Mechanics of Material
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Stresses and displacement due to torsion or inflation
Lecture – 81
Expression relating angle of twist with torsion and shear stress

Welcome to the 29 lecture in Mechanics of Materials, the last lecture you begin looking at member subjected to a twisting moment or torsion. In particular we will looked at the effect of twisting of a closed section by closed section we meant that it will would not warp.

We will see in more detail what warping is and how do you analyze for structural warp and twisted in this lecture. Basically we said that two surfaces because of twisting will rotate related to each other and there would not be any out of plane displacement like this ok. So, basically with that assumption, we came up the display field we said that the displacement field would be of the form given here are the form given here.

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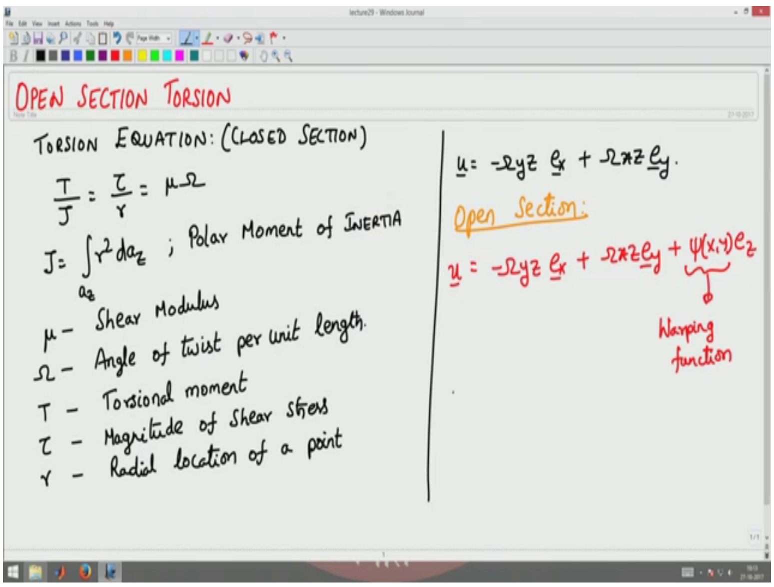

And then we went ahead and derive the torsion equation, which was T by J where T is twisting moment or torsional moment, J is a polar moment of inertia equal to stress by shear stress (Refer Time: 01:20) of shear stress by the radial location of the point equal to, your shear modulus times angle of twist per unit length.

This called as a torsion equation and we saw how we got that equation in the last class for a closed section and we worked out a couple of worked out examples, wherein we add a member subjected to a end torsion or we add two step shaft fixed at both ends subject to a end torsion. In this lecture we will look at what happens for torsion of a open section the basic difference is the displacement field. In the displacement field it will would not be like what we have assume before, that been out of plane displacement.

For example, this sheet of if this sheets represents the section of the beam or a section of the member, then because of twisting there would not be a displacement just like this there will be an out of plane displacement like this also ok. There will be an out of plane displacement like this also is being a rectangular section, this will be precisely the out of plane displacement that happens due to warping.

In this course we are not going to look at detail formulation for warping and things like that, but we will do a approximate analysis, but we will understand that the displacement field now will change for open sections for open section, the displacement field is given by u is given by the above expression sigma z e x plus sigma x z e y plus function psi is a function of x and y e z, and this function psi is called as the warping function is called as the warping function and with there is a displacement field in how you have to solve find a functional form for psi for various shape of cross sections.

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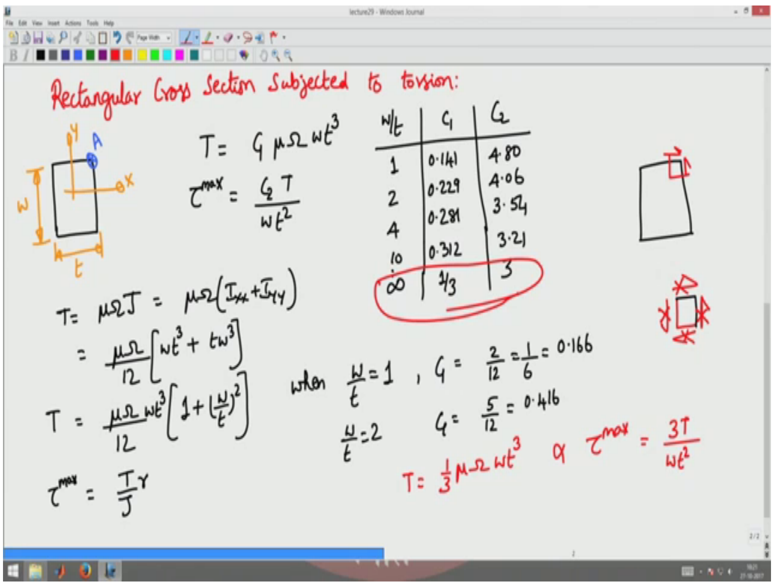

In particular we will find that for a rectangular sections, rectangular cross sections we are going to look at rectangular cross section subjected to torsion that is I have a rectangular cross section, this y x and let us say this distance is w and this distance is t.

Then you can show that the torque would be given by an expression of the form C 1 times mu omega w t cube ok. And your shear stress max maximum shear stress will be given by an expression of the form C 2 T by w t square ok. Where C 1 and C 2 are constants, which is a function of w by t. C 1 and C 2 are function of w by t in particular w by t is 1 C 1 is 0.141 and C 2 is 4.8. If this is 2 raises to 0.229 and 4.06. If it is 4, this is 0.281 and this is 3.54. If it is 10 it is 0.312 and 3.21 ok. In particular if it is infinity, this goes to 1 by 3 and this is 3.

So, now from this expression we will estimate a torque for a given rectangular cross section of a particular dimension to be avoided by this expression. If you note that what we got before was T equal to from the previous expression it was mu omega J the polar moment of inertia. For the in particular it will be mu omega I x x plus I y y because polar moment of inertia is x square plus y square d a z. So, there will be I x x and y y in particular area ok. Then what we will get is this will for a rectangular section b mu omega by twelve into w t cube plus t w cube ok. This will be the polar moment of inertia for this section.

Now, then if you pull out w t cube, which is what we have by 12 w t cube will have 1 plus w by t the whole square there ok. So, let us see what happens when w by t is 1 w y t is 1 it will be 1 by 6. 1 by 6 is this is a torque expression, when w by t is equal to 1 from this expression will get C 1 as, from this expression from this expression we will get C 1 as 2 by 12 which is 1 by 6 which is 0.166 ok. The value is lesser than that because it warps the value of C 1 is lesser than that because it is warping, usually warping will reduce the torque for a given angle of twist.

So, that is what we are observing here if w by t is 2, C 1 would be 5 by 12, which is 0.44 which is 0.416. So, basically that for two, it reduces still more drastically ok. This is what will happen because of warpi. So, having understood this, similarly what we had from the previous expression tau max would have been T by J into r where r is the maximum distance from the centre of rotation this is a c g of the cross section ok, which would be in this case this point a, but the point a has to have zero shear stress and that is why the section warps as no continuous flow of shear stress in the section there cannot be continuous flow shear stress in the section because section a I saw it is zero shear stress.

Why should the point a has zero shear stress? It is because there is no complementary shear applied on the respective top and side planes. That is in this cross section there is no shear stress applied here nor here. So, if I take a section here and look at this sectioned element, there is no complementary shear coming here, this shear is not there. So, there cannot be this shear to this shear is not there. So, this shear also cannot be there. So, that is why the shear stress at point a has to be 0, which will be satisfied when you have warping of the cross section and the hence the resistance of r the torque required to engineer a given angle of twist is lesser when the section warps ok.

So, I am seen that what is of interest is the case when w by t tends to infinity, you can see it is very close even when it is 10 ok. So, what we will do is, we will assume that w by t is tending to infinity and in which case the torque would be given by 1 by 3 mu omega w t cube, and the shear stress maximum which is of interest would be given by 3 times torque divided by w t square now let us apply this to a practical problem.