Mechanics of Material
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Stresses and deflection in beams not loaded about principal axis
Lecture – 75
Neutral axis

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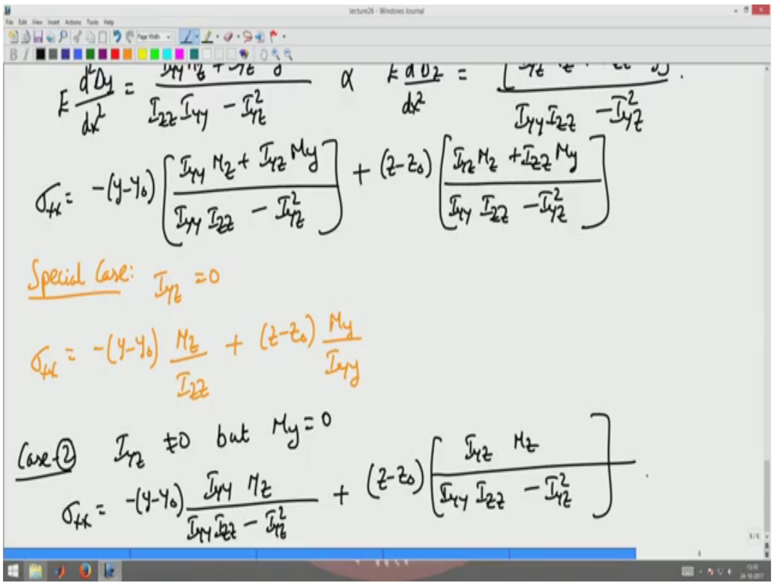

Now, in the scenario, the next thing that is of interest is to find the neutral axis of the cross section, when the beam was bending along one direction, the neutral axis was a straight and parallel to the perpendicular to the line of the action of the load. Now in the bending is about 2 directions, neutral axis will be an inclined neutral axis inclined to both the directions of the load.

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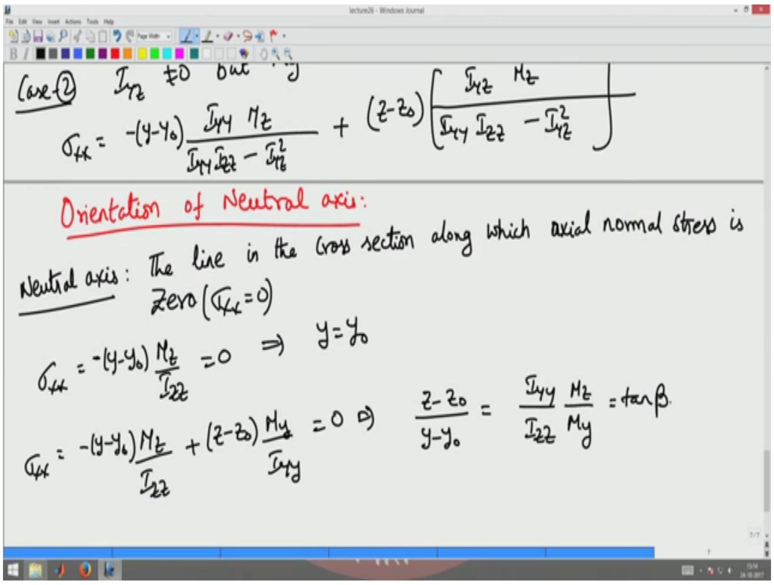

So, we want to find; what is the orientation of the neutral axis. We saw that neutral axis is the axis about which the stress is 0, we saw the definition that neutral axis is the line in the cross section.

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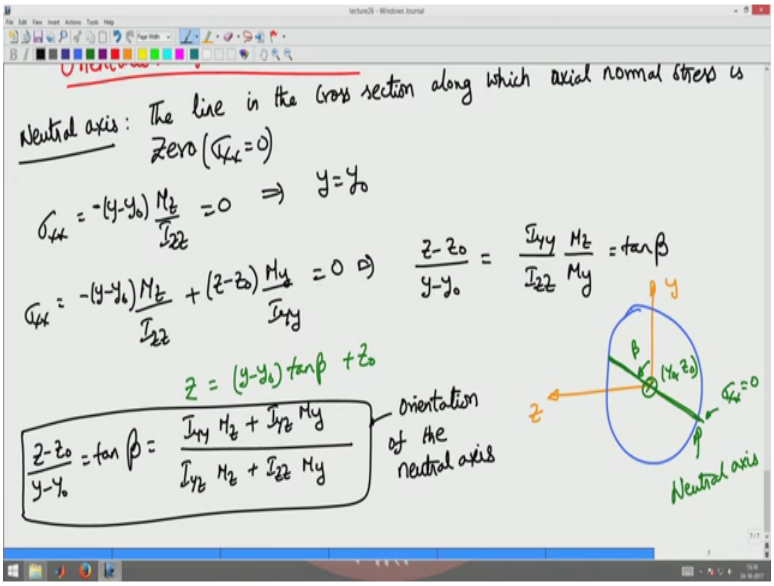

Along which normal axial normal stress is 0. So, you know that sigma xx has to be 0 along the neutral axis location, when the beam was bending along only one direction, when you have sigma xx given by y minus y naught M z by I zz you know if this is 0. This implies y equal to y naught and that was the definition of the neutral axis. Now when sigma xx is given by a complicated equation or is given by this case special case where given by the special case where I yz is 0 and thus moment along both the directions M y and M z moment, then what is the neutral axis neutral axis would be minus y minus y naught sign here y minus y naught M z by I zz plus z minus z naught M y by I yy has to be equal to 0.

This implies z minus z naught by y minus y naught must be equal to I yy by I zz into M z by M y. Now what is this y equal to M x plus c where c is the origin? Now this is the slope of the line this is tan beta that is for all cross section for all cross section with y and g like this, this equation in here this equation in here represents the equation of a line whose slope is beta equation represents equation of this line whose slope is beta measured from y like this whose slope is beta measured from y like that given by that equation in here.

So, basically you have rearranging this equation z is equal to y minus y naught tan beta plus z naught where this portion is y naught z naught that is y naught z naught, if you are taking an origin y naught and z naught was 0 and this represents the tan beta. So, this stress sigma xx is 0 about this line about this line the stress sigma xx is 0. So, this is the neutral axis of the cross section, unless unlike the shear center unlike the center area cross section the neutral axis depends upon the loading, it depends upon the bending moment M y and M z that comes in the cross section then orientation of neutral axis will change with the loading, this is the important point to be noted neutral axis is not a geometric property is a lower dependent property.

So, basically that represent the equation of that line and that is a neutral axis that is the axis about which the actual stresses are 0. Now let us consider a general case in the general case what happens the general case, I will get tan beta as again z minus z naught by y minus y naught which will be tan beta in the general case would be the general case I mean this expression, I am going to now equate this to 0 and write z minus z naught by y minus y naught as this divided by this expression in here.

So, that will be I yy M z plus I yz M y divided by I yz M z plus I zz My. So, this is the general orientation of the neutral axis this expression gives expression for orientation of the neutral axis. So, this gives a orientation of the neutral axis. So, now, what we have seen is you have seen; what happens when there is loading along the principal axis both the principal axis of the cross section, we draw the expression for stress and we generalized the case where the loading was operating in one direction not necessarily along the principal axis direction and we got an expression for that and then we found a expression for the neutral axis.

Now, in a practical problem there are 2 ways of solving a problem you can choose any axis for a given orientation of the load and then you can say that find all the moment of inertia cross moment of inertia and I yy I zz moment of inertias and then substitute in the general expression and solve for a problem in a general manner the second approach is you find the principal axis of the cross section result in loads or moments along the principal axis of the cross section and use only the principal axis the first equation as we derived in today's lecture to solve the problem in the next lecture.

What we will do is we will take a specific example for section and solve it when it is loaded about one of the lags and when it is loaded about one of the principal directions to find a displacement as well as the moment and stress expressions.

Thank you.