Mechanics of Material
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Stresses and displacement due to torsion or displacement
Lecture – 79
Torsion equation

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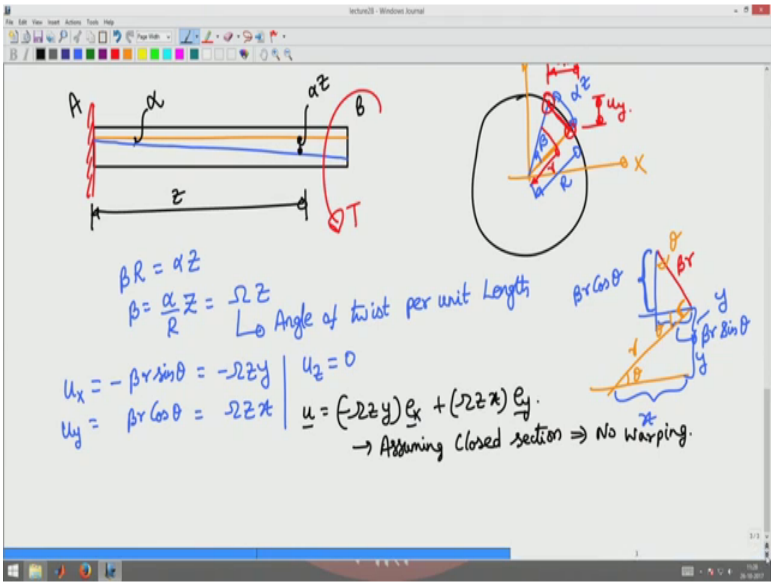

Now, I found a displacement field, next what I want to do is I want to find the gradient of the displacement field ok.

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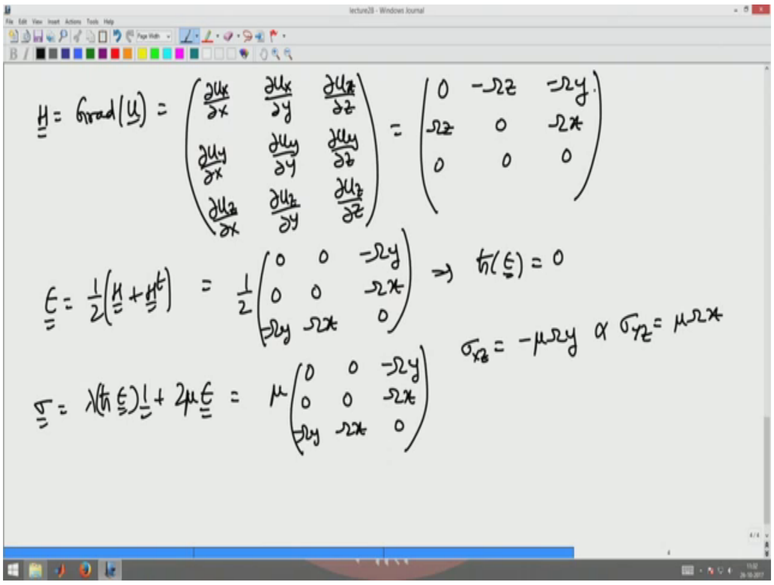

Next I want to find H, which is gradient of the displacement field U similar to what we have been doing all along. So, this would be dou U x by dou x, dou x by dou y, dou x by dou z dou y by dou x, dou U y by dou y dou U y by dou z dou U z by dou x dou U z by dou y dou U z by dou z; if I substitute the components of displacement from here into these components, I will get 0 minus omega z, minus omega y, omega z 0 omega x 0 0 0 right ok.

Now, I want to compute the linear strain epsilon, there is one half H plus H transpose. So, that will be given by the epsilon x y term will get cancelled because its q symmetric in x y component. So, I will get it as 0 0 minus omega y 0 0 omega x minus omega y omega x 0 into half.

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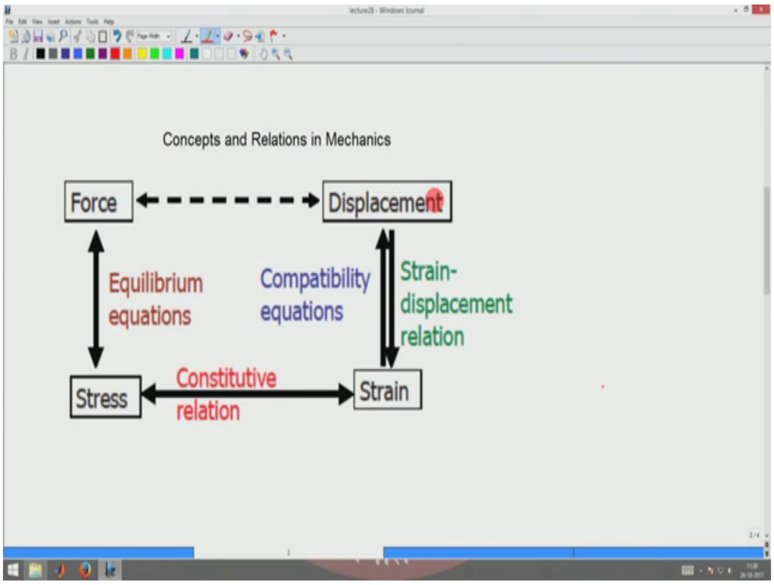

Now, next I have to from our scheme of things here I found a displacement field suitable for the boundary problem that I am studying. I use a strain displacement relationship to get this strain make sure I have to use the cast stimulization to get the stress, and then I have to check whether the equilibrium equation are satisfied or not that is the scheme of things now.

So, my stress expression is given by lambda trace epsilon identity plus 2 mu epsilon and I find here from here that trace of epsilon is 0 and hence this becomes mu times 0 0 minus omega y 0 0 omega x minus omega y omega x 0.

Now, I found this stress I found in particular sigma x z component to be minus mu omega y and sigma y z component to be mu omega x ok. Now, I will see what happens to the equilibrium equations next the equilibrium equations are I am assuming there are no body forces and the body is in static equilibrium.

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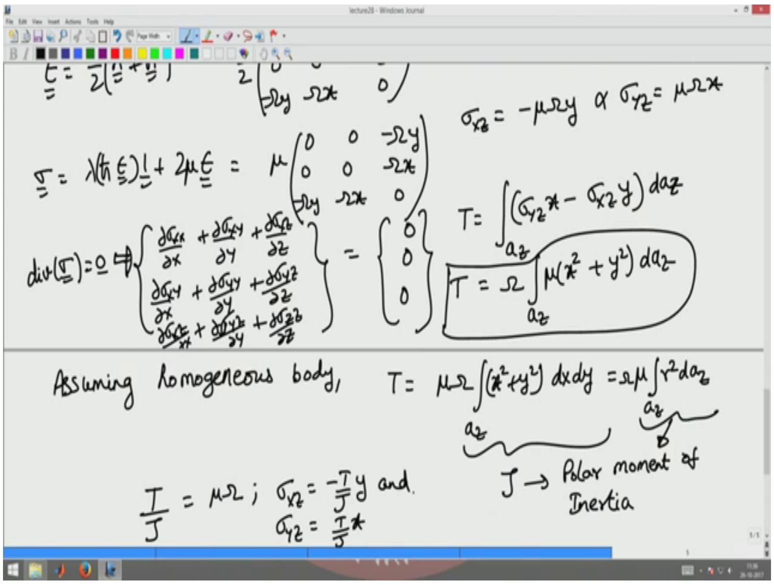

So, the equilibrium equations will reduce to recovering balance of sigma equal to 0 which is rho sigma x x by dou x plus dou sigma x y by dou y plus dou sigma x z by dou z, dou sigma x y by dou x plus dou sigma y y by dou y plus dou sigma y z by dou z, dou sigma x z by dou x plus dou sigma y z by dou y plus dou sigma z z by dou z.

This you can see that sigma x z is not a function of z or x and hence the first equation is identically 0, the other two components are 0 the second equation sigma x y is 0 sigma y y is 0 sigma y z is not a again a function of z.

So, that is 0 the third equation sigma x z is not a function of x sigma y z is not a function of y and sigma z z is 0. So, that is also 0. So, you are recommended divergence of sigma be 0 is satisfied by this stress field now the unknown is your (Refer Time: 04:56) is per unit length omega, which you will find from the requirement that the torque is given by sigma y z into x minus sigma x z into y d a z a z ok. I substitute for the expression for the stresses into this equation to get the torque to be equal to omega times mu x square plus y square d a z a z ok.

Now, I use this expression to find the relative torque to the angle of twist omega ok. Now for the simplifications possible if I assume that the member is are the structures homogeneous assuming homogenous body, I have T to be given by mu omega; I can pull them mu outside as soon the body is homogeneous the shear modulus mu it is not a function of x y or z.

So, I move that outside that would be x square plus y square d x d x d y a z ok. This is the definition of polar moment of inertia j which is the polar moment of inertia that is x square plus y square is r square. So, you can write it as mu omega mu r square d a a z and this r square d a is the polar moment of inertia J ok.

So, I have now the expression T by J equal to mu omega mu omega now I have to find what is the effective shear stress in the cross section right; I now I know that from the expression for shear stresses (Refer Time: 07:23) I know that sigma x z is minus T by J into y and sigma y z is T by J into x and sigma y z is T by J into x ok. Now what I am interested I am not interested in the components of the shear stress, but I am interested in the magnitude of the shear stress right.

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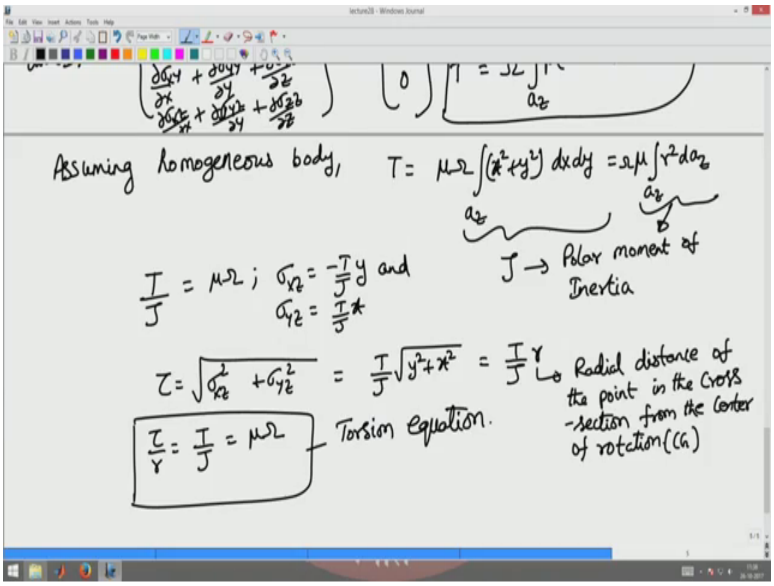

So, the magnitude of the shear stress tau is gone by square root of sigma x z square plus sigma y z square, because these are acting in the same plane ok. So, this will be T by J root of y square plus x square y square plus x square is a radial distance of the point ok.

So, this is nothing, but T by J into r, where r is the radial distance of the point in the cross section from the centre of rotation, which is usually the c g or the cross section. So, combining these equations I have tau by r equal to T by J equal to mu omega ok. So, this is called as the torsion equation this called as the torsion equation.