Mechanics of Material
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Stresses and deflection in beams loaded about one principal axis

Lecture- 68

Modified bending equation

Welcome to the 24th lecture in Mechanics of Materials, the last lecture, we saw how to solve a simply supported beam subjected to a 2-point loading to find the bending moment, shear force, reflected shape, the slope of the reflected shape and the design the cross section such that, the maximum compressive stress and maximum tensile stress are within limits. In this lecture, we look at how to find the stresses and deflection in inhomogeneous beams.

Basically, till now we have assumed that the beam is homogeneous meaning that is made of the same material throughout the cross section. And we derived that bending equation that, minus sigma axis by y minus y naught is equal to m z by I z z equal to E times d square delta y by d x square by this.

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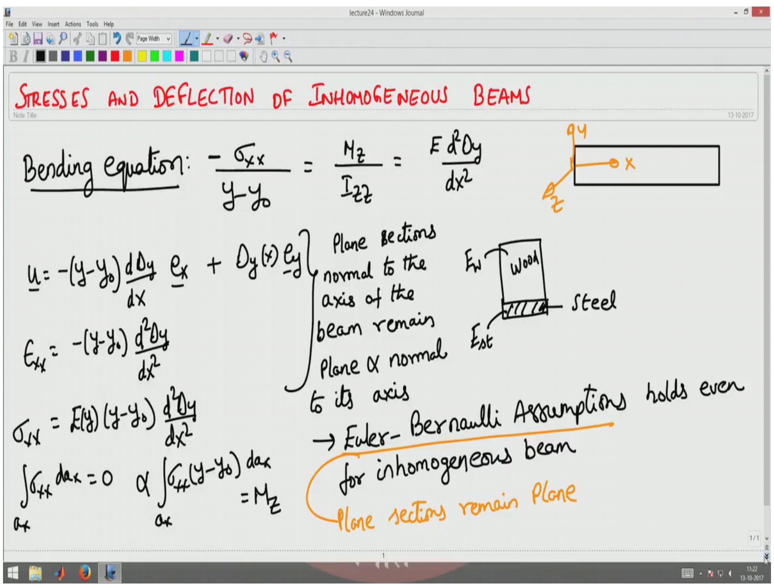

E is the Young's modulus meaning the material shows a similar response in tension and compression and that, it is made of the same material the entire cross section is made of the same material with a Young's modulus steel characteristic signal stress strain response.

Now, what are going to do is if I look at the cross section of this beam, I am going to say as the cross section of the beam is made of 2 materials, say wood and this material is say steel. How do you analyse such a cross section is we are going to see, since there is a different Young's modulus for wood, which we will denote it by E w, and a different Young's modulus for steel which we will denote it by E s t. You can not give as a single value of Young's modulus for the entire cross section as such.

So, you have to revisit the derivation that we did for this obtaining this equation talk on for a fact that the cross section is not homogeneous. So, if you recollect in the initial lecture of the beams, we said that the displacement of the beam u is given by minus y minus y naught into d delta y by d x; E x plus delta of y as a function of x; E y, right? This was that expression field that, we got by assuming that the plane section remain plane, and normal to the neutral axis of the beam. Sections were initially normal to the neutral axis of the beam remains normal to the neutral axis of the beam after deformation.

So, we based on that assumption we got this as the strain displacement field, from here we computed the epsilon x x strain as minus y minus y naught d square delta y by d x square right? Even when the beam is inhomogeneous we assume that, this is a displacement field and this is a strain field meaning we assume still plane sections normal to the axis of the beam remain plane and normal to it is axis.

So, basically this assumption which is called as the Euler Bernoulli assumption, holds even for inhomogeneous beam this assumption is called as Euler Bernoulli assumption or plane section remain plane assumption, another name for this is plane sections remain plane. Now, then if the stress is given by this expression in here then the sigma xx strain is given by expression in here, the sigma access stress should be given by e as a function of y into y minus y naught d square delta y by d x square right?

And then we employed the 2 equilibrium equations namely, integral sigma x x d a x a x has to be equal to 0. There is no net actual force is applied in the cross section, and the fact that sigma x x into y minus y naught d a x integrated over the cross-section area, must be equal to the applied moment m z at that at that the cross-sectional location. So, now if you apply this when E was a constant we pulled the Young's modulus outside the integration saying that is a constant, now you cannot do that. So, let us find out what restriction this equations place.

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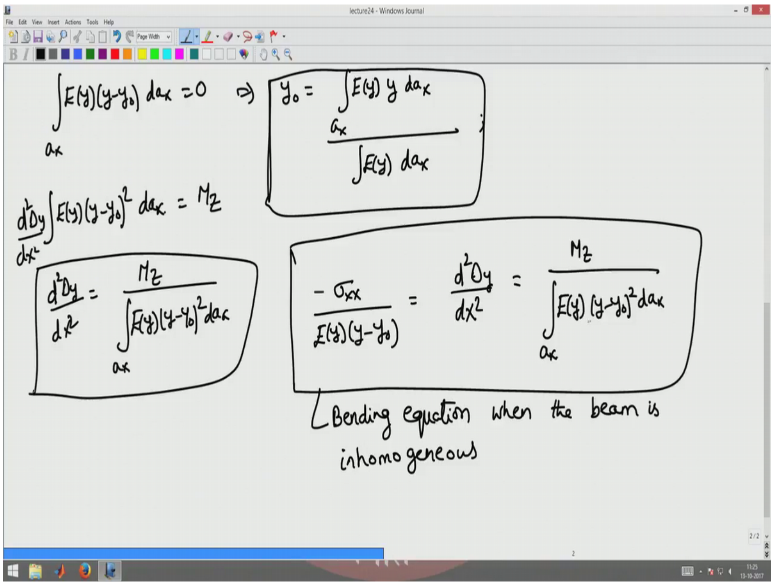

So now you have E of y into y minus y naught d a x a x has to be 0, this will imply that y naught is integral E of y into y d a x a x divided by integral E of y d a x. Now then, your substituting for sigma x x in the second equation you have E of y into y minus y naught whole square d a x into d square delta y by d x square, I left the negative sign in for a stress here. So, with the negative sign in there must be equal to your m z moment there. So now, what happens I cannot pull the E outside. So, b square delta y by d x square is m z divided by integral E of y into y minus y naught whole square d a x a x. Now substituting this in a stress expression what we get is.

So, combining you this is the equation to find y naught, and this is the equation to find d square, delta y by d x square. So, your stress expression becomes sigma x x by E of y into y minus y naught is d square delta y by d x square. Which will be equal to m z divided by integral E of y y minus y naught whole square d a x a x. So now, this is a general bending equation when the beam is inhomogeneous. This is a general bending equation, when the beam is inhomogeneous, here the y need E need not be a function of y alone E can be a function of z also, in which case I have to integrate appropriately according for a fact that, E is the function of y and z.

Now, let us go back to the specific problem that we were interested in a composite cross section with wood and steel as it is components. Now, let us see how to evaluate that, how to find y naught for such a composition composite beam and how to find this integral E of y y minus y naught whole square d a x for the composite beam.