

**Mechanics of Material**  
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**Introduction and Mathematical Preliminaries**

**Lecture – 03**  
**Part 2**  
**Concept of Force**

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The image shows a whiteboard with the following handwritten mathematical content:

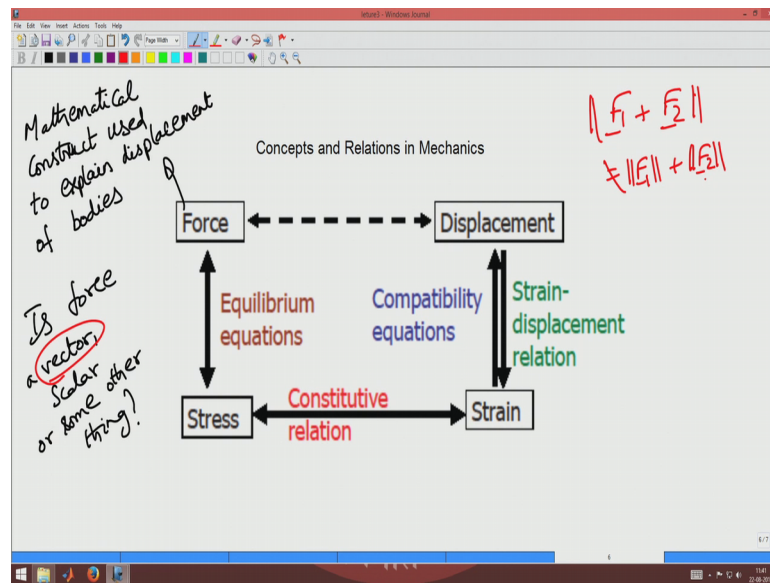
$$[Q] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad [Q][Q]^t = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 & 0 \\ 0 & \cos^2\theta + \sin^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[Q]^t = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = [I]$$

$$a = a_i e_i = a_i^* e_i^* ; \quad \{a^*\} = [Q]^t \{a\}; \quad Q_{ij} = e_i \cdot e_j^*$$

So, this is what we have seen till now and this  $Q$  is such said is inverse should be is transpose. Now, next what you are going to do is now going to move want to the concepts in mechanics we are seen a bit of map till now you want apply this map to some physical concepts that we are going to use in mechanics.

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The first concept we are going to look at is the force, the constant of force. Basically in the first search we saw the forecast mechanics which is force displacement stress and strain. And we saw that in the first lecture mechanics we saw that there are 4 concepts in mechanics which are force, displacement, stress, and strain. And then we saw that there are 4 relations connecting in this concepts displacement and strain are connected through strain displacement relationship. Stress and strain are related through the constitutive relation the stress and force are related through the equilibrium equations and then we know that there displacement and strain are related through the compatibility condition. The strain and displacement are related through the compatibility conditions.

So, basically now we are going to start looking at what is contra forces. From a first course in mechanics would have seen that what forces and what displacement is already. What is force then? Force is a mathematical concept that is used to explain a displacement, force is mathematical construct used to explain displacement of bodies. It is not intuitive concept you cannot measure force directly you can measure only displacement that the force causes and infer what the forces.

So, but we have been using this term force quite frequently in our day to day life that we think it is a physical quantity which we understand, but if you think deep about it is not a physical quantity it is a mathematical quantity which we use to describe displacement. Force is not the turn as that we feel we are going to lift up a weight, force is not the

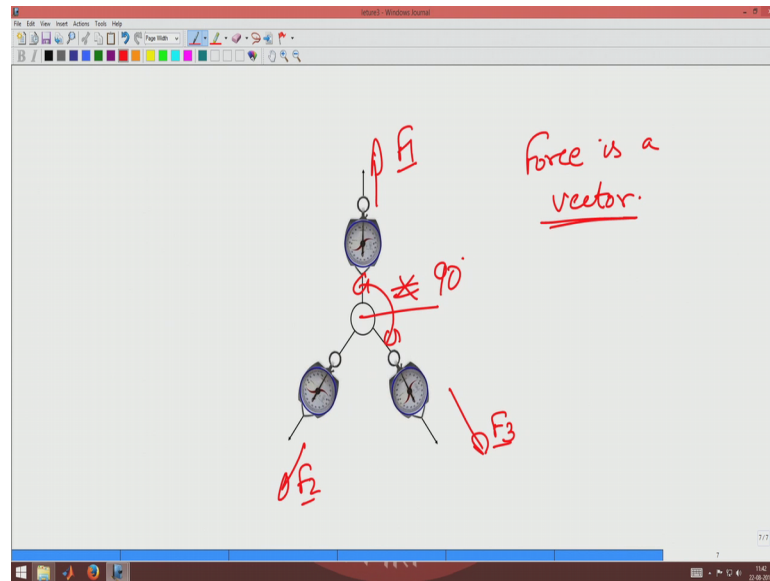
muscle contractions is not the force is not something which we can quantify it physically that is something going to understand. In equipments like load cell what you do which we used to measure the forces also you measure the displacement there will be a dia from the load cell which will displays, we would measure the displacement at dia from to infer what the load is.

In a spring balance for example, you place a mass are you calibrate the spring with respect to standard masses the elongation with the spring undergoes and standard mass and you then decide on what is a calibration factor and then you use that two measure infer the force. So, in all the circumstances force is not a physical quantity, but it is a mathematical quantity.

Now let us understand that what kind of a marginal quantities force, is it a vector, is force, a vector, scalar or some other thing how do you know what it is kind a demonstrate to experiments the it is a vector or it is a scalar. Force is a vector quantity, force is a vector quantity not because force is mass same acceleration and is a vector accelerations are vector and hence forces a vector.

But we know we can independence sure that force is a vector quantity because it follows parlogram law of addition it does not follow scalar addition rules. What it is difference? If I write if I have 2 forces  $F_1$  plus  $F_2$  the mounted of  $F_1$  is not mounted of this is not equal to mounted of  $F_1$  plus mounted of  $F_2$ . It depends upon in direction in fact it will be lesser than or equal to  $F_1$  plus  $F_2$ , unless  $F_1$  and  $F_2$  are orthogonal this equal to low one told true clown be the sum of (Refer Time: 05:08)  $F_1$  and  $F_2$ .

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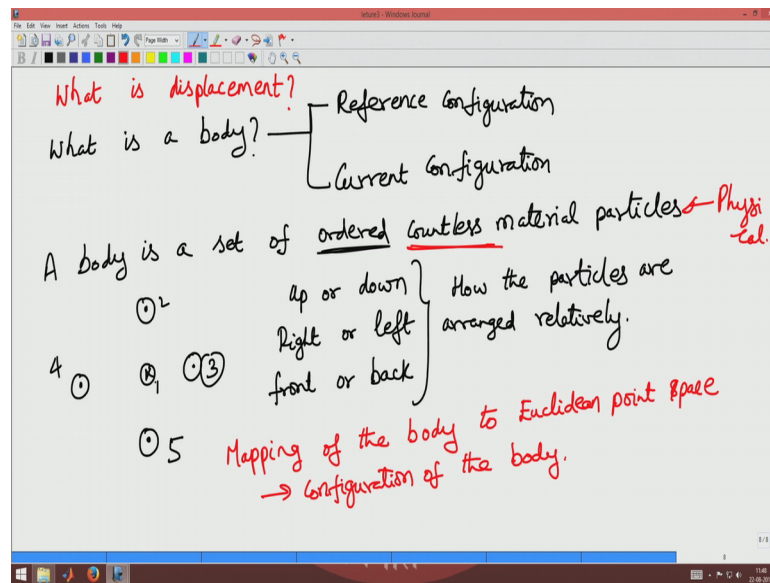


You can show this using a simple experiment which you can do 3 of you get together put a ring in between add 3 spring balances, add 3 spring balances and 3 of you pull on 3 different direction.

So, this one was  $F_1$  this is  $F_2$  and this is  $F_3$ . The angle between them is not this angle is not equal to 90, is not equal to 90, it should be different from 90 degrees then what you will find is the spring balances the resultant force that you have say you took  $F_1$  and  $F_2$  and  $F_3$  as the basic forces the resultant force  $F_1$  will be not some of the readings of the spring balances here and here, at 2 and 3 spring balances will not have direct to give value of a spring balance at 3. So, from this we can infer that force is a vector, from this experiment you infer that force is a vector quantity.

And the next chapter is that force is a vector quantity, now let us move on to the other concepts what kind of a quantity is displacement. Displacement is also a vector quantity, and let us first understand what you mean by displacement, now we are trying to understand.

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What is displacement? To understand displacement you have to understand what a body is and you have to understand to configuration that this body can be what is called as the reference configuration and current configuration. So, this is what will focus on in this class now in the remaining time and I understand what a body is and what a reference configuration and current configurations are.

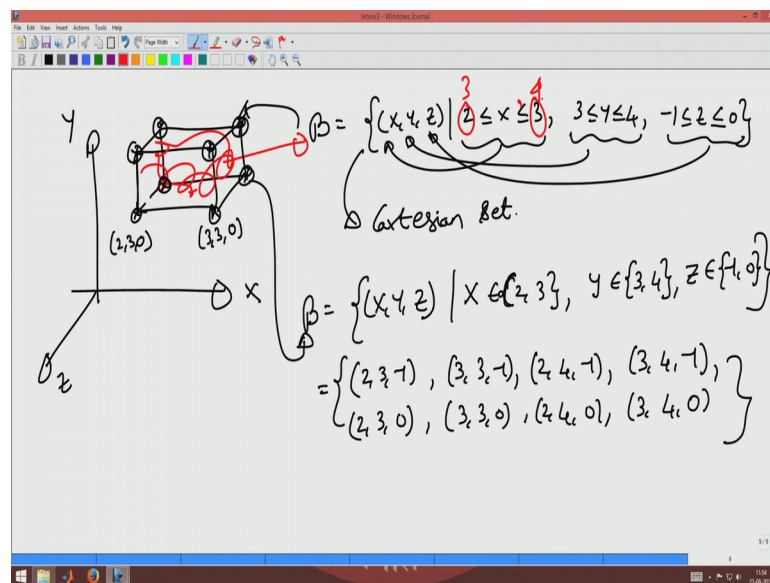
A body is a set of ordered countless material particles. What do I mean by ordered? Plus a arrangement of this material particles which you can define, that is if I have 4 material particles say 1 2 3 4 or say 5 particles I can say that this 3 years in front of 1, 2 is above 1, 4 is to the back of 1, 5 is below 1, I can say that point 6 and 7 are above are to the right and left of point 1. So, there are 3 things that you can say up or down, right or left, front or back. So, basically you can using this 3 concepts you can say how the particles are arranged relative to the other, relatively that is why it is called as an ordered material particles. Since there is arrangement of the material particles it is called as ordered.

In mechanics you want this to be countless because you want to be able to take derivatives now you want to take derivatives you have to do a limiting process and a limiting process you do not want to go outside the set that the body is made of. And you want to do derivatives you have to take a limit and you do not want that limit to result in a set go to a element is not content the set.

So, you define the body as a countless set of metal particles. This is essentially what continue a mechanics does it does not mean that a body is I mean in countless number of material particles are the body we are not ignoring the microstructure of the body. Thus is purely an assumption that you make an continuum mechanics so that we can do some meaningful approximations of the body which are useful for engineering.

So, basically a body is a set of order countless material particles, this is the physical definition of a body. What you do is you map this ordered countless material particles earn to mathematical set called as 3 dimensional Euclidian point space. So, in simply you map this body say I have this pen you map this pen onto some reason in space, in the mathematical space and that thing is called as the configuration of the body mapping of the body to Euclidean point space is called as configuration of the body.

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That is say I have a cuboid, I would a mapped it into x y z is my corner basis, I would have mapped this point on to sum 2 comma 3 comma 0 and this point would have been 3 comma 3 comma 0. And all this set up points within this cube which will denote this by a Cartesian set as x comma y comma z where 2 less than x less than or equal to 3 and y is 3 less than y less than equal to 4 minus 1 less than z less than 0. This is the representation of this body.

What you are saying here is we take one element one number from this range for X you take one number in this range real number in this range for Y and we take one number in

this range for  $Z$ ,  $X$  and  $Y$ , and you permute all the possible combination that are there for those 3 number to form a set which we call it as a Cartesian set, this is called as this representations called as Cartesian set.

So, basically now we represent the body using Cartesian sets. I can represent the body using the Cartesian set as a continuous set like what we have here are again represent it as a discrete elements if I say that the body is representatives in this  $X$  belonging to 2 comma 3,  $Y$  belonging to 3 comma 4 and  $Z$  belonging to minus 1 comma 0. Here I can enumerate the elements of the set this will be 2 3 minus 1, 3 3 minus 1, are 2 4 minus 1 3 4 minus 1, and 2 3 0, 3 3 0, 2 4 0 and 3 4 0. So, all this 8 corner points are what this body is consisting of, this 8 corner points this is what this body consist, of where as this body consist of the entire region it consists of the entire region occupied by this cube is this body.

So, basically now we can represent the body as countless ordered set of particles which automatically you map it on to a Euclidian point space where in you map it on to one of this Cartesian sets, where  $x$   $y$   $z$  this values can be different for different bodies, this values can be different for different bodies. So, for us a body is to recap, a body is ordered countless set of material particles which are mapped on to a Euclidian point space this called as the configuration of the body.

Now, at different instances the body will occupy different regions of the space, in one time it will have been occupying this in a subsequent time the  $x$  should have varied from 3 to 4 or something like get if the body is getting this place right. So, there is a difference between reference configuration and current configuration. Reference configurations is such some given time you find what is the cross sectional area of the body, what is the length of the body and so on. Whereas, the current configuration because the current instant of time. So, we will revisit this idea of reference and current configuration in the next class.

Basically, what you are seeing today is you are seeing how to represent vectors basically we represented vectors has a  $i$   $e$   $i$  and that a  $i$  the 3 numbers are represent a vector there are set of them you will represent the same vector  $a$ , because set of them will be related through a transformation matrix  $Q$  is an orthogonal matrix and then we saw that what a forces we saw and experiment through which we can show that force is a vector quantity.

And then we are looking at what a displacement is for that we said we have to understand what a body is, and we understood that body is a set of ordered particles countless set of ordered particles and then we saw how to represent that countless set in Cartesian set format.

So, in the next class we will what a reference configuration is, now what a current configuration is and then we will taken from there to see what a displacement is.

Thank you.