Mechanics of Material
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Stresses and deflection in beams loaded about one principal axis
Lecture- 69
Bending of a composite beam

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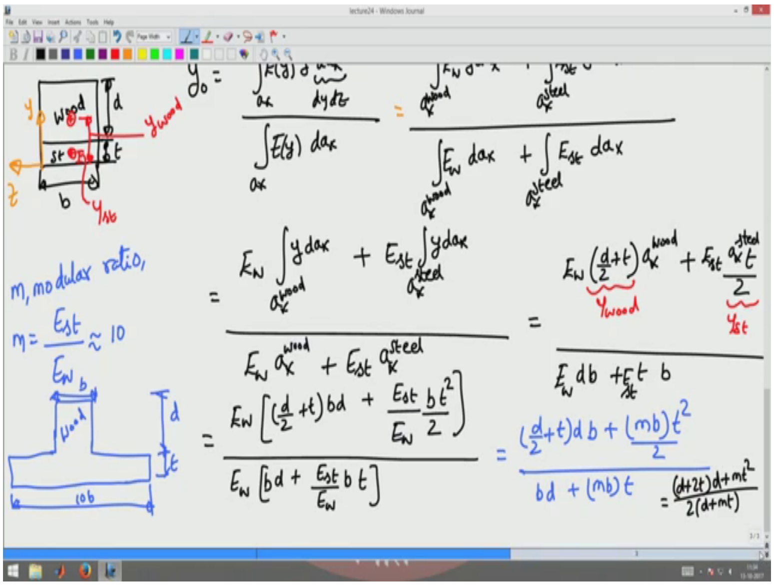

So, I have this beam, this is steel, this is wood for dimensioning purposes let me say this is of depth d, this is of thickness t, this is of width both are of width b ok. Now what happens I have to find y naught, y naught integrate E of as a function of y into y d a x, d a x is dy dz divided by integral E of y d a x a x. Let me assume that the y and x of the cross section is measured from y and z of the cross section is measured from here y and z ok. Now I define y naught which is given by that expression, I decompose since integration I decompose the area over which I integrate additively, I can rewrite this as this integration as integral E wood y d a x a x of wood plus integral E st into y d a x, a x of steel divided by integral E of wood d a x, a x of wood plus integral a x of steel E st d a x ok.

Now, this I can pull out now I can pull out d w and E st outside the integration, because they are constants. So, I will get it as E w into integral y dax ax of wood plus E st integral y dax, ax of steel if I do the same thing at the bottom I will get E w into a ax dax of integrating dax over ax of wood would be the area of the wood plus E st into ax of steel area of steel ok. Now this y dax is nothing, but the center of the wood region y ax is center of the wood region times the area of the wood region. So, that will be E w to center of the wood region is measured from the origin assumed would be d by 2 is center from the wood bottom surface, but from the steel bottom surface it will be d by 2 plus t into ax of wood area of wood that will be an integration of y dax of ax of wood plus E st into ax of steel area of steel into t by 2 into t by 2 is the distance of the center of the steel from the bottom panel, bottom fiber.

So, this is yst and this distance is Y wood this is Y wood and this is Y steel ok and then divided by area of wood which will be d times b plus t times b ok, now I do a simplification I will pull out E w outside. So, I will get d by 2 plus t into area of wood is b into d plus E st by E w into b into t square by 2 divided by if I pull out E w, this will be d plus E st by E w bd into bt ok. Now what I want to define is I want to define some factor called m m stands for modular ratio which is defined as in this case I am defining it as E st by E w Young's modulus of steel by Young's modulus of wood ok. Then what happens is I going rewrite this equation as d by 2 plus t into d b plus m times b into t square by 2 divided by bd plus m times b into t ok.

So, what I am going to do now is, I am going to change the width of the steel to m times the width of the steel where m is a modular ratio. In that sense I can make this beam a homogeneous beam as that is made up of wood ok. So, typically this modular ratio E st by E w will be of the order of 10 this ratio would be of the order of 10. So, I have to increase this by the width of steel by 10 times. So, I will have a cross section something like this ok. This remains as t this remains as d this is 10 times b and this width is b that width remains as b ok. This is a equivalent wood section for this from this analysis in other words. So, I simply my y 0 I will obtain this as y 0 would be b plus 2 t into d plus mt square by 2 times d plus m times t ok. I consider the b throughout and that is what I will get as y naught ok.

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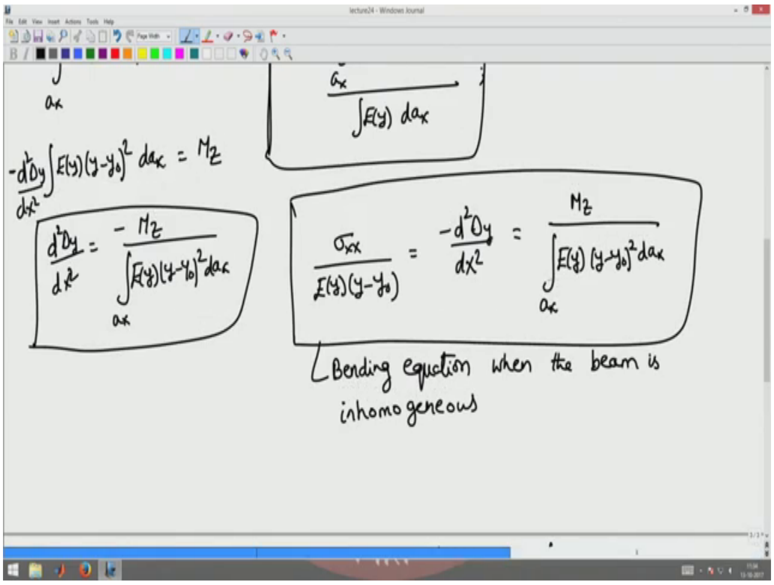

Now, I have estimated y naught now let us go out and estimate the other term which appears in the denominator to M z moment, let us now compute this term E y into y minus y naught whole square da ok.

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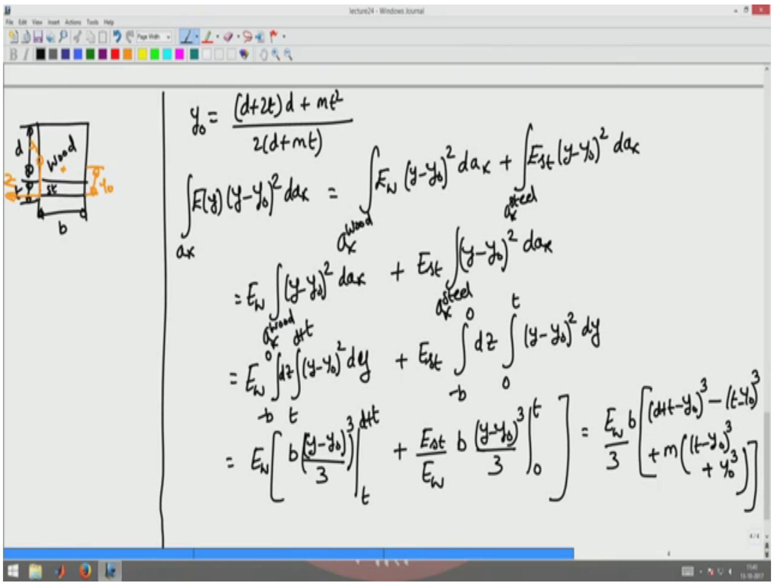

So, for this beam you have assumed we have computed y naught as d plus 2 t d plus m times t square divided by 2 times t plus m times t ok. Now for the same section steel and wood of depth d t and width b, I am interested in computing I zz. Not I zz I am interested in computing e times y y minus y naught whole square dax ax ok. Where y naught was some distance here this is y naught and remember we took the axis as this as y and this was z ok. So, now, what will this be? This as usual I can demark it into ax of wood E w of wood that is y minus y naught whole square dax plus E st of steel integral y minus y naught whole square dax ax of steel region ok.

Now since E w is a constant I can pull E w outside integral y minus y naught square dax, ax of wood plus E st integral y minus y naught whole square dax ax of steel ax of steel ok.

Now, this will be E w into if first I will do by integration, then I will do it by another method wherein I I will use the flat parallel axis theorem to shift the areas and do that. So, for a of wood it will be from y minus y naught whole square d y, integrated from t to d plus t and d z integrated from minus b to 0 ok. Integrated from minus b to 0 because that is what is, the area of wood is plus E st into dz of steel is again minus b to 0 into y minus y naught whole square dy integrated from 0 to t ok.

So, this will be E w if I pull it out it will be b dz is integrated dz is z, when I apply the limits it will be b into this will be y minus y naught whole cube by 3 from t to d plus t plus E st by E w into b. Again integrating dz we will get it as z 0 minus b to 0 will be b, and this will be y minus y naught whole cube by 3 from 0 to t ok.

This is nothing, but E w in to b, this will be d plus t minus y naught whole cube minus t minus y naught whole cube into m times t minus y naught whole cube minus minus plus y naught cube ok. Now what I want to do next is instead of doing the integration, if I want to use the parallel axis theorem for finding this integration integration or doing this y minus y naught whole square dax of ax of wood and this of steel.

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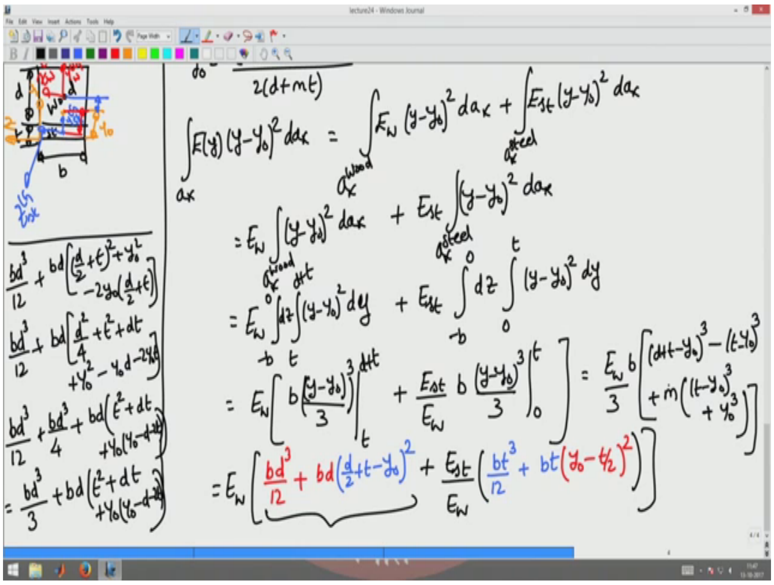

I will get the same thing as E w into about the central axis of the wood, which might be here y wood CG and is that wood CG you know that it is bd cube by 12. So, I will have b d cube by 12 plus I will have the area of the wood which is b into d times this shift in the CG of the wood to the CG of the cross section or the effective CG of the cross section.

So, that would be this that distance would be d by 2 plus t minus y naught whole square y naught whole square that will for the first integration that will be for this integrating the wood, that will be integrating for the wood now doing it for the steel I will have plus E st by E w since I have pulled out the Young's modulus of wood outside that will be the for this steel again, I will have the CG of the steel to be here this is y steel CG, that will be this will be z steel CG ok. So, about that CG axis I will get it as, I will get it as b t cube by 12 plus b into t which will be the area of the steel into this distance square I have to find this distance I have to square that distance by use of parallel axis theorem ok. So, that distance squared would be y naught minus t by 2 the whole square minus t by 2 the whole square.

So, that is this expression ok. Now you have to show that these two are equivalent. So, I have to show basically bd cube by 3 plus bd into d by 2 plus t minus y naught the whole square is equal to that expression I will show that and I will assume that for the shear also the same thing happens ok. So, I I have to simplify this I have to simplify this expression. So, I will have bd cube by 12 plus bd into d by 2 plus t the whole square, plus y naught square minus 2 y naught d by 2 plus t right ok.

So, this will be bd cube by 12 plus b d into d square by 4 plus t square, plus plus dt plus dt plus y naught square minus y naught d minus 2 Y naught t ok. So, this is nothing, but bd cube by 12 plus bd cube by 4 plus bd into t square plus dt plus y naught into y naught minus d minus 2 t ok.

So, this will be nothing, but bd cube by 3 plus bd into t square plus dt plus y naught into y naught minus d minus 2 t ok. Now let us expand this term lets expand d plus t minus y naught whole cube into b minus b into t minus y naught whole cube.

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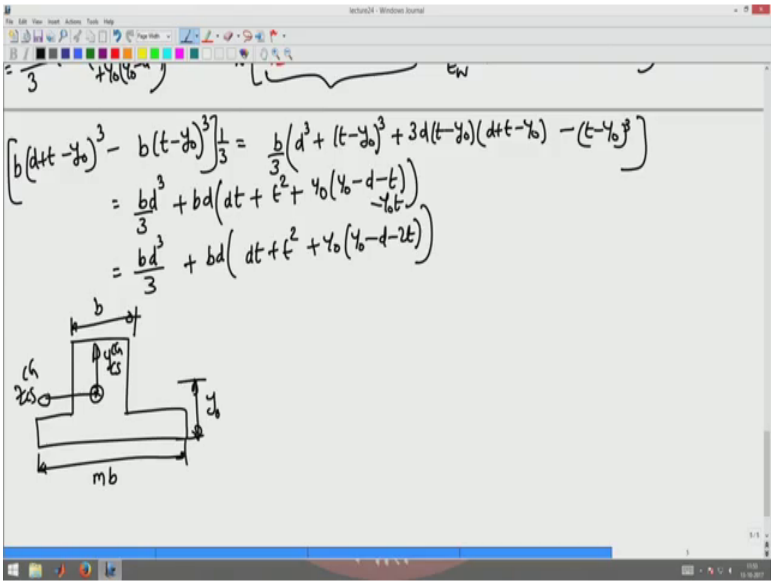

This will be b into d plus t the whole cube minus y naught cube or let me do it the other way round. It will be d cube plus t minus y naught whole cube plus 3 d into t minus y naught into d plus t minus y naught minus t minus y naught the whole cube ok. So, this will be now equal to I left that 3 outside 1 by 3 is there, this will be b by 3, this will be b by 3 d cube plus d into plus bd into I move a t minus y naught inside. So, it will be dt plus t square minus y naught or plus y naught into y naught minus d minus t minus y naught t.

So this will be equal to bd cube by 3, plus bd into dt plus t square plus y naught into y naught minus d minus 2 t ok. This is same as what we got before. So, these two expression are identical. So, there are couple of ways by which you can compute this integral E of y, y minus y naught dax one the simplest way is what I did the last basically you pull out the Young’s modulus which is constant for a region and then use a parallel axis theorem to compute the corresponding moment of inertias of y minus y naught square ok. And here again the thing to be noted is the modular ratio comes about here ok, where it will affect the width of the cross section alone ok.

So, if I add a cross section which is of this form as an equivalent steel section equal root section or I make this m times b, when this is b I will get since the heavier area more area is there at the bottom, the CG will shift towards the bottom. So, allow it the CG as here for the cross section Y CG of cross section and this will be Z CG of the cross section and this distance would be y bar y naught, this distance would be y naught as we computed before and for this cross section we compute the moment of inertia be same as what you have got here ok.

So, we have computed essentially the ingredients that are required to solve the problem. So, essentially we have computed y naught for a cross section and this integration for the cross section. So, remaining steps remains the same as it is for the homogeneous cross section. To get delta y we have to integrate this equation where M Z is a function of x to get delta y, to get the stress we use the fact that it is E y times M Z into y minus y naught divided by E y times y minus y naught whole square da ok. So, use this facts to compute the moment or distress respectively.