Mechanics of Material
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Stresses and displacement due to torsion or inflation
Lecture – 82
Example problems: Open sections

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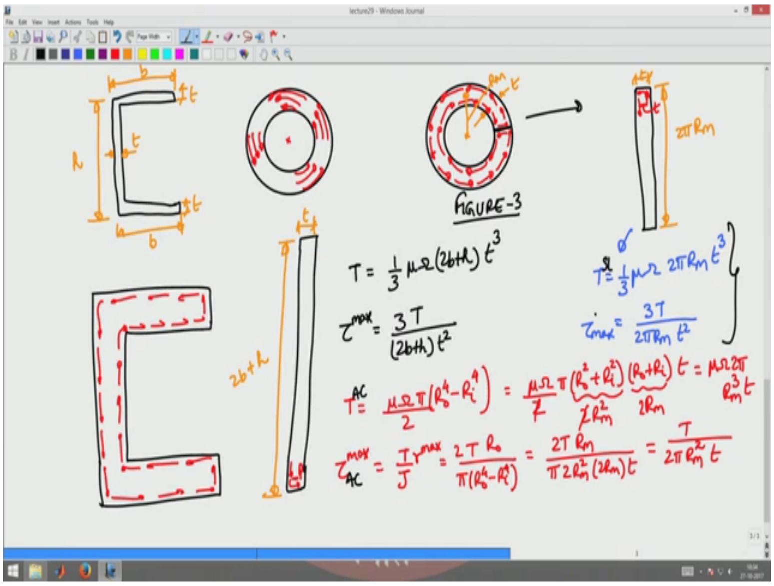

Now, let us say I am interested in 3 sections, one is a channel section, the other one is an annular cylinder and next section a diameter (Refer Time: 00:40) this annular cylinder with a cut in between with the cut here, as the cut their as a longitudinal cut that is it is something like this. Its some cube bent like this where there is this cut along the axis of the member. So, that is a last one.

Now, let us assume that the channel as the following dimensions, the thickness of the flange is t, thickness of flange is t the width of the flange is b both top and bottom, and depth of the web is h with the thickness t again.

Now, I order to analyze it in one of the forms that we have seen till now, this is a open section again because there is no continuous flow of shear stresses. The shear stress flow in this case has to be something like this. I (Refer Time: 01:47) the figure ok. The shear stress flow in this case due to torsion would be something like this, would be something like that.

So, what I can do is, I opened this section up and consider it as a equivalent rectangular section, this is a simplified analysis not an exact analysis, but a simplified analysis which will give a quick estimate of what is the maximum stress at this section will see for a given torque and what is the angle of twist at the section will see for a given torque. So, what I do is I open this up and think of it as being a rectangular section of thickness t, of thickness t and of depth 2 b plus h ok. That is I open the 2 flange is to become straight and I am thinking of it as an equivalent rectangular section of this dimensions.

So, now the torque for this section would be 1 by 3 mu omega, the depth is 2 b plus h and t cube is the width of the cross sectional. So, that will be the torque relating to the angle of twist per unit length omega, and tau max would be 3 times T divided by 2 b plus h into t square. So, that will be your tau max for this.

Similarly, the final figure here the figure 3 for this also is an open section and it will warp because there is no continuous flow of shear stress is possible here, in this case the shear flow would be something like this because there is a cut there the shear stress cannot be continuous in the cross section will be a shear flow something like this and the hence this also will open it up and think of it as being a rectangular section, this think of it as being a rectangular section of thickness t. If this initial thickness was t and if its mean radius from the centre was R m, this will have a thickness t and the depth of this rectangular section will be 2 pi R m ok. In this case in figure 3 case a torque could be for this case similar to here the torque could be 1 by 3 mu omega 2 pi R m into t cube and tau max would be 3 times torque divided by 2 pi R m t square.

Note in all this approximations the thickness is, the thickness over which the lever arm of the shear stress acts that is a thickness ok. It is not because it is thin in that dimension the shear flow is such that, it will it is it will go around in the inner dimension, but not always ok. In all this if we open it up the thickness is where the shear stress changes its direction, this lever arm between the shear stresses is what is the thickness of the cross section ok. So, even here the shear flow is something like this if I open it up and the lever arm between the shear stresses is the thickness of the cross section that is why you have to judge which is a thickness and which is a depth of the cross section.

Now, coming to the annular cylinder here, we saw in the last class that it is a closed section and we saw that the torque is related to the angle of twist by expression mu j was pi by 2 R naught power 4 minus R i power 4 right.

So, now that is the expression for the torque there and in this case the shear stress will wont go around inside the cross section, on the other hand it will be something like this at every section there will be a shear stress, there is point where there will be zero shear stress in this cross section ok. There will be all increase in shear stress as we go outside from there because tau is given by T by J into R where R is a radial distance from the centre of the cross section known as c g of the cross section ok. Now let us simplify this will be mu omega by 2 pi into R naught square plus R i square into R naught plus R i into t right.

I have used a square minus b square is a minus b into a minus b formula twice and I got that expression in here now if the cross section is such that r naught and r I are close to each other and if the thickness of the cross section is small, then I can replace this with R m the mean radius 2 times R m and this I can replace with 2 times R m square ok. Then this expression for torque reduces to mu omega 2 pi R m cube into t ok. One 2 cancel this 2 and 2 cancel. So, I have left with 2 pi R m cube t.

Now, similarly tau max in this case is given by T by J into r max, which is T by pi 2 pi R naught power 4 minus R i power four into R naught.

Now, this again I will simplify like what I did before, to get 2 times T this will become R m divided by pi 2 times R m square into 2 times R m into t. So, this will be T divided by 2 pi R m square into t R m square into t I.

Now, let us compare what happens when you introduce a longitudinal slit in this annular cylinder, that is you got this expressions when you introduce a annular slit you got this expressions when you introduce a annular slit. So, small a slit just a axa blade slit into the cross section a depth, but the shear flow changes and hence you have a such a reduction in the torque required to induce a given angle of twist ok. Now let us say this is with this slit this is annular cylinder a c a c and this will be the slit ok.

Now, let us take the ratio of T annular cylinder divided by T the torque required for engineering a given angle of twist with the slit.

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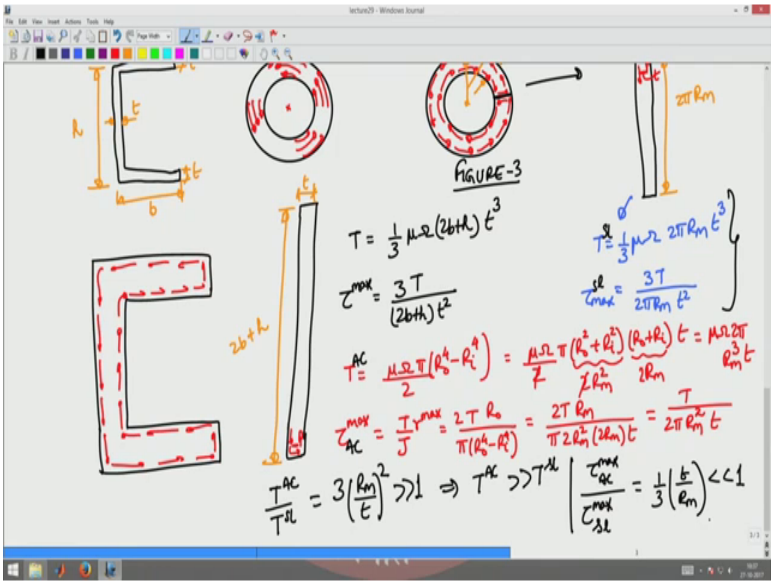

What will this b shall be the ratio of these 2 shall be the ratio of this ok. So, that will I will give as 3 times R m by t the whole square. So, R m by t is greater than 1 significantly greater than 1. So, you can see that the annular cylinder torque to engineer a given angle of twist is much greater than this implies angle of twist for a given angle of twist the torque required for angular cylinder is much greater than torque required for a split cylinder.

So, this is because the liver arm changes for the torque to be generated. In one case it is the entire cross section liver arm plays now other one it is only the thickness of the cross section plays the role of the liver arm ok. Similarly for completeness let us do tau max of annular cylinder divided by tau max of split cylinder this will be nothing, but 1 by 3 t by R m is much less than 1.

So, this will be much less than 1 which means for a given torque the stress develop in the angular cylinder is much less compared to the stress developed in the split cylinder ok, which is recognisable, because it is going to deform more for a given torque, it will develop more stresses because of that this split cylinder.