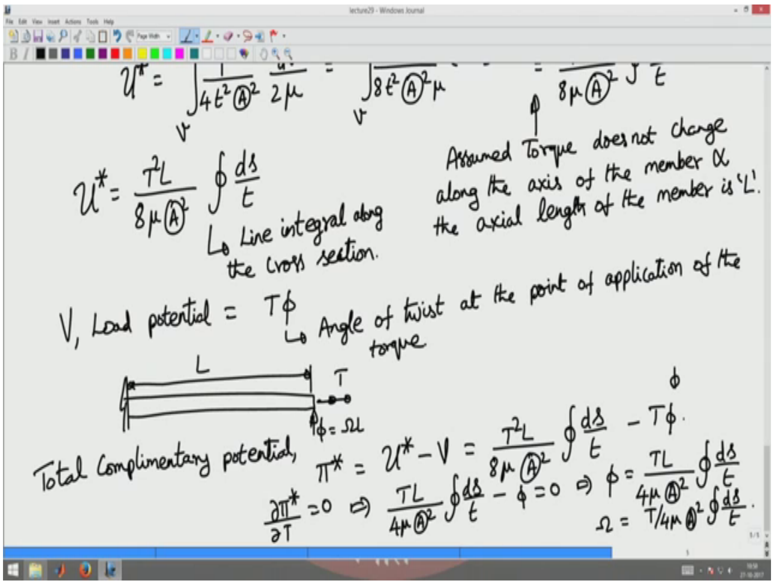

Now, load potential V is defined as T times phi because I am applying a torque T and phi is the angle of twist at the point of application of the torque, that is I have this cross section where I am applying a torque T here at a distance L from this fixed end. So, that phi would be the angle of twist that this section. The angle of twist of this section the angle of twist of this section is phi that will be nothing, but omega times L where omega is angle of twist per unit length ok. So, that is phi ok. Now the total complimentary potential pi star is u star minus v which is T square L by 8 mu a square line integral d s by t minus t into phi.

Now, I want to get this phi I know that in energy method, I have to minimize the potential with respect to the independent variable I am writing in terms of complementary strain energy. So, the independent variable is the torque or the force. So, I have to minimize this with respect to the torque which means I have to set dou pi star by dou t to be equal to 0 ok. So, that will give me T L by 4 mu A square into d s by t minus phi equal to 0 ok. In other words this will imply that phi is T L by 4 mu A square into d s by t ok. Now you know that phi is omega into L. So, from here I get omega angle of twist per unit length as T by 4 mu A square into line integral d s by t.

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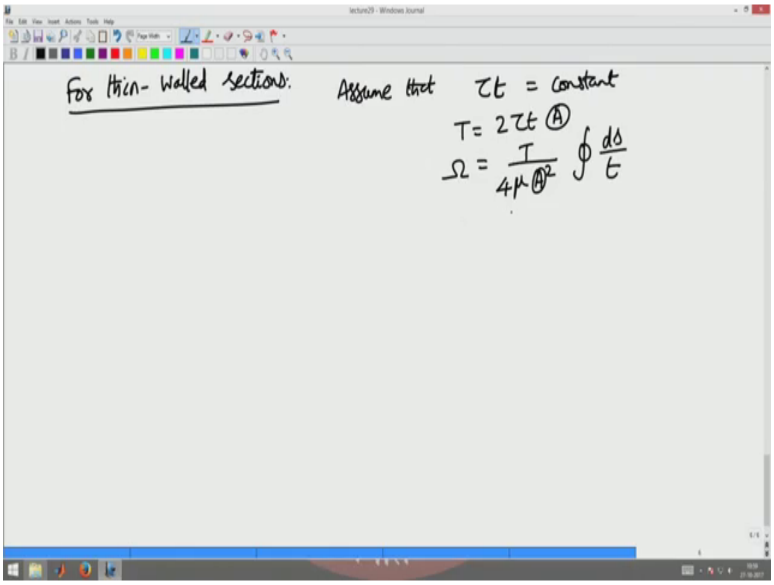

So, essentially what we are shown is, for thin walled sections sections we assume that tau times t is a constant. That is thickness times the shear stress is a constant it comes from the equilibrium requirement of the cross section in other words and then we showed that the torque is given by 2 times tau t into the enclosed area of the cross section and we have shown that the angle of twist per unit length is given by T by 4 mu A square into d s by t the line integral along the circumferential area of the cross section divided by t thickness of the cross section.

So, this is the result that you have to remember from here.