Mechanics of Material
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Stresses and deflection in beams not loaded about principal axis
Lecture – 76
Load not about principal axis

Welcome to lecture 27 of mechanics of materials, the last lecture we saw how to derive general equation for bi-directional bending that is, when the load is not applied along one of the principal axis the cross section. We saw how to get the deflections and stresses in particular, we saw that the expression for stress is given by this expression in here and the expression for the corresponding deflection along the y and z direction is given in here.

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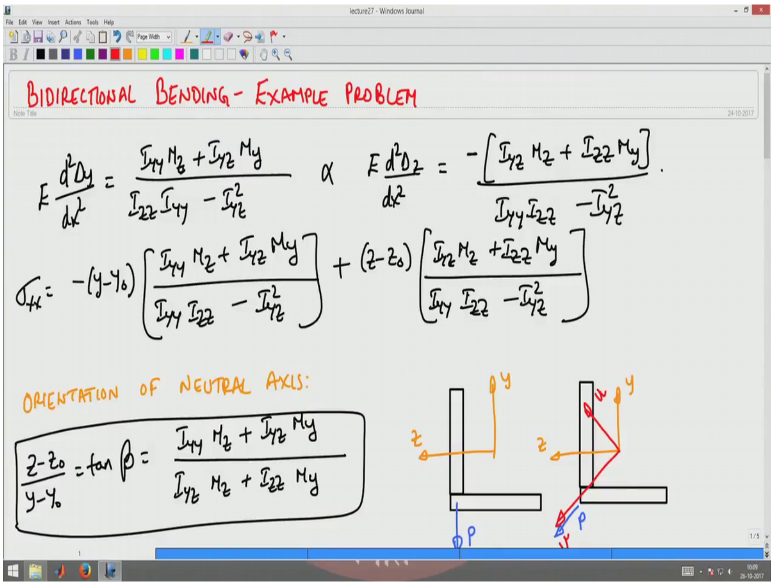

So, we also saw that now, the neutral axis will not be aligned along with the axis of the problem that we have chosen, but it will be inclined to the axis of the cross section that we have chosen and that expression is given by, the expression given here tan beta equal to I yy into M z plus I yz into My divided by I yz into M z plus I zz into M y. In today's lecture will apply these concepts to solve 2 problems, one is an equal angle loaded about one of it is legs like this and an equal angle loaded about it is principal axis one of the principal axis like that.

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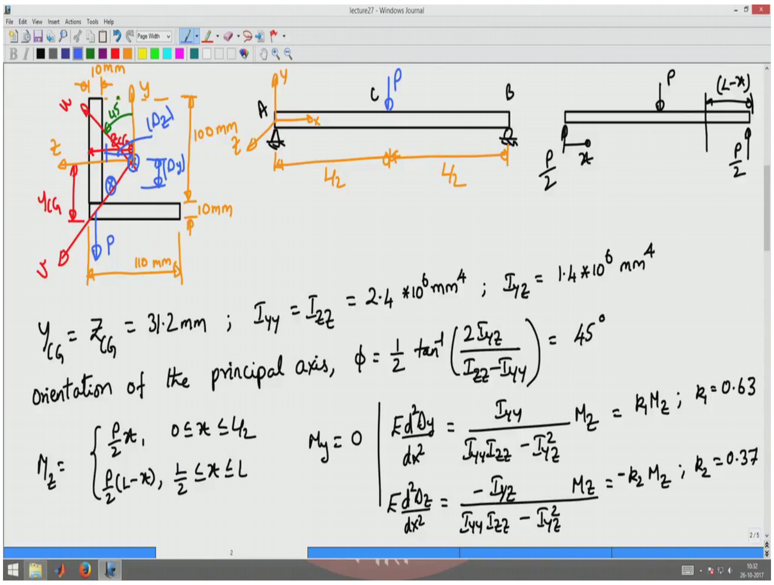

So, this what we are going to do in today's lecture. In particular we are interested in solving the problem, where in you have a simply supported beam subject to a point load in the middle, and this point load is acting on one of the legs in the first problem of the equal angle section. The dimensions of the angular given here, each of the angle is like this 100 mm in length and 10 mm in thickness. In lecture 25 we saw how to compute the moment of inertia on the center of this cross section. So, I am not going to repeat how I am going to compute the center and the moment of inertia of this cross section, but give the end the cells.

So, for this cross section the y CG is since is an equal angle cross section, you know that the centroid of y centroid and z centroid would be same, and that is equal to 31.2 mm that is this distance is y CG and this distance is it z CG what are the mark 31.2 mm.

Now, similarly for this cross section for the oriented axis I yy to be equal to I zz, again because of a sequel angle and that will be equal to 2.4 into 10 power 6-millimeter power 4 and then I yz is going to be 1.4 into 10 power 6-millimeter power 4, for this oriented orientation of the coordinate system y NC directions. So, basically these are the section properties of the cross section.

Now, a first thing we want to find this, we want to find what is the orientation of the principal axis? So, to find the orientation of the principal axis, we saw and lecture 25 again this is given by 1 half tan inverse of 2 times I yz divided by I zz minus I yy since I zz and I yy are same this happens to be 45 degrees, because tan inverse of infinity is 90 degrees and this happens to be 45 degrees. So, the principal axis is oriented like this at the rotation of 45 degrees to this, this will be your u and this will be your v for this angle is 45 degrees the phi. So, that is the orientation of principal axis.

Now, for this loading for this orientation of the coordinate system, you know that the moment that is produced is a M z moment and you can find what is the support reaction, the support reactions are going to be, the simply supported beam the support reactions are going to be P by 2, I am not going to go into the direction of this, the load is acting and the center. So, you take a moment about one of the edges one of the supports say, at A you get that from the moment balance will get the reaction at B to be P by 2 and from the vertical first equilibrium, then the reaction at a has to be P by 2 the same thing can be inferred from the symmetry of the problem.

So, there is a free body diagram of this beam and hence, the variation of M z moment Mc variation of M z moment with x is given by, P by 2 into x for 0 less than x less than L by 2 and P by 2 into L minus x for regents L by 2 less than x less than L, here x is measured from the end A x is measured from here, this is x any section here would be x. So, any section on this segment section here, will have a distance from the same d as L minus x.

So, basically now for a section greater than L by 2 it will be P by 2 into L minus x will be the moment, that comes in at that section point, by now you should be familiar with writing this equations you have done that thrice in this course. So, you have to be conversant in writing these equations. So, M z is this now going back to our expression for the deflections, you will find that M y is 0 for this M z is this and M y is 0, because a load is acting along the y direction. So, M y is 0 for this case you will find that E times d square delta y by dx square is given by I yy divided by I yy I zz minus I yz squared into M z M y is 0.

So, I have used this fact and put the bending moment equations, into this equation in here I put the bending moment M z and M y into the equation in here, and hence got that this I will write it has k 1 times M z, where k 1 for our cross section has 0.63. Similarly, E times d square delta z by dx square from the equation above will be, minus I yz divided by I yy I zz minus I yz squared times M z this will write it as k 2 times M z bar k 2 is 0.37 for our cross section.

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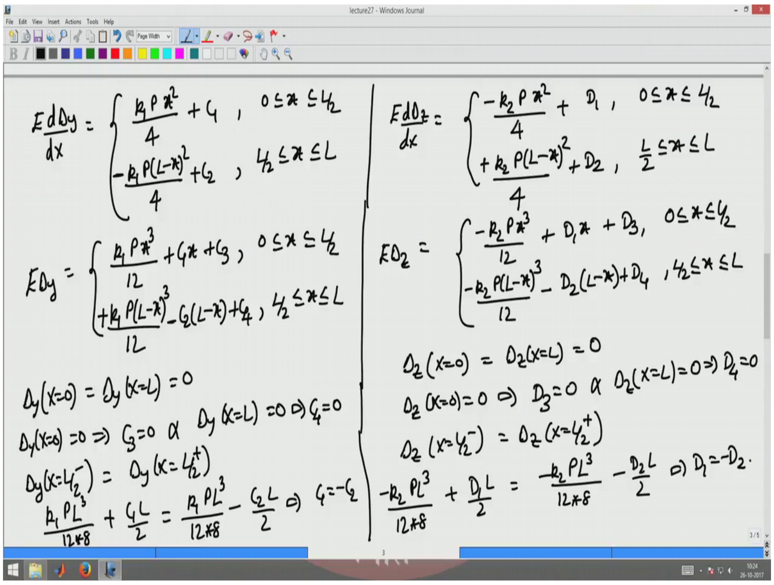

Now, proceeding further we got d square delta y and d square delta z dx square. So, we want to find the deflection and the stresses. So, the deflection is given by, to get the deflection d delta y, by dx is given by I substitute for M z into this equation. So, that will be P x square by 4 plus C 1 for 0 less than x less than L by 2 P L minus x square by 4 minus plus C 2 for l by 2 less than x less than L. Similarly, E d delta z by dx would be that should be a negative sign there minus k 2 M z, because it is a negative sign in there will be minus k 2 P x square by 4 plus D 1 for 0 less than x less than L by 2 minus k 2 P L minus x whole square by 4 negative sign for the differentiation makes is positive plus D 2 for 0 less than for l by 2 less than x less than L.

Now, integrating this once more, I get E times delta y to be k 1 P x cube by 12 plus C one x plus C 3 0 less than x less than L by 2 minus k 1 P L minus x cube by 12 and negative sign there coming in, because of the integration. So, that become positive I write C 2 x as minus C 2 the L minus x plus C 4 for convenience that, you will see later shortly now this is same as C 2 x, right? I have added C 2 L. So, C 4 would be C 4 minus C 2 L will adjust itself that is a constant.

Similarly, E times delta z is given by minus k 2 P x cubed by 12 plus d 1 x plus d 3, for 0 less than x less than L by 2 minus k 2 P to L minus x cube by 12 plus, I do the same trick here minus D 2 into L minus x plus D 4 for L by 2 less than x less than L. Next, I have to find the constant C 1 to C 4 and D 1 to D 4 for which I apply the boundary conditions, I have simply support boundary condition means, delta y at x equal to 0 is equal to delta y at x equal to L, that has to be equal to 0 same thing in z direction I do not allow displacement in y or z. So, delta z at x equal to 0 and x equal to L has to be 0.

Here I am assuming it is simply supporting both y and z direction, but it is not always necessary, it can happen that it can be simply supported in y and fixed in z or vice versa are fixed in both. So, that depends upon the problem at an so, this is 0. So, these conditions will imply that delta y at x equal to 0 equal to 0 implies C 3 is 0 and delta y at x equal to L equal to 0 would imply C 4 is 0, similarly here delta z at x equal to 0 equal to 0 implies D 3 is 0 and delta z at x equal to L equal to 0 implies D 4 is equal to 0.

So, now we have to find constant C 1 and C 2, that comes from the continuity condition, the first continuity condition that will n forces delta y at x equal to L by 2 minus should be equal to delta y at x equal to L by 2 plus. So, what does this mean? This means k 1 P L cube by 12 into 8 plus C 1 into L by 2 C 3 0, must be equal to k 1 into P L cube by 12 into 8 minus C 2 into L by 2 this implies C 1 is equal to minus C 2.

Similarly, here delta z at x equal to L by 2 minus must be equal to delta z at x equal to L by 2 plus this is the continuity condition a del by 2, this will imply minus k 2 into P L cube by 12 into 8 plus D 1 into L by 2 must be equal to minus k 2 into P into L cube by 12 into 8 minus D 2 l by 2 this again implies D 1 must be equal to minus D 2.

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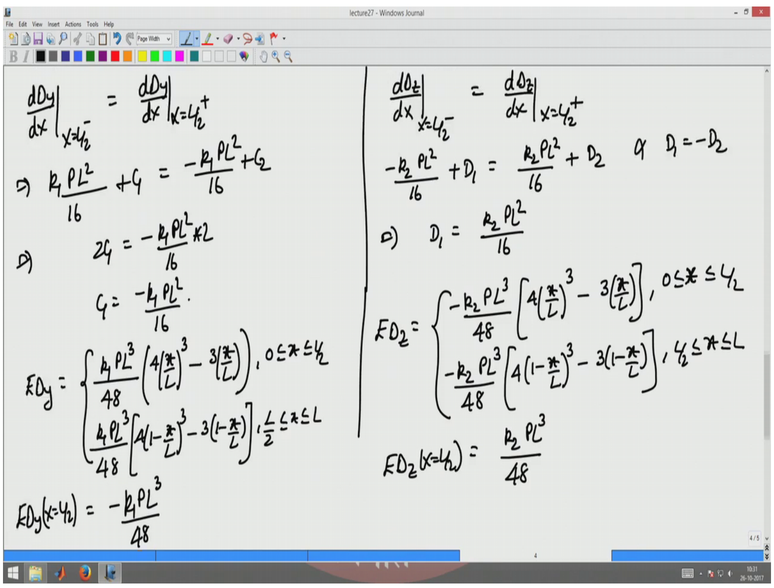

Now, moving forward we want to apply continuity condition on d delta y by dx. So, continuity condition second continuity condition that will be using is, d delta y by dx at x equal to L by 2 minus, must be equal to d delta y by dx at x equal to L by 2 plus. So, this condition will tell us that, from the equation here it will give us this implies k 1 into P L square by 16 plus, C 1 must be equal to minus k 1 p L square by 16 plus C 2, already you have C 1 equal to minus C 2. So, this would imply that this with this implies, 2 C 1 is minus k 1 P L square by 16 into 2 or C 1 is minus k 1 P L square by 60.

Similarly, applying the condition here, d delta z by d y d delta z by dx at x equal to L by 2 minus, must be equal to P delta z by dx at x equal to L by 2 plus, you will get minus k 2 P L square by 16 plus D 1 must be equal to k 2, P L square by 16 plus D 2, here the sign differs because there is a negative sign in here you can see that this as differing sign from, what you add here hence the sign will differ.

So, this implies along with the condition that, D 1 the condition D 1 is equal to minus D 2 from the continuity of the displacement from, where you got that condition D 1 equal to minus D 2 here you got the condition D 1 equal to minus D 2 these 2 conditions together will tell you that d 1 is k 2 P L square by 16.

Now, substituting all these values of constants into the deflection equation, you have E times delta y to be given by k 1 P L cube by 48 into 4 x by L whole cube minus 3 x by L, in the region 0 less than x less than L by 2 and I have k 1 P L cube by 48, 4 into 1 minus x by L the whole cube minus 3 into 1 minus x by L in the region L by 2 less than x less than L.

Similarly, E times delta z is given by minus k 2 P L cube by 48, 4 into x by L the whole cube minus 3 into x by L in the region 0 less than x less than L by 2 and minus k 2 P L cube by 48 into 4 into x by L whole cube minus 3 into 1 minus x by L in the region L by 2 less than x less than L.

Next, we are interested in finding what is E delta? Y at x equal to L by 2 this will find is minus k 1 P. So, you will find this minus k 1 P L cube by 48 and similarly here, you will find that E of delta z at x equal to L by 2 is k 2 P L cube by 48 because, it is 1 by 2 minus 3 by 2 1 by 2 minus 3 by 2 is minus 1. So, this one with the negative sign becomes positive, but delta z and it is negative for delta y, because there is no negative sign in front of delta y.

So, what does this mean, let us interpret this result with respect to the cross section, let us go back to the cross section, what deflection we got was, negative displacement in y and positive displacement in z. Which means the CG from here, will move to a point from the CG from here moves to a point somewhere here, where in this displacement is delta y absolute value of delta y and this displacement is absolute value of delta z that is the displacement.

So, what that says it moves down and it moves in this a direction, in the is a direction it moves down and out of plane out of the board white board. So, that is what it happen so now, this is one way of solving this problem, was another way of solving this problem and I can resolve this loads into the principal axis and I can do repeat the same thing, then what I will get this I will get the deflection along u and v directions..

I have to then dissolve the deflection along u and v directions. So, y and z direction, now what we are going to do is. So, I was going to resolve these forces along the principal directions. So, this P when this all along y u and v would be undersold along u and v it would be along v, this is going to be P by root 2, because it is 45-degree rotation that is what we saw for the principal axis from the given axis.

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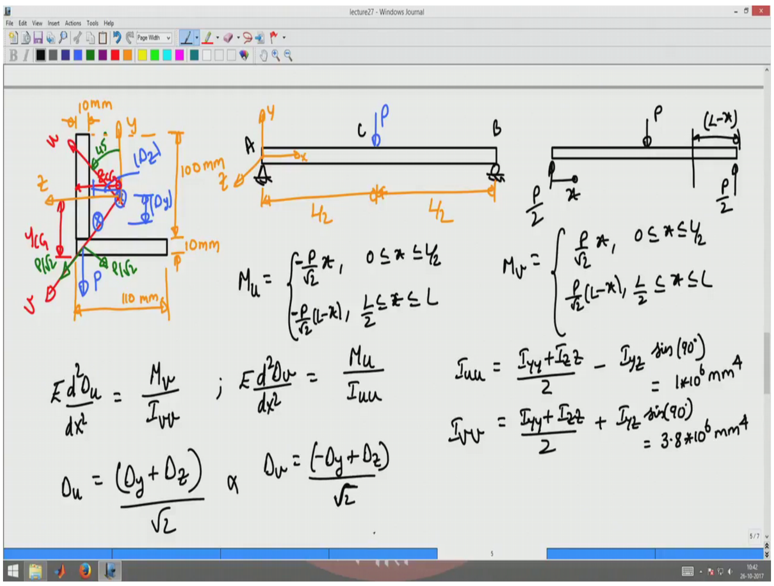

So, this will be P by root 2 and along the u is going to be P by root 2. So, we will have moments Mu and Mv Mu is given by P by root 2 into x for 0 less than x less than L by 2 and it will be given by, P by root 2 into L minus x for 0 less then, for x L by 2 less than x less than L. Similarly, Mv moment given by P by root 2 into x again, 0 less than x less than L by 2 minus P by root 2 into L minus x for L by 2 less than x less than L.

Now, let us see how I got the Mu moment, Mu moment would be the load along u direction, load along v direction multiplied by the liver arm along the x direction, that will give me Mu moment. So, I have a load I have this angle cross section, in which the load is acting like this. So, this load is going to produce a Mu moment like this, the lower arm is along x that is a load acting like this. So, there will be a moment Mu produce, because of that load and that will be a negative moment and Mv moment is, the moment produced you to the load acting along u due to a lever arm along x direction, that will be the Mv moment that will be given by this expression in here.

So, that is the Mv moment there, now you know that I uv is 0 since there is a principal axis, and hence E times d square delta E square delta u by dx square, would be given by Mv divided by I vv. Now, we have to find I uu and I vv. We will do that shortly and similarly E times d square delta v by dx square is given by Mu divided by I uu Mu.

Now, I know that I can use more circle equations to transform my to find I uu and I vv I zz and I zz and I yy was same for this cross section. So, the expression for I uu would become, I yy plus I zz by 2 minus I yz in to sin 2 phi, which is sin 2 times 45 degrees. So, it will be 2 times 90 degrees that is 1 so, this is I yy and I zz are same. So, I did not have a cast to feed term. So now, this boils down to 1 into 10 power 6 mm power 4. Similarly, I vv is given by I yy plus I zz by 2 plus I yz into sin 90 degrees. So, this will be 3.8 into 10 power 6-millimeter power 4.

Now, I have to repeat the steps that, I did for integrating the moments, similar to what I did in the previous case and find delta u and delta v, and you can show them the delta u would be given by delta u would be given by delta y that, we found in the previous expression as delta z by root 2 and delta v would be given by minus delta y plus delta z by root 2.

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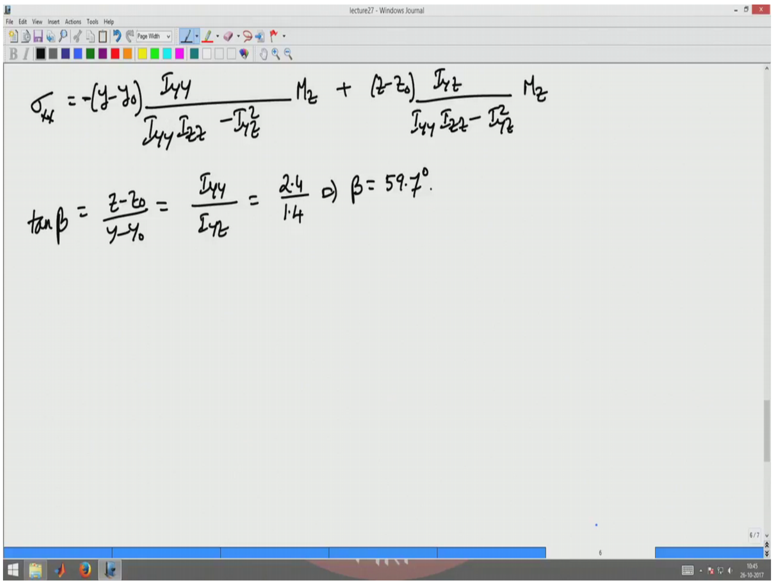

Now, I leave this as an exercise for you to work it out and show. Now, moving on to the stress expression the stress sigma xx, now would be given by from the equation that we had before from this expression we will get this stress to be given by, y minus y naught in to I yy divided by I yy into I zz minus I yz squared into Mz the negative sign in here, plus z minus z naught into I yz divided by I yy I zz minus I yz squared Mz, we got the expression for Mz.

So, you know how the actual sets will vary along the axis of the beam and there is a variation along the cross section of the beam, in the cross section of the beam off interest is the neutral axis, that is the axis about which sigma xx is 0. So, you will find that the orientation of the neutral axis tan beta, is given by z minus z naught by y minus y naught. Which is nothing but I yy divided by I yz and this is 2.4 divided by 1.4, this would imply that beta if you do the inverse of tan it will be 59.7 degrees.

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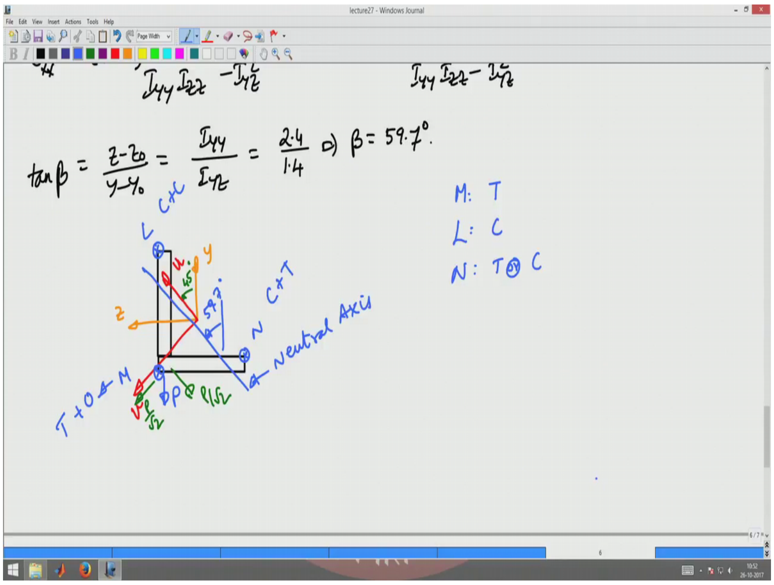

So, now if you look at the angle section the principle axis is oriented at 45 degrees this is 45 degrees but the neutral axis is oriented at 59.7 degrees that is the neutral axis is oriented in some line like that this is the neutral axis and its orientation from the vertical is 59.7 degrees. So now, what happens below the neutral axis, you are operating low like this and which points will be in tension and compression is a next in the we want to see. So, it is convenient to see that, when you resolve the forces into the principal axis orientations.

So, basically when I apply a lower like this, we are interested in finding what are the nature of stresses at points P at points L M and the N? To find the nature of stress at M, when I resolve a load along the principal axis direction, you find that the P by root 2 acting along v will produce compression at M, will produce tension at M and the load acting along the u direction, P by root 2 load acting along u direction that be the neutral axis. So, it low on produce any tension at M so, at M would be T plus 0.

Similarly, at L the lower acting along v will produce compression, load acting along the direction of the principal direction v will produce compression. Load acting along the direction of u will produce this is the P by root 2, load acting like this as a P by root 2 load acting like that. So, that is going to also produce compression at L. So, that we compression and compression at L whereas, at N the load acting along v produces compression because, it is about the u axis and load acting along u, will produce tension because it is below the v axis, it will produce tension there.

So, basically now you will find that M would be in tension L would be in compression and the N can be in tension or compression depending upon which is dominating because, it is combination of a branch cell session the compressive stress, that depends how the orientation of the neutral axis is, what is orientation of neutral axis is. So, this kind of analysis you should be able to do to quickly tell whether, a given point will be in tension or compression due to bending stresses.