Mechanics of Material

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Stresses and deflection in beams not loaded about principal axis
Lecture - 71
Moment of Inertia about arbitrarily oriented axis

Welcome to the 25th lecture in mechanics of materials. Till now, we have been looking at problems are and the loading was confined to one of the axis and the axis was what is called as a principal axis. So, that the bending or the displacement takes place along in a particular plane or the displacement predominant displacement is along the y axis ok

So, now we saw that in when we introduced bending moments that can be bending either in the z direction or y direction, depending upon what is the deflection would be in y direction or the z direction, right. So, now, we will go about and see what happens when the loading is on both the axis or the loading is not about the principal axis ok.

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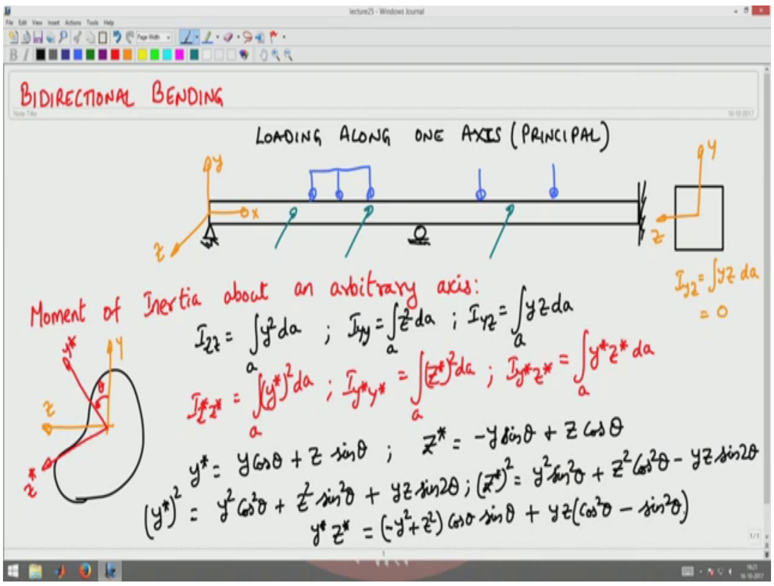

So, till now what we have been looking at this case? Till now what you have been looking at this case and you are beam with some support conditions inch roller or a fixed end subjected to some loading along one direction. Now what we will do is we will apart from there being loading on the y direction that can be load coming on the z direction also that is there can be loads acting like this in this beam, there can be loads are acting like that on that beam, then we will see, what happens, how do you analyze such a scenario and also in all these cases that we construct; till now the cross moment of inertia are I y z defined as I y z defined as integral y z d a was 0, it need not be always true for some oriented axis.

So, let us first understand, how to compute moment of inertia about an arbitrary axis; not necessarily a principal axis and then we will see that there will exists always an axis in about which I y z will be 0. So, we will determine that axis and then we will see how to analyze the beams subjected the bending moment through that axis loading on through that axis.

So, first what we are going to do now is I going to look at moment of inertia about an arbitrary axis. So, let me assume that the cross section is in the shape of a potato and I have some axis y and z taken like this, for this I can compute I for this, I can compute I zz that is a moment of inertia about the z axis which is y square d a area of the cross section and compute I yy, what is this integral z square d a where integrated over the area of the cross section and I can define I y z, what is this integral y z d a integrated over the area of the cross section.

Now, say as usual like, what we did for the stress component strain component? This moment of inertia about an axis oriented in a particular direction, but to offers did not agree on this orientation of the axis. So, let us ask the question, what happens if someone comes and rotate this axis the anticlockwise sense to y star and z star where this angle is theta?

Now, what happens; I z star z star would be integral y square y star square d a same area of cross section and I y star y star would be integral z star square d a, again over the area of the cross section and I y star z star is going to be integral y star z star d a area of the cross section.

Now, what is a mapping? I will take from y to y star or y star to y and z y star can be written as y star can be written as y cos theta plus z sin theta, just I taking the component of forces y star. So, y cos theta plus z sin theta and z star z star would be minus y sin theta plus z cos theta, I substitute for y star and z star into the expressions here. So, what I do is I substitute this expression for y star in here this expression for z star in here and y star and z star in there and I expand these equations.

Then what do I get? I get I have to find y star squared that will be y square cos square theta plus z square sin square theta plus y z sin 2 theta that is 2 y z cos theta sin theta is yz sin 2 theta and similarly, I get the z star squared as y square sin square theta plus z square cos square theta minus y z sin 2 theta and y star z star would be y square minus y square plus z square cos theta sin theta plus y z cos square theta minus sin square theta minus y square sin square theta a cos square theta minus sin square theta.

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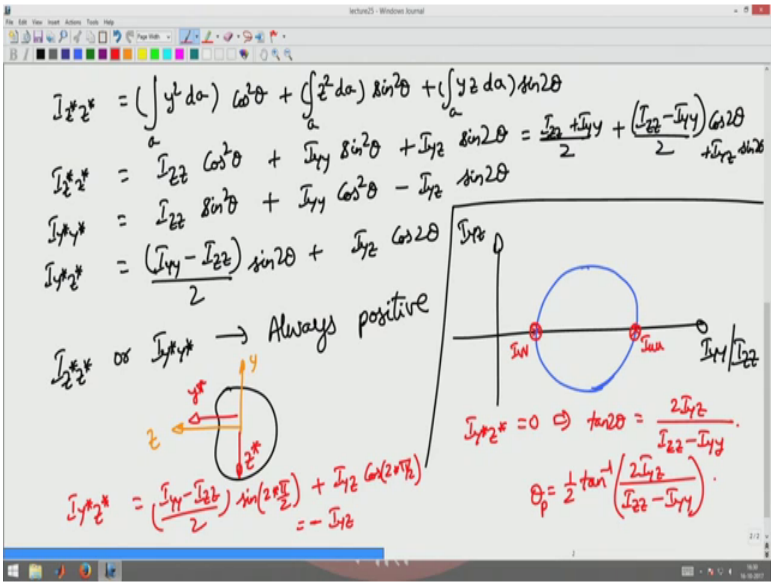

Now, so, what does the expression I z star z star become this will become integral y square d a a cos square theta plus z square d a sin square theta plus integral y z d a sin 2 theta this is nothing, but I zz cos square theta plus I y y sin square theta plus I y z sin 2 theta, right, this I z star z star.

Similarly, you can see that I y star y star what is this z star d a which is this is z square d a is square of this what is this. So, if I substitute in there what I will get is I will get I zz sin square theta plus I yy cos square theta minus I y z sin 2 theta.

Similarly I y star z star is z star is I yy minus I zz by 2 sin 2 theta plus I y z cos 2 theta where I use the fact that I use the following equations, I used the fact that cos square theta minus sin square theta is cos 2 theta. Now I use the fact that cos theta sin theta is sin 2 theta by 2 and minus y square d a be I zz minus I zz and z square d a would be I yy that is what I used to find this expression in here.

Now, what we have to understand next is we have to answer the question; how does this transformation look like? This looks like similar to the Mohr circle equations, right. So, this equation, I can rewrite it as I zz plus I yy by 2 plus I zz minus I yy by 2 cos 2 theta plus I yz sin 2 theta, this is exactly the Mohr circle equation right and this is also same form. So, I z star z star I y star y star plays the role of the normal strain or the normal stress whereas, I y star z star plays the role of the shear stress or the shear strain right. So, these transform just like the Mohr circle equations.

Now, what is the property of I normal what is the property of I z star z star or I y star y star they are going to be always positive this will be always positive right because it is a integration of a positive function y square is a positive function. So, they will always be positive. So, the Mohr circle equations if you look at if I plot I x I y y or I zz along the x axis and I y z along the y axis then my circle would be something like this my circle would be something like that where in, there will be positive I y z and negative I yz positive for a certain orientation of the axis.

Now, let us understand why that happens physically why there is a positive I y z for some orientation of the axis and negative I y z for the some other orientation of the axis. Now let us say I have this cross section some potato with y and z initially oriented like this.

Now, say I change the cross section to some orientation y star pointing like that and g star pointing like this that is I rotated the coordinate system by ninety degrees we are rotate the coordinate system by ninety degrees what happens what is I y star z star substitute ninety in there it will because, it will be I yy minus I zz by 2 the sign of a theta rotation the coordinate system is true theta in the Mohr circle. So, theta is pi by 2. So, 2 into pi by 2 plus I y z into cos 2 into pi by 2 this is minus I y z.

So, because sin pi is 0 cos pi is minus one. So, this minus I y z. So, if I y z is positive I y star z star will be negative if I y z is negative I y star z star will be positive there is a flippin sign by for a ninety degree rotation of the coordinate system there is a flipping sign for a 90 degree rotation of the coordinate system.

So, what this means is before that that had be a point where I yz will go to 0 that is between 0 to 90 degrees that will exist a orientation of the axis where I yz is 0 because I y, this is a continuous function of theta and it changes sign as the theta varies from 0 to ninety degrees and the ends there will be orientation of the axis for which I y z will be 0 that I y z axis where it is 0 is the principal axis.

So, there will be corresponding to the principal axis I will be a point which has say a principal axis is denoted by u and v there will be a point where it has a minimum moment of inertia iv v in the maximum moment of inertia. So, I uu. So, we are interested in finding this I uu and I v v and the orientation of u and v with respect to x and y and z. So, that is the principal moment of inertia the orientation of the axis of the principal moment of inertia.

Now, what is that the question is what is the orientation for which I y z will be 0 or I y star z star is 0 implies tan 2 theta is 2 times I y z divided by I zz minus I yy. So, the orientation of the uv axis with respect to the y and z axis is given by this theta's principal axis is 1 half tan inverse 2 times I y z divided by I zz minus I y y.

This is similar to finding the principal stresses or principal strains and the orientation of those principals stress and principal strains, again you have to be careful when you do this times operation the angle has to be between 0 and 360 degrees. So, it matters whether numerator is positive denominator is positive numerator is negative and denominator is negative or both are negative all these four things will determine which quadrant the axis is what is the theta for that corresponding. So, you have to be careful there.