Mechanics of Material
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Stresses and deflection in beams loaded about one principal axis
Lecture- 66
Deflected shape and rotation of cross section

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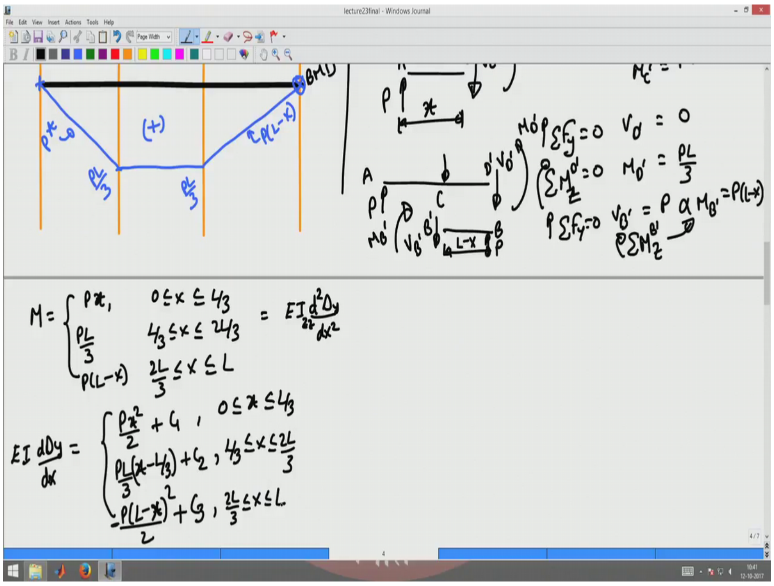

So, let me write the bending moment diagram in the 3 regions. So, I have bending moment M, I have bending moment M given by P into x for region 0 less than x, less than or equal to L by 3, PL by 3 for region L by 3 less than x less than 2 L by 3 and P into L minus x in the region 2 L by 3 less than or equal to x less than or equal to L. That is the variation of the bending moment in the 3 regions. To find a deflected shape you know that this bending moment is nothing, but EI this bending moment is EI d square delta y by d x square from the bending equation ok.

So, to get the slope that is EI d delta y by dx I integrate this equation once to get P x square by 2 plus C 1 in the region 0 less than x less than or equal to L by 3 and I get it as pl by 3 into x plus C 2. I will write it as x minus L by 3 plus C 2 where I have not made anything the C 2 constant now, will be the actual constant plus PL square by 9. So, I can do this I am doing this because if I do this the calculation becomes simpler because I want this term to go to 0 and x equal to L by 3 that is why I am doing this L by 3 less than x less than 2 L by 3 and this will be P into L minus x whole square by 2 and with a negative sign because x has a negative sign plus C 3 in the region 2 L by 3 less than or equal to x less than or equal to L.

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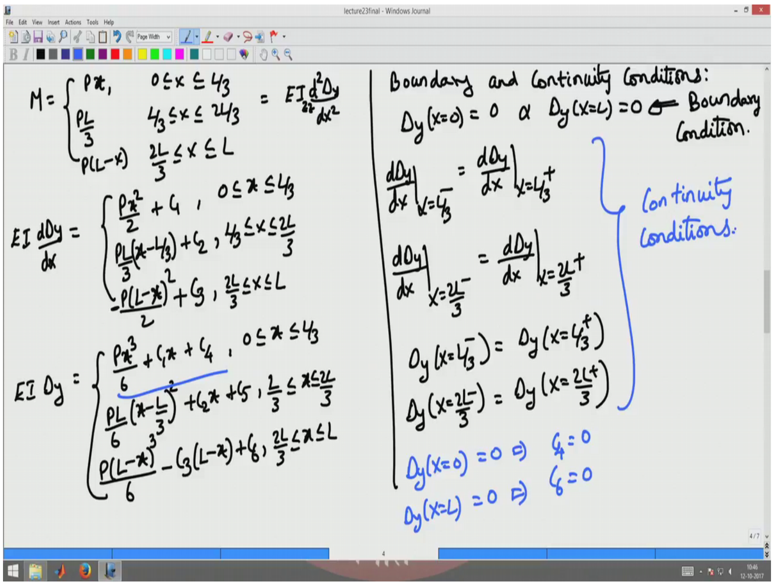

Now, I want integrate this equation once more to get E i delta y. So, that is going to be P x cube by 6 plus C 1 x plus C 4 the region 0 less than x less than L by 3 PL by 6 x minus L by 3 the whole square plus C 2 x plus C 5 the region L by 3 less than x less than 2 L by 3. Now, minus minus will become plus L minus x the whole cube by 6.

Now, again I will make it minus C 3 into L minus x plus C 6 I will be C 3 x, C 3 x I am getting here to C 6 would be the actual integration constant will be C 6 minus C 3 l, but I want it to go to 0 and x equal to l, so I am writing it like this. So L which will be 2 L by 3 less than x less than or equal to L. Now, what are the boundary conditions for this problem? The boundary condition for this problem are the boundary and continuity conditions, conditions I know that the there is no vertical deflection at A and there is no vertical deflection at B the right and left supports. So, delta y at x equal to 0 is equal to 0 and delta y at x equal to L is 0 this is the boundary condition.

Now, coming to continuity conditions I need a second derivative to exist which means up to the first derivative there should be a continuous function at least since the second derivative exist the first derivative has also has to be continuous which means d delta y by dx at x equal to L by 3 minus that is to the left of L by 3 must be equal to d delta y by dx at x equal to L by 3 plus that is to the right of L by 3. That is to the left means I use this equation to compute d delta y by dx L by 3 plus means I use this equation to compute dy by dx at L by 3. Now, similarly I need d delta y by dx at x equal to 2 L by 3 minus must be equal to d delta y by dx at x equal to L 2 L by 3 plus that is at L by 3 minus I have to use this equation 2 L by 3 minus and 2 L by 3 plus I have to use this equation to get the restriction due to this. Then I have delta y at x equal to L by 3 minus must be equal to delta y at x equal to L by 3 plus, similarly I have delta y at x equal to 2 L by 3 minus must be equal to delta y at x equal to 2 1 by 3 plus ok.

Now, let us look what restriction these conditions plays. The first condition delta y at x equal to 0 equal to 0 tells me that from this equation from this equation I get C 4 to be equal to 0. Similarly delta y at x equal to L equal to x will imply that C 6 is equal to 0 from this equation, from this equation x equal to L this two terms goes off. So, C 6 becomes 0. Now, this are the constants found from the boundary conditions.

The other constants I have to find from this continuity conditions these conditions are called as continuity conditions these conditions are called as continuity conditions. Now, let us apply the first continuity condition d delta y at x equal to L by 3 minus would be I have to use this equation L by 3 minus this should be equal to at L by 3 plus which is this equation.

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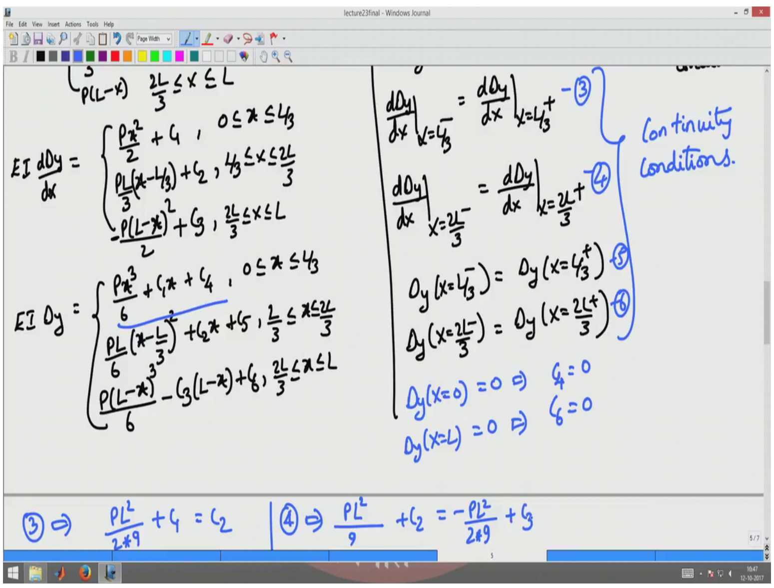

So, that gives me basically this is equation 3 say this is 4 this is 5 and this is 6, 3 gives me PL square by 2 into 9 plus C 1 must be equal to C 2 because of L by 3 this term goes off at x equal to L by 3 this term goes off. So, that will be C 2. Similarly equation 4 gives me that a 2 L by 3 minus it is PL square by 9 plus C 2 must be equal to minus PL square by 2 into 9 plus C 3 that will be coming from equation 4. Now, let us see what happens from equation 5.

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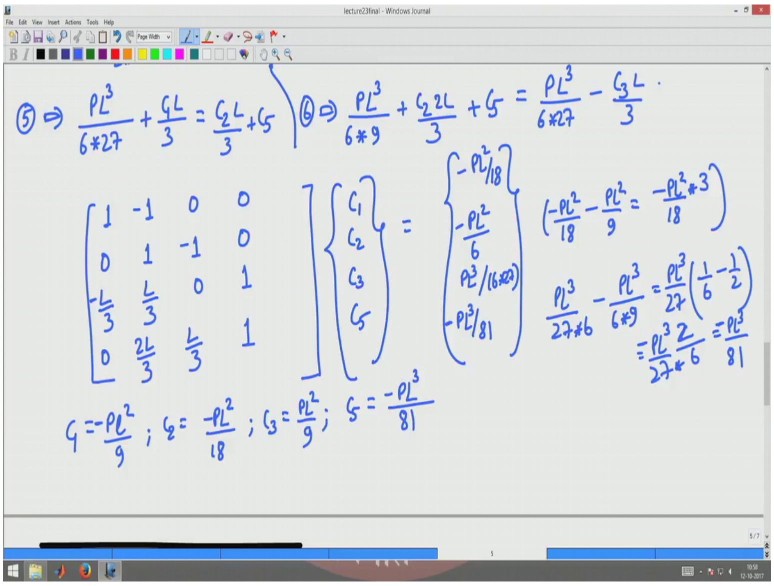

Equation 5 gives me, 5 gives me I have to substitute x equal to L by 3 in the equation C 4 I know is 0. So, I can drop that and x equal to L by 3 in this equation that will be at L by 3 plus. So, that will be PL cube by 6 into 27 plus C 1 into L by 2 L by 3, C 4 is 0 must be equal to at L by 3 plus this is 0 so that will be C 2 into L by 3 plus C 5.

Student: C, C 1 L by 3 (Refer Time: 09:57).

Yeah, it will be C 1 substituting L by 3 in the first equation we will get this as the result ok. Now, using 6 from 6 we will get it as I have to substitute x equal to 2 L by 3 in this equation and in this equation then I will get it as PL cube by 6 into 9 plus C 2 into 2 L by 3 plus C 5. Now, as B equal to now, as B equal to twelve by 3 minus in this equation will give me P into L cube by 6 into 27 minus C 3 into L by 3 and C 6 is 0 C 2 into 3 L by 3.

Now, this are linear equations involving 4 constants C 1, C 2, C 3 and C 5 I have to solve this system of linear equations, we solve this system of linear equations I will get C 1 to be whom I will write in the matrix form and then I will ask you to invert and solve it. So, it will be C 1, C 2, C 3, C 5 or a 5 constants this should be equal to the right hand side whatever we are getting it as. The first equation the coefficient of C 1 is 1 coefficient of C 2 is minus 1. So, that is minus 1 C 3 is not there so this is 0; C 5 is not there, so that is 0 ok. So, that should be equal to minus PL square by 18.

Now, equation 4 gives me that, equation 4 gives me that coefficient of C 1 is 0 I bring C 3 to the side of C 2. So, C 2 coefficient is 1, C 3 coefficient is minus 1, C 5 is not there so that is 0 and that should be equal to PL square by 6 minus because I have minus PL square by 18 minus PL square by 9 this will be minus PL square by 18 into 3 into 3 by 18. So, that will be PL square by 6.

Now, the fifth equation as C 1 coefficient is L by 3. I will take it to the other side of the equation. So, that will be minus PL by 3 C 2 coefficient is L by 3 there is no C 3, C 5 is 1 and this will be equal to PL cube by 6 into 27. The 4th equation which is a 6th equation that we have here we will bring C L by 3 onto the left hand side. So, I have no C 1 contribution there. So, C 2 has coefficient 2 by 3, C 3 has coefficients L by 3 and C 5 has coefficient 1 and I have to take PL cube by 6 into 9 to the other side. So, this will be PL cube by 27 into 6 minus PL cube by 6 into 9 which is PL cube by 27 into 1 by 6 minus 1 by 2 right ok. So, this will be PL cube by 27 into 6 into 2 so that will be PL cube by 81. So, this will be PL cube by 81 ok.

Student: (Refer Time: 15:17).

Yeah minus, minus PL cube by 81. Now, we solve for this constants I will get C 1 to be we solve for this constants I will get C 1 to be PL square by 9 minus PL square by 9, C 2 would be minus PL square by 18, C 3 would be PL square by 9, C 5 would be minus PL cube by 81. Now, let us go ahead and substitute these values of constants into the equations and see what the slope at different points is an or a deflected shape at different points is.

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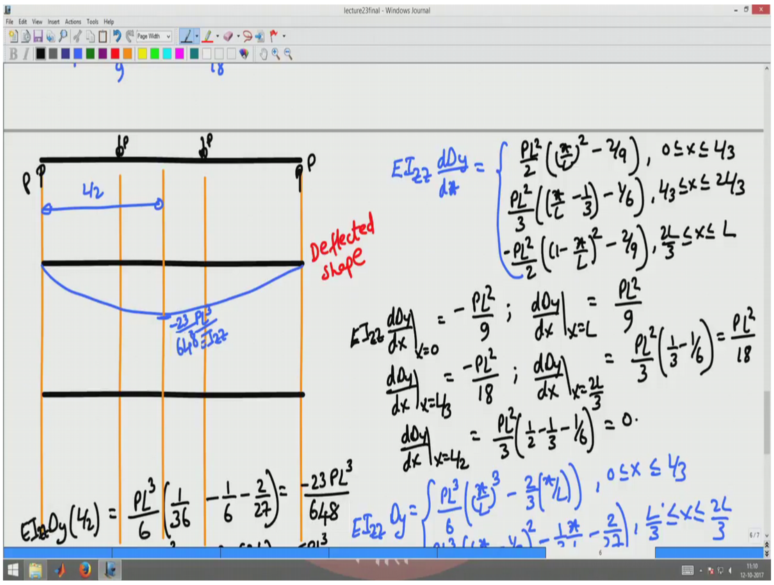

So, I we have E times E times I zz into d delta y by dx given by P into L square by 2 x by L the whole square minus 2 by 9 into PL square by 3 into x by L minus 1 by 3 minus 1 by 6 PL square by 2 minus 1 minus x by L the whole square minus 2 by 9. This is for 0 less than x less than L by 3, L by 3 less than or equal to x less than or equal 2 L by 3 this is 2 L by 3 less than or equal to x less than or equal to L ok.

Now, let us find the specific values of this slope. Let us say this is d delta y by dx at x equal to 0 would be minus, minus PL square by 9 and d delta y by dx at x equal to L this are the slopes or the supports this will be substituting in the final equation, substituting in this equation x equal to L I will get that this is nothing, but PL square by 9. They have to add the same angle, but opposite in direction because one rotates in the clockwise direction whereas, the other one rotates in the anticlockwise direction it will be evident when we draw the deflected shape.

So, now, similarly d delta y by dx at x equal to L by 3 would be minus PL square by 18 we obtain this by substituting x equal to L by 3 in this equation and d delta y by dx at x equal to 2 L l by 3 would be substituting again in the equation substituting again x equal to 2 L by 3 in this equation it will be PL square by 3 into 1 by 3 minus 1 by 6 which will be PL square by 18 it will be PL square 18.

This again should have opposite sign, but same in magnitude because of symmetry and because the deflected shape is slope has to be same at equidistance from the centre of the beam. Now, finally, d delta y by dx at x equal to 1 by 2 will be PL square by 3 into 1 by 2 minus 1 by 3 minus 1 by 6 which is 0. That should happen because the maximum deflection will occur at the midspan because of symmetry again.

Now, let us see what is E I zz into delta y this is now, PL cube by 6 x by L the whole cube minus 2 by 3 into x by L for 0 less than x less than L by 3 PL cube by 6 into x by L minus 1 by 3 the whole square minus 1 by 3 into x by L minus 2 by 27. This is at L by 3 less than x less than or equal to 2 L by 3 and final equation would be P L cube by 6 into 1 minus x by L the whole cube minus 2 by 3, 1 minus x by L for 2 L by 3 less than or equal to x less than or to L.

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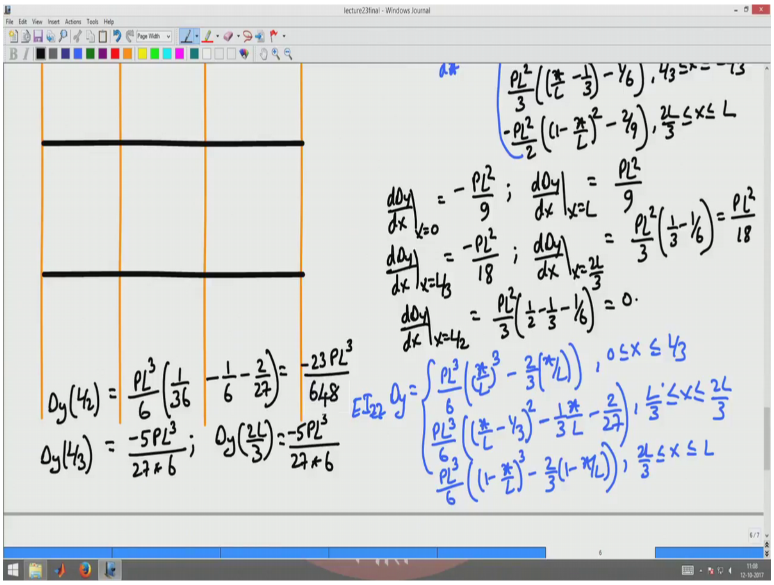

Now, you are interested in finding delta y at 0 is delta y at 0 is from this equation you can see that it is 0 delta y at x equal to L is 0 from this equation. Then we are interested in finding the delta y at L by 3 and delta y at L y 2 at delta y at 2 L by 3. So, delta y at L by 2 would be PL cube by 6 substituting in this equation it will be PL cube by 6 into 1 by 2 minus 1 by 3 is 1 by 6 it will be 1 by 36 minus 1 by 6 minus 2 by 27 which will be minus 23 PL cube by 648.

Similarly delta y at del by 3 is going to be substituting x equal to L by 3 in this equation or in this equation we will get it as minus 5 PL cube by 27 into 6. Similarly delta y at 2 L by 3 would be substituting x equal to L by 3 in either this equation or this equation you will get it as again minus 5 PL cube by 27 into 6, 27 into 6 ok.

So, now, let us the deflected shape for this beam subjected to two point loads P P and the reaction force P P here. First we are plotting the deflected shape. We can find that all the deflections are negative right from L by 2 and L by 3. So, the deflected shape that we get would be of this form, would be of this form wherein the maximum deflection will occur at L by 2, maximum deflection will occur at L by 2, this distance is L by 2, if this distance is L by 2 the maximum deflection occurs here which is minus 23 by 648 PL cube by E times I zz.

All the things I left E times I zz should come under denominator because E times I zz is that. So, in all this expressions I should have E times I zz. So, now, that is minus PL cube by 648 E I zz. Similarly at L by 3 and 2 L by 3 it will be here it is going to be minus 5 PL cube by 162 times EI zz. Same value here too here also it is going to be the same as this ok.

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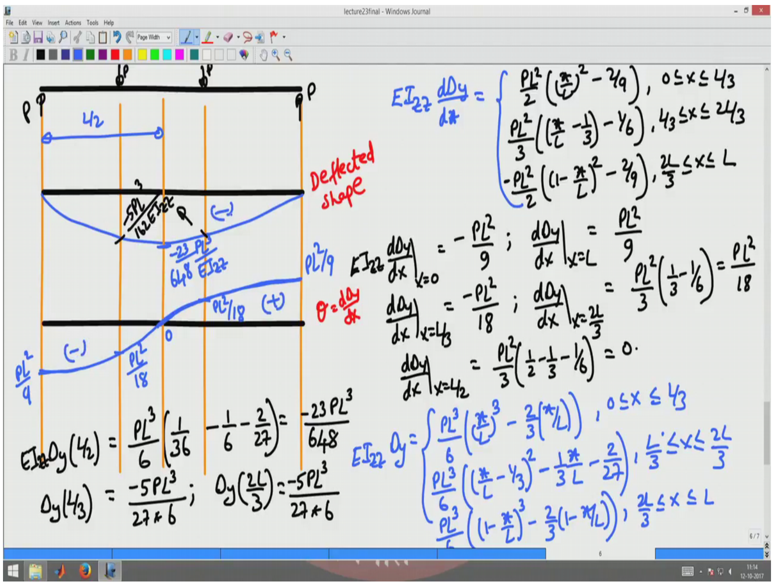

Now, let us draw the slope that is this is variation of theta that is t delta y by dx. We find that the slope is 0 at L by 2 because as expected the deflection is maximum there and then it varies from in the negative in the left hand side of the midsection to positive the right hand side of the midsection. So, the variation is something like a parabolic curve. So, it does that.

Wherein here it is PL square by 9 this is a straight line, this also a straight line there. Slope of this has to be the bending moment diagram, so it has to be 0 at the supports.

This is PL square by 9, this is 0 here, at L by 3 it is PL square by 18, PL square by 18, this is positive, this is negative, this all negative deflected shape.