Mechanics of Material

Dr. U. Saravanan
Department of Civil Engineering
Indian Institute of Technology, Madras

Stresses and displacement due to torsion or inflation
Lecture – 78
Displacement field

Welcome to the 28th lecture in Mechanics of Materials. Till now we have solved two kinds of boundary problems one involving an axial member, where in the axial stresses were uniformly distributed across the cross section. The second was a beam problem bending problems were and we add both the axial stresses and shear stresses coming in the cross section.

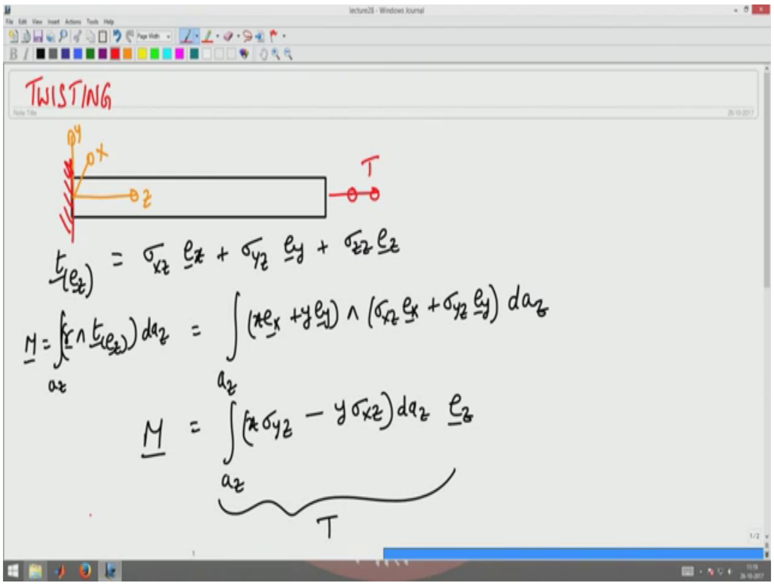
We saw to compute the actual stresses in the cross section which will vary linearly across the depth of the cross section and then we saw the shear stresses will vary parabolically across the depth of the cross section, for certain types of cross sections. So, basically now we will move on from bending problems to twisting problems, were and we saw even bending problems that there will be a torsional moment or twisting happening in the cross section were load is not applied through the shear centre of the cross section right.

So, now we look at what happens when the load is not applied to the shear centre of a cross section are when there is twisting. We will would not go into the specific case of load not being applied to the shear centre, but we will look up problem simple problems and twisting where your shaft which is rotating at a some RPM or you applying a twisting moment to a beam to eccentrically apply load and so on ok.

Now in contrast to what we have been seeing till now were in the predominant stress were the actual stress now the predominant stress are the shear stresses in the cross in the in the structure ok. Now due to twisting what happens is, you have a bar which you twist by applying twisting moment at one end. So, you can see it forms kind of an helix kind of a shape here. So, this is what happens when you apply a twisting moment to a bar ok. That is I am fixing this end and I am rotating this section and then this bar is getting twisted.

So, what happens now is two sections which are like this rotates related to each other ok. Two sections are like this rotates related to each other producing a shear stresses in the plane of the cross section, and we are interested in finding what these shear stresses are independent of the cross section ok. Now to analyze this, I am going to now change the coordinate system from what I have been using till now to a different coordinate system this is just to make you conversion that any coordinate system that you use you have to be able to adapt to a different coordinate system in a problem.

(Refer Slide Time: 02:55)



So, till now we have been using x y z over in a particular fashion, but now I will orient x y z in a different manner ok. I have this cross section and for me now z is the axis of the beam y still remains the vertical, but the x is oriented like this. So, this is the axis that I am going to use.

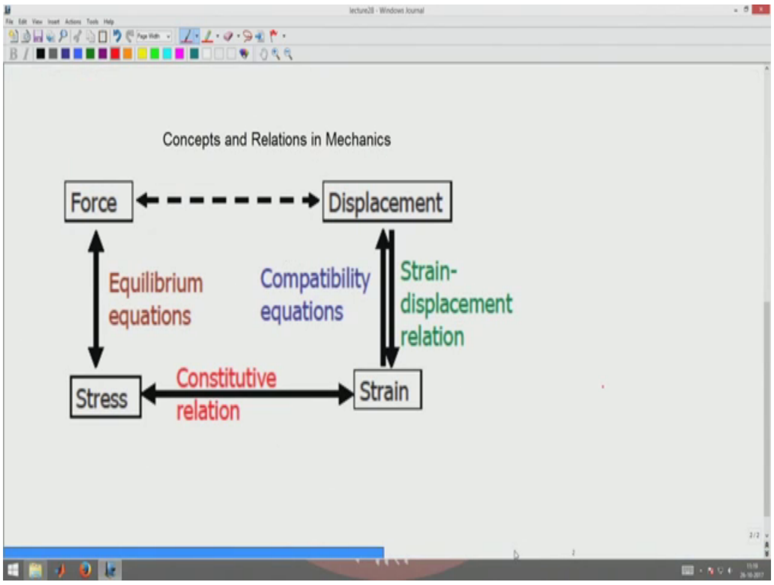
Now, what is the what was the torsional moment in case of the axis ordinate like; when the x was oriented along the axis of the beam the moment along x direction was a torsional moment, the moment along the z direction was the bending moment. On the other and now the moment around the z axis is a torsional moment and moment around the x axis is the bending moment ok.

Now we have to find the expression for the torsional moment when they orientation is like this. So, as I did before for the bending moment, I cut a section exposed the torsion along the z direction and find what is the torsional moment ok. So, now ts of e z; I am not going to go into step by step procedure, you have to refer back to our first lecture and beam bending were we did this is respect to x to get this expressions t e z will be given by sigma x z, e x, y z, e y plus sigma z z, e z ok. Now at a given cross section I want to find the moment due to this exposed traction. So, the moment equation would be r cross ts of e z integrated over d a d a z now because a z is the is that is the axis of the beam ok.

So, now in this case r would be now it will be a z, r would be x e x plus y e y. I am not bother about z in this case because I am interested only in the torsional moment otherwise I would have plus z naught e z cross sigma x z, e x plus sigma y z, e y and I am also going to ignore sigma z z e z because I am not interested in the actual stresses either I assume there are no actual stresses in this phase d a z. So, this will be integral x sigma y z minus y sigma x z, d a z, a z, e z this is the torsional moment m z moment this is a torsional moment or this is the torsional moment in the b.

Now, let us assume that I fix this end for torsion, now I am applying a torsional moment t here the double arrow indicates the torsional moment ok. When applying a torsioanl moment t in there and I am interested in finding what will be the stress distribution in this structure now as always what we do?

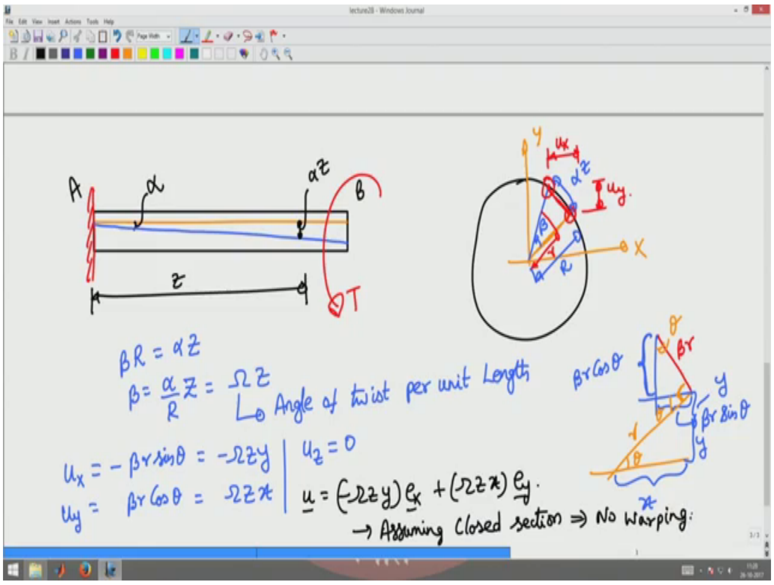
(Refer Slide Time: 06:33)



We start with a displacement field, we start with the displacement field whose the same displacement relationship to get the strain who is a consolation to get this stress equilibrium equations to get the forces or moments or what are unknown displacement component that we have.

So, to assume a displacement field we have to I have put the (Refer Time: 06:54) is on a possible more of displacement. So, to us set what we are going to do is, I am going to see what happens when I twist a bar. So, when I twist your bar and I fixed this end and I twist the bar like this.

(Refer Slide Time: 07:07)



I apply a torque t like that then what happens to a straight line here what happens to the straight line there ok. This straight line is now going to deform into an inclined line, essentially an inclined line because I am assuming the displacements are small; we saw that we derive a strain we always assumed that the component of displacement is small displacement gradient is small. So, that I can do whatever I am doing right I can use a linear extend measure to do the calculations.

So, since the displacement are small unlike what I showed in the beginning of this lecture why the displacements where large where you can see the displacement, see now you cannot see the displacement and hence when the displacement is small assume that the straight end deforms in to under a straight line like that which is inclined, basically since it is fixed at the end A since it is fix at the end A that end will wont rotate the section, which is farther away from the fixed end will rotate more and more as the length increases ok.

So, basically let us assume that this rotation is given by omega let that rotation be alpha, then at any section this rotation of the cross section from here to here would be this rotation would be alpha times z, again I am assuming that the rotation is small. So, I can write this as alpha times z because my axis are again oriented along this is the z distance my axis of the beam is oriented along the z axis this is z distance. So, that will be alpha times z.

Now, let us look at the cross section what happens? I have some arbitrary cross section I have x and y the cross section, this is surface of the cross section what you see as elevation. So, basically what happens to a line element this line element is it rotates in to, it rotates goes in there ok, but this rotation is alpha times z ok. And if this angle were to be beta this angle were to be beta and if the same distance were to be capital R if this distance were to be capital R then beta times R capital R should be equal to alpha times Z right ok.

Now, then beta would be alpha by R into Z this alpha by R is called will replace that symbol as omega Z and this alpha is called as angle of twist per unit length ok.

Now, this is what has happened in the process what has happened is a point which is here the point which is here has moved to a point here let us move to a point there. So, it has a u x displacement and it has a u y displacement which we have to find ok. Again since beta is small I wrote it as beta times R which means this distance is beta times R. So, I know that this distance is any point (Refer Time: 11:10) also would be beta times R this is a some point in here which is at a distance of r small r ok.

So, that will be the displacement now that displacement I have to resolve into u x and u y displacement basically by assuming this displacement as beta times r I am approximating the arc of a circle to this secant length. The arc length to the secant length ok; So, now, I am interested in finding this distance this distance and this distance and what I know is I know that this makes an angle theta and this distance is r if that distance is r and this is 90 degrees I know this is 90 degrees ok.

Then I can say that this angle would be theta right because this angle is theta this angle is theta by parallel lines and this angle will be 90 minus theta and that angle will be theta ok. So, this distance here would be given by beta r into cos of theta right. And this distance here would be given by beta r into sin of theta

Now, so, u x component or displacement that is a negative displacement it moves opposite to the orientation of the axis hence it is minus beta r sin theta r this is minus omega z r sin theta is nothing, but y r sin theta is this distance y ok. So, that is minus omega z y.

Similarly, u y displacement is given by beta r cos theta this a positive displacement because it is displacing along the direction of the coordinate system and this will be omega z substituting for beta and r cos theta is nothing, but x r cos theta is nothing, but x this distance is x, and this distance is y, and hence that is what you got as u x and u y displacement ok.

You are assuming that there is no displacement along the z direction u z is zero because I am assuming that there is no warping essentially ok. What I am assuming is there is no warping as there is no out of plane displacement and the hence u z is 0. Putting all this together you get your displacement field u to be minus omega z y e x plus omega z x e y because I assume no warping in other sense what we are doing is we are doing deriving a equation for what is called as a closed section ok. Assuming closed section implying no warping.