Mechanics of Material

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Stresses and deflection in beams not loaded about principal axis
Lecture – 72
Example: Angle section

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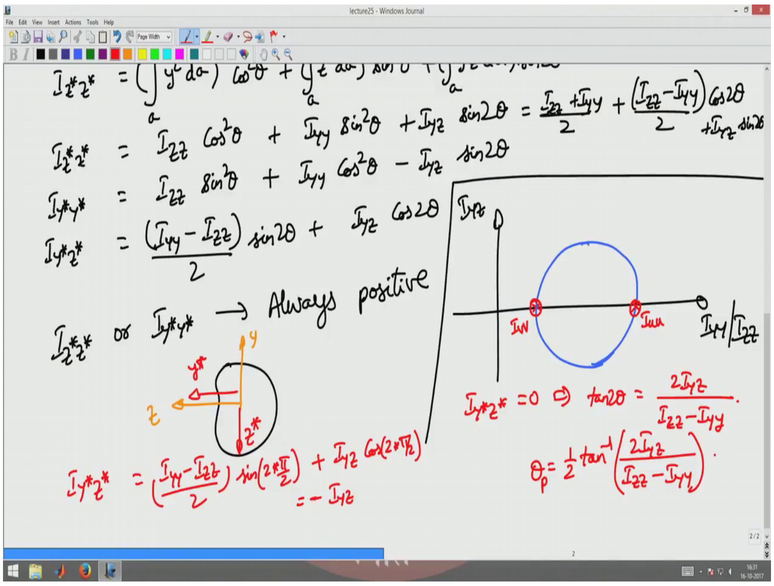

Now let us do an application of this for an angle section. Let us say the cross section is in the shape of an angle.

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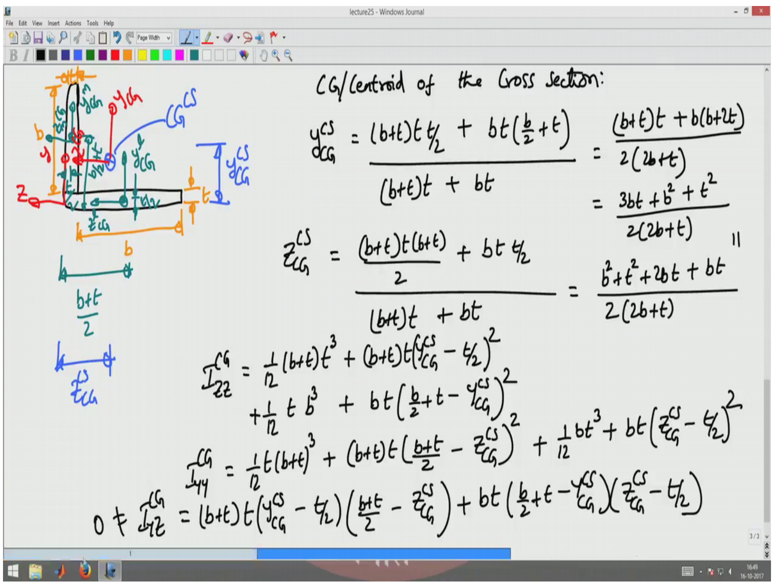

In the shape of an angle let us assume that this distance is b as this distance this is also b let us assume that this thickness is t and this thickness is also t. Let us assume that thickness is also t. Now the axis is first you have to find where the CG of the cross section is right, first you have to find the CG of the cross-section CG or centroid of the cross section of the cross section is something you have to find first.

Let us assume for that initially your y and z axis is oriented like this this is y and this is the z axis, but rest of this i am finding the centroid of the cross section. Now I know that for let me close this let me assume that i am closing this and the centroid of this leg of the angle section would be somewhere here and that would be let us say that this centroid is y leg centroid and let us say this is z leg centroid.

And for this now for this portion of the leg there will be another centroid which is here. So, let us say this is y m CG is that m CG. Now the distance of this from this end the centroid of this leg from this end would be t plus b by 2. And then the distance of this centroid from here to there would be b by 2 plus t. Now this portion of the centroid this distance would be t by 2 as just this distance this distance also is going to be t by 2.

So, to find the y centroid yCG of the cross section, what should i do? I have to take the leg area that is b plus t into t times the center of the leg region along the y direction is this distance is along the y direction that is t by 2 as we are estimated here. So, yCG would be the area of the leg b plus t into t into the y distances would be t by 2 plus the area of the web that is this area. That will be b into t into the y distance from the origin that is the corner most point to the centroid of that that will be this distance, that will be that distance which will be b by 2 plus t.

So, we are (Refer Time: 04:54) for 2 areas and hence entire cross section divided by a total area of the cross section, which will be b plus t into t plus b into t. Now I can simplify this equation to be given by i can cancel a t throughout. So, that will become b plus t into t plus b into b plus 2 t divided by 2 into 2 b plus t. So, this will simplify to further 3 bt plus b square plus t square divided by 2 into 2 b plus t.

Now, similarly let me find the z centroid of the cross section. Now what is that we have found the y centroid, what is that? That is nothing but this location that is centroid of the cross section this is CG of cross section and this distance is y CG cross section. And now what you are going to do is move this axis from that to here y CG to z CG you want to move it to there. Now similarly I am interested in finding what this what this distance is that CG of cross section is from that end. So, towards that what i have to do again? I have to find the area of the bottom leg b plus t into t into the z distance of with centroid which would be this distance b plus t by 2.

So, it will be b plus t by 2 plus for the vertical member for this member for this vertical member it will be plus b into t which is the area of that member into it is z distance of the centroid which is this distance in here that is this t by 2 in there so into t by 2 divided by b plus t into t plus bt. So, this again cancel the t throughout and this will become b square plus t square plus 2 bt plus bt divided by 2 into 2 b plus t which is nothing but same as that that is what we expect because the angle is a symmetrical angle. So, let us see g and y CG has to be same.

Now, next what I am interested is i am interested in finding I zz about the CG axis CG of the cross-section axis. Now I am going to use parallel axis theorem, i know that the moment of inertia of this horizontal leg would be 1 by 12 into b plus t into t cube that is about this is z axis. Now I have to that add this y CG CS squared the area of that leg area of this leg. So, that be plus using parallel axis theorem area of the leg is b plus t into t into y CG cross section minus t by 2 the whole square. That is for the horizontal leg for a vertical leg i will have plus 1 by 12 t into b cube plus b into t is a area of that into i have to look at the shift from here to here.

This distance would be b by 2 plus t this entire distance is b by 2 plus t. That is this distance minus y CG cross section will give me this distance in here. So, that will be b by 2 plus t minus yCG cross section whole square. I am not going to simplify this i am going to leave it as it because it is has no meaningful purpose now for our discussions, similarly I yy is for the horizontal leg it will be 1 by 12 t into b plus t the whole cube plus b plus t into t into the horizontal shift into r 2 multiplied by this distance squared that distance square would be b plus t by 2 is this distance minus is that CGs CS will give me that distance.

So, that will be b plus t by 2 minus z CG CS will be that distance squared plus for the vertical leg it will it is going to be 1 by 12, this is bt cube instead of bd cube by to do db cube, now because i am looking at moment of inertia about the y axis. So, it is going to be bt cube plus that area b into t into their shift, that is i have to look at this distance for this shift along the z axis along the y axis moment of inertia along the y axis about the y axis i have to look at this shift.

So, basically that will be is that CG CS minus t by 2 the whole square. You know all these things it did not matter for us why they are the shift was positive along the positive direction or the negative direction because you are squaring the distance. So, I always took the distance to be a positive number, but when it comes to Iyz and i want to compute Iyz over the centroid, i want to compute Iyz about the central axis. Now this will be the moment of inertia about the centroid of the cross section which is 0 for this case and there will be only the shifting term. So, it matters whether the shift is positive or negative along the x and y direction y and z direction.

So, let us look what it is for the horizontal member let us look what is a shift for this horizontal member. So, it is area b plus t into t because the basic moment of inertia Iyz about the centroid of the cross section is 0 for this orientation of the axis for this rectangular section. So, that is that is 0. So, I am left only shifting term, shifting term would be for the horizontal leg you add this and this are the shifting terms. So, basically now that will be y cross section CG minus t by 2 into b plus t by 2 minus z cross section CS.

Now, should this be positive or negative is the question. So, what happens? Both the shifts are in the negative y direction and in the negative z direction. So, negative will balance out to give positive direction positive value, similarly next what you have to do is you have to look at the shift in the vertical leg. So, the area that vertical leg is b times t. Now what is the distance you have to use reuse this distance b plus t minus yCGs and zCGs minus t by 2 is the distance you have to use. So, that will be b by 2 plus t minus y cross section CG into z cross section CG minus t by 2.

Now, what happens now both the shifts are in the positively positioned coordinate of that of the vertical member. So, hence this also remains positive so the shift terms are positive so you get Iyz to be given by this expression in here by the angle section equal angle section. Now so the point is Iyz is not equal to 0 it has some finite value. Now you have to find the orientation such that Iyz goes to 0 that will give you the principal orientation of the a principal axis.

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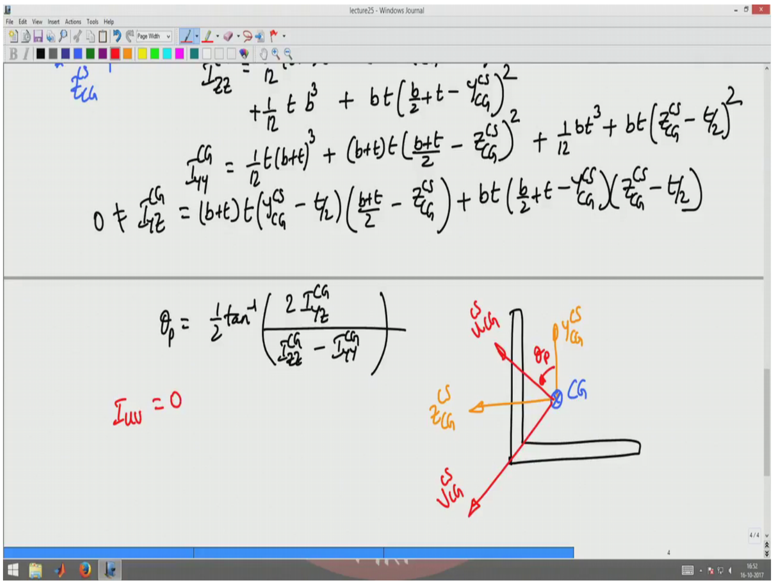

So, that axis would be theta P the orientation of the principal axis will be a 1 half tan inverse 2 times Iyz of the cross centroidal axis divided by Izz a central axis minus Iyy a centroidal axis.

So, for this angle section you will find that such an axis would be oriented like this, such an axis would have orientation this was y cross section with CG z cross section CG and the centroid was this is the CG of the cross section. Now what happens is the principal axis this oriented at an angle theta P given by there that is a clockwise rotation. So, the principal axis this is u CG this will be v CG cross section this angle would be theta P and for this angle I uv would be 0. So, for equal angle that is how that is the orientation of the principal axis.

Thank you.