Mechanics of Material
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Stresses and deflection in beams not loaded about principal axis
Lecture – 74
Bending equation about arbitrary axis

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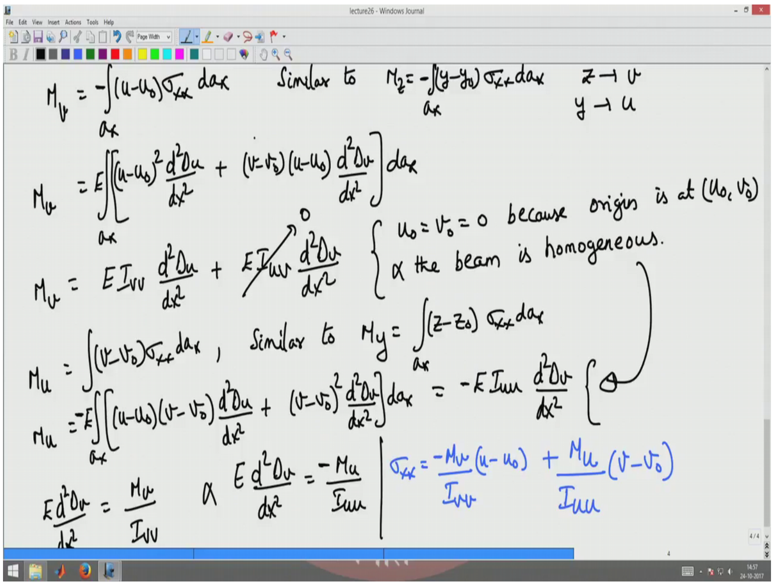

Next what do you want to do is, see what happens when Iuv is not 0 when Iuv is not 0 that is i am dealing with the given axis y and z what happens.

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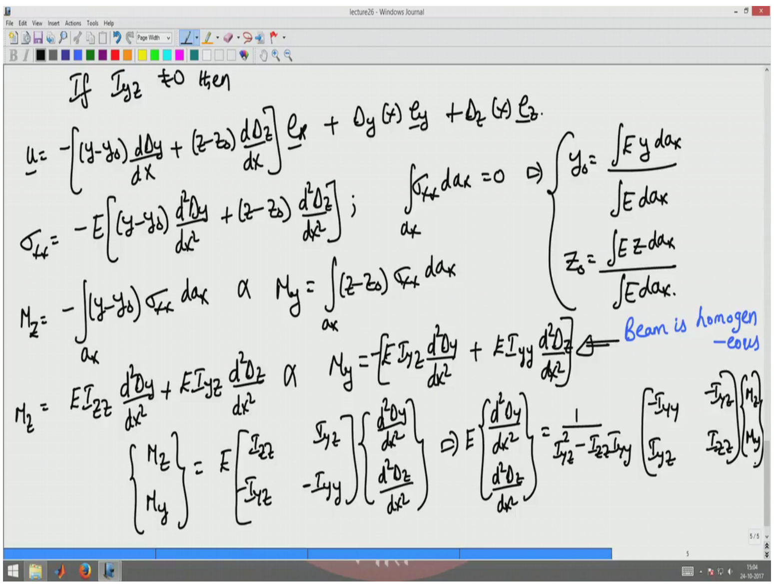

If Iyz is not equal to 0, then what are we did till here holds this equation still holds and still i will get y naught as this z naught as this. And then whatever we did till their holds. So, when Iyz is not 0 i have u in general given by minus y minus y naught d square delta y d delta y by dx plus z minus z naught d delta z by dx ex plus delta y function of x Ey plus delta z function of x Ez, expression field similar to u and v i replace u and v with y and z here remains are same.

Then My sigma xx would be minus e times y minus y naught d square delta y by dx square plus z minus z naught times d square delta z by dx square. This will be My stress and then the condition that integral sigma xx dax ax has to be 0 would imply that y naught is given by integral E times y dx here are not assumed the beam is homogeneous E dax and My z naught would be integral E times z dax by integral E times dax. This will get from this equation and then you have moment equation Mz to be given by minus integral y minus y naught sigma xx dax from our definition the very first class when we began the beam bending problem, this will be the definition of Mz and you are definition of My as integral z minus z naught sigma xx dax right ax.

Now, this will tell us that Mz is given by E times I zz d square delta y by dx square plus E times I yz d square delta z by dx square and M y would be given by E times I yz d square delta y by dx square plus E times I d square delta z by dx square. This assumes beam is homogeneous. That is made of the same material there is no difference in tension and compression or there is no difference of materials in tension and compression.

Now, what I have to do is i have to solve these 2 equations to get delta y by dx square d square delta z by dx square in terms of My and Mz. So, there will be a negative sign in here. So now, to solve this equation i class it in this following form Mz My is equal to E times Izz Iyz minus Iyz Iyy into d square delta y by dx square d square delta z by dx square.

Now, I invert this 2 by 2 matrix from here i get d square delta y by dx square d square delta z by dx square into E is 1 by Iyz squared minus Izz Iyy into minus Iyy minus Iyz, Iyz to Izz into Mz My.

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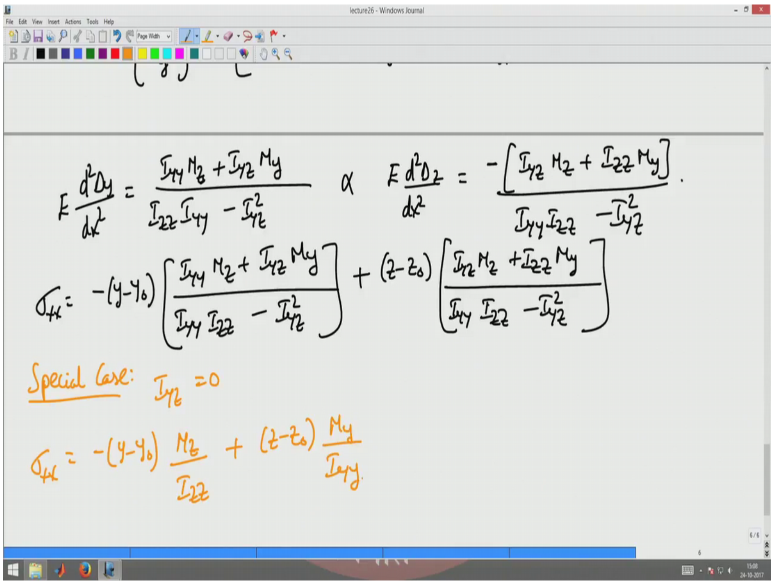

Now from here I get that d square delta y by dx square into E is Iyy into Mz plus Iyz into My divided by Izz into Iyy minus Iyz squared and E times d square delta z by dx square would be minus Iyz into Mz plus Izz into My divided by Iyy Izz minus Iyz square.

So, here found these 2 now going back to the expression for the stress i will get it as sigma xx would be going back to the expression for the stress will be minus y minus y naught into Iyy to Mz plus Iyz into My divided by Iyy Izz minus Iyz squared plus z minus z naught into Iyz Mz plus Izz My divided by Iyy Izz minus Iyz square. Here and I comes because it is minus y minus y naught into E times d square delta y by dx square that negative sign comes here. Here this is minus E times z minus z naught d square delta y by delta z square and hence this negative sign does not propagate into here. So, that is the expression was stress.

Now, let us look at special cases let us look at the case where Iyz is 0. Special case where Iyz is 0 then your expression was sigma xx becomes minus y minus y naught Mz by Izz plus z minus z naught into My by Iyy. This is the same expression that we got in your it is same expression that we got in your for sigma xx except that u and v roles are change to y and z. Now except that u and v roles are change to y and v, now u is y v is z. So, this Mz by Izz as a negative sign from see that Mz by Izz as a negative sign in here. So, this same as what we got there. So, it is consistent now.

The second special case that we are going to consider is when there is only along one direction case 2.

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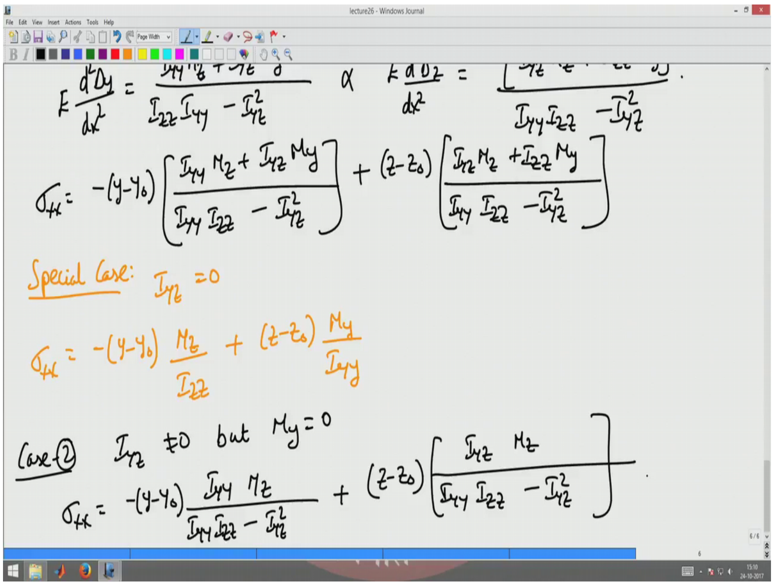

Iyz not equal to 0, but My is 0 applied force along only one direction, but i did not apply it about the principal direction now sigma xx would be minus y minus y naught My is 0. So, I have Iyy divided by Iyy Izz minus Iyz squared into Mz plus z minus z naught into Iyz into Mz divided by Iyy Izz minus Iyz squared. So now, this is where you have only one moment Mz moment, but the stress where is both along y and z direction.