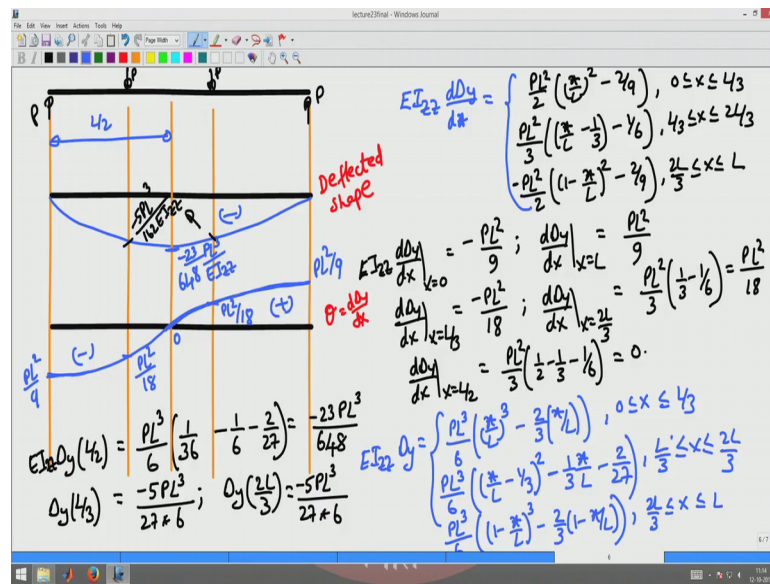


Mechanics of Material
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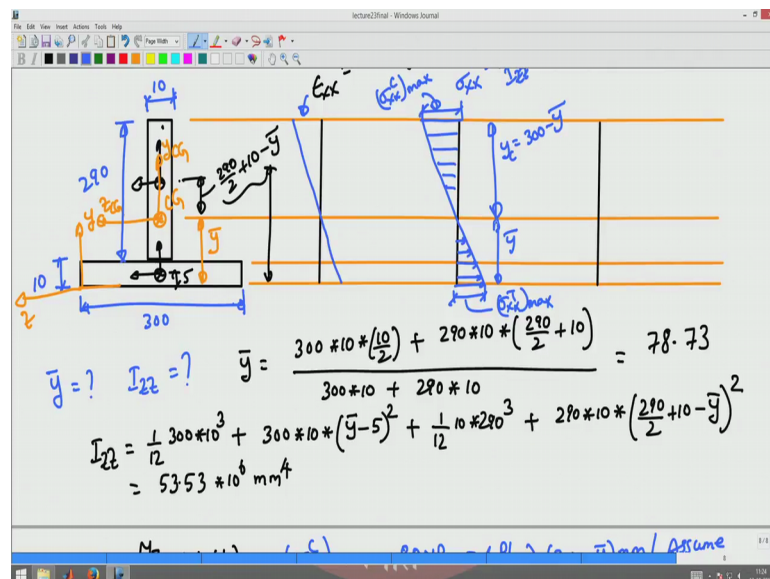
Stresses and deflection in beams loaded about one principal axis
Lecture - 67
Finding allowable load

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Now finally, let us find what are the stresses in this cross section.

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Now, we have assumed that the dimension of this cross sections to be the following. The cross section is as the following dimensions, 290 all the dimensions are in mm. So, your first job is to find y the centre of this cross section is and the second moment of inertia of this cross section. The centre would be y bar we have to find centre of cross section and we have to find I_{ZZ} about y bar. So, let us assume that initially the axis is located at these locations (Refer Time: 01:22) y .

Now I have to find the CG of this section. I have to move this axis from here to some point y CG to z CG where this is CG of the cross section and let this distance be y bar. Then we can see that y bar will be given by area of the bottom, area of this bottom flange which will be 300×10 and the centre of this from the assumed origin which, the centre of this flange from the assumed origin which will be 10×2 plus for the web the area is 290×10 into the centre of the web from the bottom of this flange which will be 290×2 plus 10 . So, that will be that divided by this divided by the area of the entire cross section 300×10 plus 290×10 is going to give me 78.73 , that is y bar.

Now to find I_{ZZ} , I use the parallel axis theorem I find the I_{ZZ} about the central axis of this flange about which will be $1 \times 12 \times 300 \times 10$ cube plus the shift from here to the CG, plus the shift from here to the CG of the cross section which would be 300×10 into this shift which will be y bar minus 5 the whole square because the CG of this section is at a 5 mm from the bottom of the flange. Plus I will (Refer Time: 03:44) for the web $1 \times 12 \times 10 \times 290$ cube plus the area of the web 290×10 into the shifting term from the CG of the web which is just here. Here to here I have to find this distance this distance would be 290×2 plus 10 minus y bar 290 plus 2×2 plus 10 is this distance is this total distance. So, that will be 290×2 plus 10 minus y bar the whole square the whole square. So, this is going to be 53.53×10^6 millimetre power 4.

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The image shows handwritten mathematical derivations on a whiteboard. At the top, the moment of inertia I_{ZZ} is calculated as $53.53 \times 10^6 \text{ mm}^4$. Below this, the maximum compressive stress $(\sigma_{xx})_{\text{max}}^C$ is set to 80 MPa and equated to $\frac{PL}{3}(300 - \bar{y})$ divided by I_{ZZ} . A note on the right says "Assume Length of beam, $L = 10\text{m}$ ". The maximum deflection Δ_{comp} is then calculated as $3 \times 80 \frac{\text{N}}{\text{mm}^2} \times 53.53 \times 10^6 \text{ mm}^4$ divided by $(300 - 78.73) \times 10^3$ and L , resulting in 5.8 kN . Finally, the maximum tensile stress $(\sigma_{xx})_{\text{max}}^T$ is calculated as $40 \text{ MPa} = \frac{PL}{3}(\bar{y})$.

Now, we use expression σ_{xx} is $M Z$ by I_{ZZ} into y minus y naught to compute the stress at any location. So basically all of strain vary in this cross section all of ϵ_{xx} vary ϵ_{xx} varies as b square Δy by dx square into E times I_{ZZ} this variation is given by minus y minus y naught d square Δy by dx square this a constant. So, basically there is going to be very linearly with respect to y . So, the strain variation is going to be linear like this. This is a strain variation.

Similarly this stress σ_{xx} would vary as $M Z$ by I_{ZZ} into y minus y naught that variations are also going to be linear with a negative sign in here, a negative sign in here. So, that is going to be compression of the top and tension of the bottom something like that. So, we are interested in finding what is this maximum compressive stress that will come here and what will be the maximum tensile stress that comes in here.

So, basically I am interested in finding this stress $\sigma_{xx} C$ max and I am interested in finding this stress which is σ_{xx} in tension maximum and that has to be limited to what are limiting value we add before. So, σ_{xx} compression maximum was assumed to be 80 MPa this is $M Z$ which is PL by 3 that is a maximum moment PL by 3 into y minus y naught is I have to get this distance this distance is $y C$ which is 300 minus \bar{y} because this distance is \bar{y} . So, I will have into 300 minus \bar{y} divided by I_{ZZ} which is 53.53 into 10 power 6 mm power 4 this units also in mm ok.

Now, I am interested in finding the maximum load that it can. So, the P allowable in compression would be 80 Newton per mm square that is mega Pascals into 53.53 into 10 power 6 into mm power 4 into 3 divided by 300 minus 78.73 this is in mm and length is I assume it to be 10 metre length beam. So, that will be 10 into 10 power 3 this is L. Now, I am assuming that the length of the beam is assume length of beam L to be 10 metres assuming the length of the beam to be 10 metres. So, this will give me the compressive load it can take to be 5.8 kilo Newton.

Now, similarly I want to find what is the maximum tensile stress that it can withstand sigma xx t max is forty MPa that will again PL by 3 which is a maximum bending moment that is coming in the beam into y bar now because I am interested in this stress (Refer Time: 10:00) at a distance y bar from because I am interested in this stress which is at a distance y bar from the total axis. So, I will get it as 5.8 into y bar divided by I ZZ which is 53 which is 53.53 into 10 power 6.

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The image shows handwritten mathematical derivations for allowable load in compression and tension. The derivations are as follows:

Stress distribution: $\sigma_{xx} = \frac{M_z}{I_{zz}} (y - y_0)$

Maximum compressive stress: $(\sigma_{xx})_{max} = 80 \text{ MPa} = \frac{PL}{3} (300 - y) \text{ mm}$

Assume Length of beam, $L = 10 \text{ m}$

Allowable load in compression: $P_{All, Comp} = \frac{3 \cdot 80 \frac{\text{N}}{\text{mm}^2} \cdot 53.53 \cdot 10^6 \text{ mm}^4}{(300 - 78.73) \cdot 10 \cdot 10^3}$

Result: $P_{All, Comp} = 5.8 \text{ kN}$

Maximum tensile stress: $(\sigma_{xx})_{max} = 40 \text{ MPa} = \frac{PL}{3} (y)$

Allowable load in tension: $P_{All, Ten} = \frac{3 \cdot 40 \frac{\text{N}}{\text{mm}^2} \cdot 53.53 \cdot 10^6 \text{ mm}^4}{78.73 \text{ mm} \cdot 10^4 \text{ mm}}$

Result: $P_{All, Ten} = 8.16 \text{ kN}$

Final allowable load: $P_{All} = 5.8 \text{ kN}$

So this will be this will give me P allowable in tension to be 3 into 40 into Newton per mm square into 53.53 into 10 power 6 millimetre power 4 divided by 78.73 which is in mm, mm into 10 power 4 which is the length of the beam in mm. So, it will evaluate to be 8.16 kilo Newton. So, now, you have two load estimates one is P allowable in tension to be 8.16 and P allowable in compression to be 5.8, which is a load that will allow on the beam. The load that will allow on the beam would be the least of these two which is

5.8. So, allowable load on the beam is 5.8 kilo Newton's. So the allowable load on the beam is 5.8 kilo Newton's.

So, this is how you solve a beam problem right from the starting to the finding the allowable load and finding the allowable deflections. In the next class what will do is we will look at the shear stresses variations at t section and how do you manufacture the t, how do you weld the t to get the appropriate behaviour to get integral action of t action.

Thank you.