

Mechanics of Material
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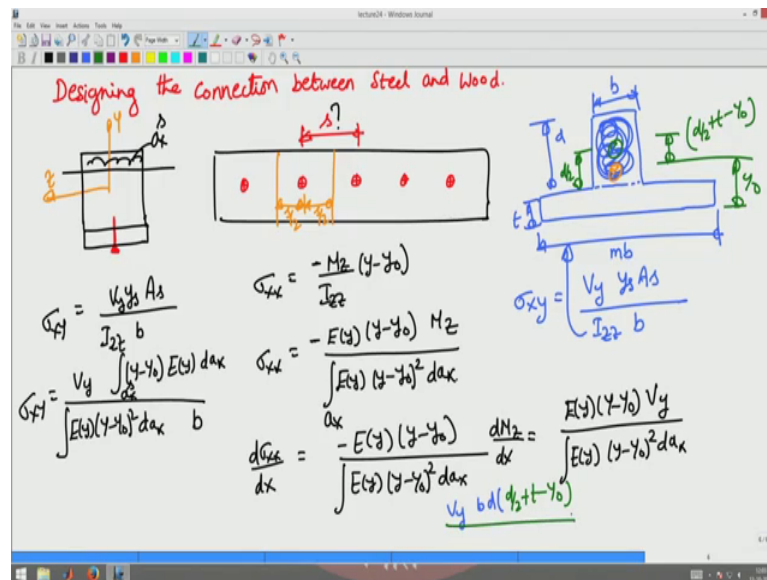
Stresses and deflection in beams loaded about one principal axis
Lecture – 70
Connection design

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$$\begin{aligned}
 \left[b(dt - y_0)^3 - b(t - y_0)^3 \right] \frac{1}{3} &= \frac{b}{3} \left(d^3 + (t - y_0)^3 + 3d(t - y_0)(dt - y_0) - (t - y_0)^3 \right) \\
 &= \frac{bd^3}{3} + bd(dt + t^2 + y_0(y_0 - d - t)) \\
 &= \frac{bd^3}{3} + bd(dt + t^2 + y_0(y_0 - d - 2t))
 \end{aligned}$$

Now, in the remaining few minutes, what I want to do is I want to find out; how the bonding between steel and wood have to be how the bonding between steel and wood have to be to ensure that they act integrally.

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So, say I add this is designing the connection between steel and wood. So, I add the cross section in here with the steel and wood. So, now, say let us say, I nail through the steel and I penetrate to the wood like this, this is the cross section. Now in the elevation or this is in the y z plane, if I look from bottom what will happen is I will have the steel plate with some screws on it at equal distances apart screws or nails nailed on to it at some equal distances apart.

Now, let us say the spacing of this nails is s what am I interested is what should the spacing be. So, that what is this spacing? So, that I can get an integral action between the steel and the wood that I do not want we saw the shear stresses develop in the beam to maintain the integral action the steel and the wood were to deform differently, then what will happen is there will not be this integral action between steel and the wood the wood will deform that will be top surface will be in compression for the wood bottom surface will be in tension for the wood and the steel also will have top surface in compression bottom surface in tension and there will be a separation at the interface between the steel and the wood that is not what we want right.

So, how do you connect how do you design this connection is a question now for that we have to find the shear stresses that will come at the steel wood interface. So, we saw that the shear stresses σ_{xy} is given by V into y s a s divided I_{zz} into b , right. Now we derive this expression for a homogeneous section where in we said that σ_{xx} is given

by M by I M z by I z z into y minus y naught this is what we use the expression for the bending armistice. Now what will be the expression σ_{xx} would be $E y$ times y minus y naught mz , there should be a negative sign here divided by integral $E y y$ minus y naught whole square dax ax with the negative sign right now or looking at the magnitude is throughout the magnitude of the actual bending stress would be.

Now, what I was interested was $d \sigma_{xx}$ by dx this; what gave us this v_y here, right. So, that will still be minus E of $y y$ minus y naught divided by integral E of $y y$ minus y naught whole square dax into dm z by $d x$ that will be minus $V y$ which will be which will mean that this will be $E y$ times y minus y naught $V y$ divided by integral E of $y y$ minus y naught whole square dax .

Now that is $d \sigma_{xx}$ by dx . So, now, either I have to do it like this, wherein, I have to go from first principles and rewrite the σ_{xy} expression as $V y$ divided by integral E of $y y$ minus y naught whole square dax b of y you know into b into integral y minus y naught dax E of y dax up to the section where I am interested in finding the shear stress from y to d by 2 say up to and are dax section that is up to if I am interested in the shear stress at this location this will be dax s .

So, that is how this is from the first principles on the other hand if I use the equivalent section approach then I need not change this expression for shear stress it will remain the same except that now I have to use this as the cross section of this beam this as the cross section of the beam where this will be M times b this is b this is d and this is t and my expression for σ_{xy} remains as $V y$ into y s into a s divided by I z z of this section times b of that section.

So, now, then the problem boils down on this is the same thing as before you have section you can find y s a s for that up to this point and to find y s a s up for this entire region for that entire region and then you have to find what is the tributary area of this nail the tributary area of this nail would be the width of the cross section times s where in this is s by 2 this is s by 2 and this is s by 2 and you have to multiply this σ_{xy} with s by s times b and you have to equate it through the nail allowable shear force in the nail ok.

So, σ_{xy} then would be $V y$ into that entire area is b d into the center of that cross section would be measured from the neutral axis y s should be measured from the neutral

axis. So, it will be neutral axis is somewhere here this is the neutral axis the centroid of that cross section would be this distance is d by 2 and I know that this distance is y naught. So, I am interested in this distance that distance would be d by 2 plus t minus y naught. So, this will be d by 2 plus t minus y naught divided by I_{zz} is for this section would be the expression given in here I_{zz} for this cross section would be.

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$$\begin{aligned}
 &= E_w \int_0^t \int_{y_0}^{y_0+t} (y-y_0)^2 dy + E_w \int_0^d \int_{t/2}^{t/2} (z-d/2)^2 dz \\
 &= E_w \left[\int_0^t \left(\frac{(y-y_0)^3}{3} \right)_{y_0}^{y_0+t} dt + \frac{E_w t}{E_w} \left(\frac{(z-d/2)^3}{3} \right)_{t/2}^{t/2} \right] = \frac{E_w b}{3} \left[(d+t-y_0)^3 - (t-y_0)^3 \right] \\
 &= E_w \left[\frac{bd^3}{12} + bd \left(\frac{d}{2} + t - y_0 \right)^2 + \frac{E_w t}{E_w} \left(\frac{bt^3}{12} + bt \left(\frac{y_0 - t/2}{2} \right)^2 \right) \right] \\
 &= \frac{bd^3}{3} + bd \left(dt + t^2 + y_0(y_0 - d - 2t) \right)
 \end{aligned}$$

This expression in here with E of wood as the Young's modulus this will be the effective moment of inertia of that ok.

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$$\begin{aligned}
 \sigma_{xx} &= \frac{-M_z (y-y_0)}{I_{zz}} \\
 \sigma_{xx} &= \frac{-E(y) (y-y_0) M_z}{\int E(y) (y-y_0)^2 dx} \\
 \frac{dQ_{xx}}{dx} &= \frac{-E(y) (y-y_0)}{\int E(y) (y-y_0)^2 dx} \frac{dM_z}{dx} = \frac{E(y) (y-y_0) V_y}{\int E(y) (y-y_0)^2 dx} \\
 \sigma_{xy} &= \frac{V_y y_0 A_s}{I_{zz} b} \\
 \sigma_{xy} &= \frac{V_y b d \left(\frac{1}{2} t + y_0 \right)}{I_{zz} b} + s * b = F_{shear} \text{ Nail}
 \end{aligned}$$

So, divided by I_z into b s, I am looking at the wood interface I have to look at the shorter re region. So, it will be b this into s into b must be the Shear force allowable in the nail.

So, once we know this we can rearrange to find b that is how you solve this problem. So, that is it for today's class what you have looked at is how do you analyze in homogeneous beam how do you find the axial stresses how do you find the shear stresses and how do you analyze for the deflection of the beam.

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The image shows handwritten mathematical derivations on a digital whiteboard. The derivations are as follows:

$$\int_{a_x} E(y)(y-y_0) \, dx = 0 \Rightarrow y_0 = \frac{\int E(y) y \, dx}{\int E(y) \, dx}$$

$$-\frac{d^2y}{dx^2} \int_{a_x} E(y)(y-y_0)^2 \, dx = M_z$$

$$\frac{d^2y}{dx^2} = \frac{-M_z}{\int_{a_x} E(y)(y-y_0)^2 \, dx}$$

$$\frac{-\sigma_{xx}}{E(y)(y-y_0)} = \frac{-d^2y}{dx^2} = \frac{M_z}{\int_{a_x} E(y)(y-y_0)^2 \, dx}$$

Below the last equation, it is noted: "Bending equation when the beam is inhomogeneous".

The basic equation is basically there are 2 approaches one is you can modify the basic equations to get these expressions for y naught and the bending equation or you can transform the cross section by using a modular ratio times the corresponding width of the cross section when the variation in the cross section is only along the one direction y direction, then you can do this modular ratio approach whereas, this bending equations are general when the variation in the cross section of the properties is both in the y and z directions ok.

Thank you.