Mechanics of Material
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Stresses and displacement due to torsion or inflation
Lecture – 80
Example problems

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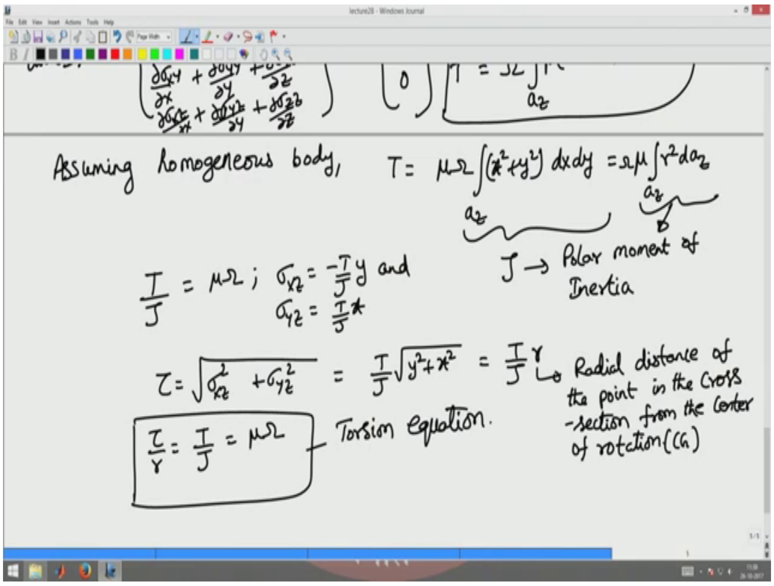

Now, let us apply this torsion equation to study some problems ok. First problem that I am interested is in is in the following. I have this bar fixed at one end, the torsion applied at the other end and the cross section is cross section is a circular cross section is x and this is y ok.

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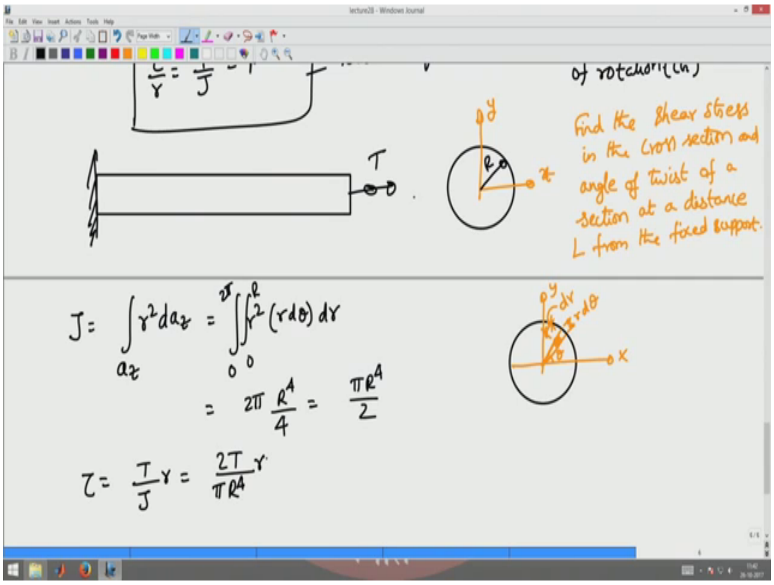

Now, I want to find what are the stresses find the in particular; I want to find the shear stress in the cross section and angle of twist of a section at a distance L from the support from the fixed support. So, this is what I am interested in finding. So, to find the shear stresses I have to first find the polar moment of inertia torsion is given as t.

So, I have to use this equation to find the shear stresses in the cross section. So, to find the polar moment of inertia J as integral r square d a z a z. Now for the circle for the circle, I can use the polar integration. So, basically x y I have an area this area is given by this length is d r and this length is r d theta, but theta is measured from here to there. So, d a z would be integral a z r square r d theta into d r by the limits of integration would be 0 to 2 pi for theta and 0 to r, where r is this radius of the circle r of the radius of the circle. So, this will be theta integrates to 2 pi and r integrates to r power 4 by 4.

So, this is pi r power 4 by 2. So, my shear stress expression becomes T by J into r the radial location. So, that will be T by 2 T by pi R power 4 into r ok. So, now, let us see how this variation of the shear stress occurs in the cross section ok.

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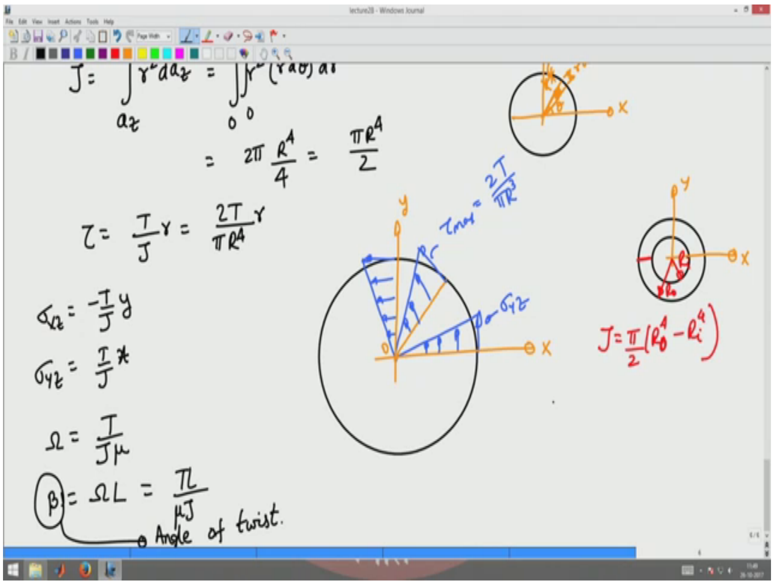

So, I have this cross section which I have zoomed out. So, the centre of the cross section is this x, this is y. I know that the shear stress is varying linearly radially which means that the centre where r is 0 here stress is 0 here and then if I look at some radial distance r I the shear stress is going to vary linearly that is going to vary something like that going to vary linearly like that.

The magnitude is going to increase linearly from the origin to maximum on the periphery by this maximum tau max would be 2T by pi R cube. Because r is the outer radius now ok. So, it varies linearly from 0 to 2 pi by 2T by 2 T by pi R cube ok. Now what is the orientation of this, here I drew it incline why did I do that? Because now I what I have is the magnitude of the shear stress, but the direction of the shear stress will vary depending upon the location because there is sigma x z and sigma y z component coming in here. Because of this sigma x z and y z component being there, it will vary the direction will vary the magnitude of shear stress will remain the same at a given radial distance by the magnitude will vary depending upon its x and location.

In particular when y is 0 let me rewrite that expression that we had before, you add sigma x z as minus T by J into y and sigma y z is T by J into x right ok. Now, in particular when y is 0 for example, when I am on the x axis, the shear stress will vary there will be only sigma y z shear stress this is a sigma y z shear stress and this will vary linearly like that and when I am at x equal to 0 line this is a y axis I will have only sigma x z coming there will be only sigma x z coming in and the shear stresses will vary linearly like this along that length ok.

Now, inclined line or a inclined line when this both x and y the variation would be will depend upon what x and y values are and it will be tangential to the point the circle at that point that is how it will be and hence you get the variation of tau along the inclined line to be to be as shown here, this will be a direction of the tau in a inclined line with the arbitrary x and y directions.

Now, let us see what happens when the cross section becomes an annular cylinder cross section ok. The cross section becomes an annular cylinder that is for annular cylinder cross section as shown here say is x y and the dimensions of the cross section is R I inner surface radius R naught is the outer surface radius.

Now, all the things remain the same except that now J becomes pi by 2 R i power R naught power 4 minus R i power 4 ok. The integration is the same except there is a limits for R becomes R i to R naught (Refer Time: 07:33) 0 to R and hence you get j to be different or the other (Refer Time: 07:40) remain the same the variation of the stresses will be in the region where the material is the variation of the stresses will be in the region were the material is that is only along this thickness region there will be shear stresses developed.

Now, coming to the second part of the question, how does the angle of twist vary along at a what is the angle of twist at a section l from the fixed end. Now, for that you have to use the second part of this equation T by J equal to shear modulus x angle of twist after per unit length and you have to compute that angle of twist at a cross section at a distance here.

So, basically omega is T by J into mu and angle of twist is what do you want is you want beta that is the angle of twist that beta is omega times z, which is now that the portion of the section is at a distance l from the fixed end. So, that g is L. So, you want this L. So, this is TL by mu J. This beta is angle of twist beta is the angle of twist ok. So, we found the angle of twist and we found the stresses. So, you know what is the rotation of the cross section undergoes and what are the stresses that are developed in the cross section.

Next let us do one more problem were and now, what I have is wherein I am applying a torque t in a step shaft like this ok.

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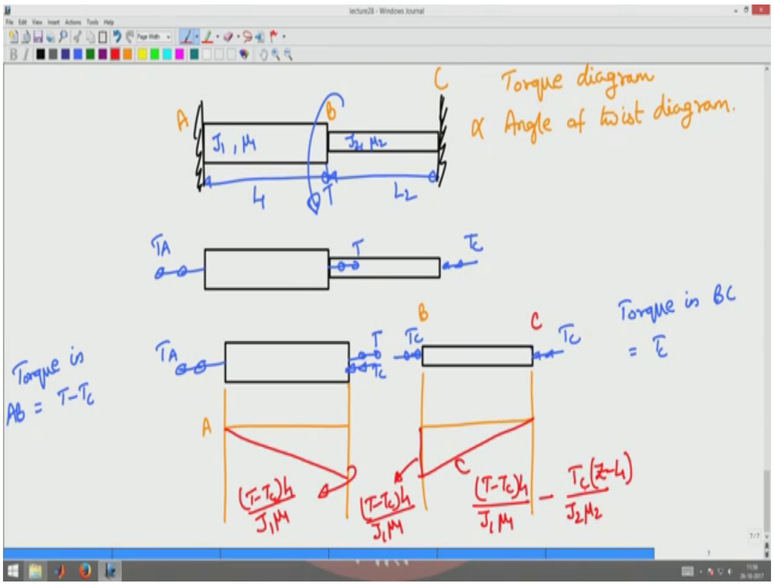

Now, let us assume that this has J 1 as a polar moment of inertia, this has a length L 1 and mu 1 is the shear modulus of this material let us assume this as having a length L 2 J 2 and mu two are the polar moment of inertia and the shear modulus of this second shaft, let us also assume that this is A B and C. I am interested in finding what is the torque diagram and angle of twist diagram I am interested in finding these two ok.

Now, for this I know I operate torque T this is statically indeterminate problem, because there are two fixed ends draw the free body diagram of the structure there is a torque T as a torque t applied here there will be a reaction torque TC there and there will be a reaction torque TA here ok. Further separating out these blocks for the separating out these blocks I will have TC acting here. So, to maintain a equilibrium there will be a TC acting like this and here there is a T and TC opposing that because T C has to be equilibrate there and then there will be a TA acting like this TA acting like that.

So, in this member effectively in member A B the effective torque in member A B is torque in A B is T minus T C and torque in B C is T C. So, A is fixed. So, how does the angle of twist vary angle of twist will vary from here in this member the angle of twist is going to vary like this end is fixed A is fixed.

So, the angle of twist is going to vary linearly like this, where this value will be given by T minus T C by J 1 into L 1 1 and now this end is fixed. So, basically in this section it should vary here the angle of twist at this end C and this end B the angle to twist at the end B is the same as this value T minus T C L 1 by J 1 mu 1.

And then this T C is opposing that angle twist this angle of twist is anti-clockwise T C a clockwise rotation. So, it will decrease down to 0 because that end is fixed in there ok. And this slope of decrease is given by the slope of this decreasing function is given by T minus T C into L 1 by J 1 mu 1 minus T C into z by J 2 mu 2 ok. Z if I minus z from a it will be z minus L 1 z minus L 1.

Now, I know that c twist has to be 0, I use that condition to find what T C is ok.

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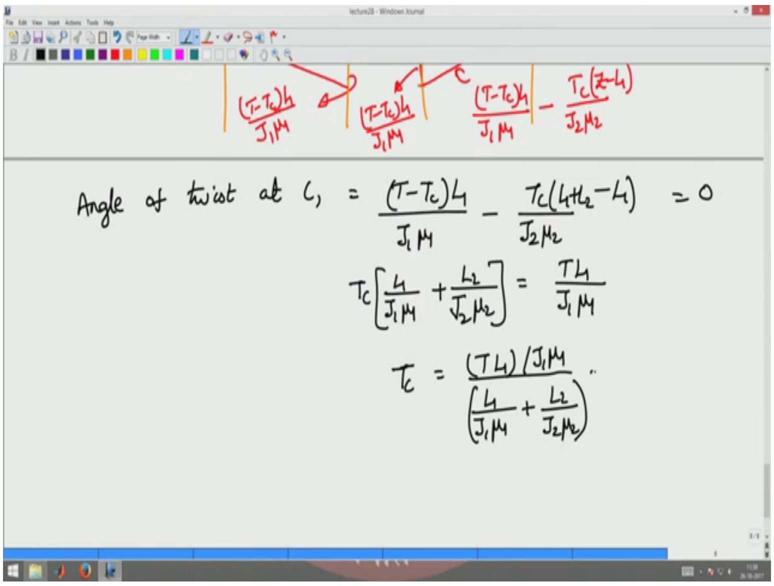

So, to twist at C angle of twist at C at C is given by T minus T C into L 1 by J 1 mu 1 minus T C by J 2 mu 2 into L 1 plus L 2 minus L 1 must be equal to 0. So, that gives you T C into L 1 by J 1 mu 1 plus L 2 by J 2 mu 2 must be equal to T into L 1 by J 1 mu 1 right ok.

So, that gives me T C to be T into L 1 by J 1 mu 1 into L 1 by J 1 mu 1 plus L 2 by J 2 mu 2. So, we have drawn the angle of twist diagram here combining these two will get the angle of combining these two commanding, these two diagrams superposing these two will get the angle of twist diagram for the entire member; now angle of twist diagram would be the following.

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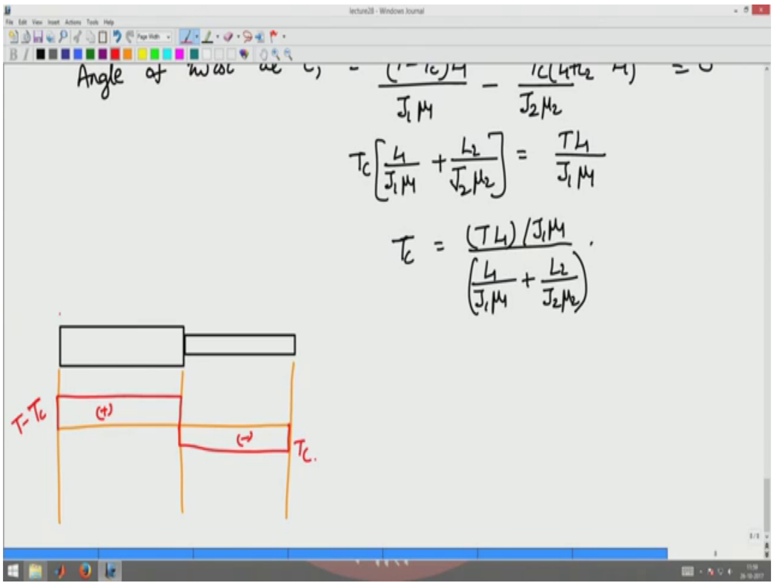

The torsion diagram would be the following the torsion diagram would be the following in this member there is a negative torsion T C is an ideal torsion T C and here there is T minus T C. So, basically this is a torsion diagram for this structure which is fixed at both the ends ok.

So, what we have seen is, you have seen how to analyze for twisting of a closed section in this class, we worked out to solve problems one of a one end fixed the operator at the other end and the second problem is step shaft which is fixed at both the ends ok.

Thank you.