Mechanics of Material
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Stresses and deflection in beams loaded about one principal axis
Lecture - 65
Shear force and bending moment diagram

Welcome to the 23rd lecture on Mechanics of Materials. The previous 3 lectures we looked at the bending equation which was minus sigma xx by y minus y naught is equal to M x by I zz equal to E by r.

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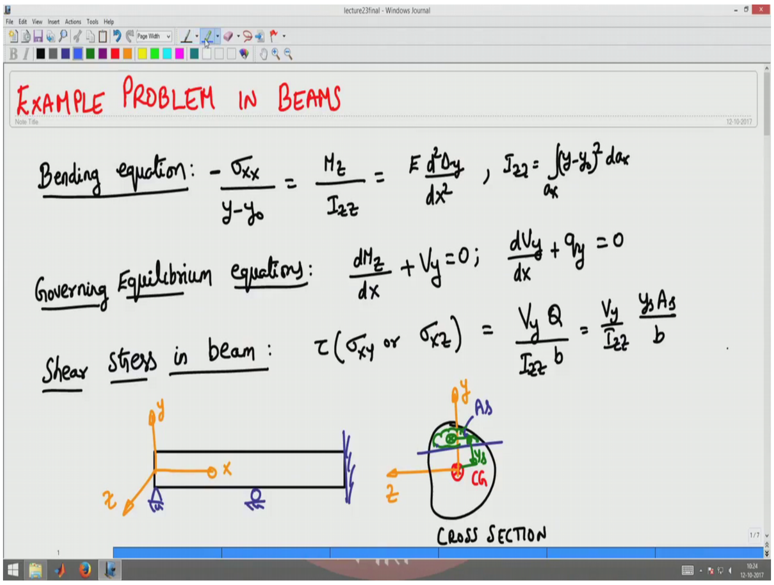

We looked at this equation, we looked at the derivation of this equation where I zz was y minus y naught the whole square integrated over the cross sectional area. We also saw that the governing equilibrium equations for the beam bending problem was the derivative of the bending moment with respect to the axial distance of the beam plus the shear force of V is equal to 0 and derivative of the shear force plus the applied load per unit load q y has to be 0 here.

We also saw that the shear stress in the beam is given by the shear force times Q by I zz times b, where Q is nothing but the area of the cross section up to the section point that which you are interested in computing the shear this is the As indicated in this diagram times the centroid of this cross section from the neutral axis of the cross section. So, we saw that this is given by ys times As by b this is the shear stress in the beam ok.

Now, what we will do is we will apply this equations to solve a practical problem which is a two point beam bending problem. We want to solve this two point beam bending problem and find the bending moment diagram, shear force diagram and draw the deflected shape of this beam.

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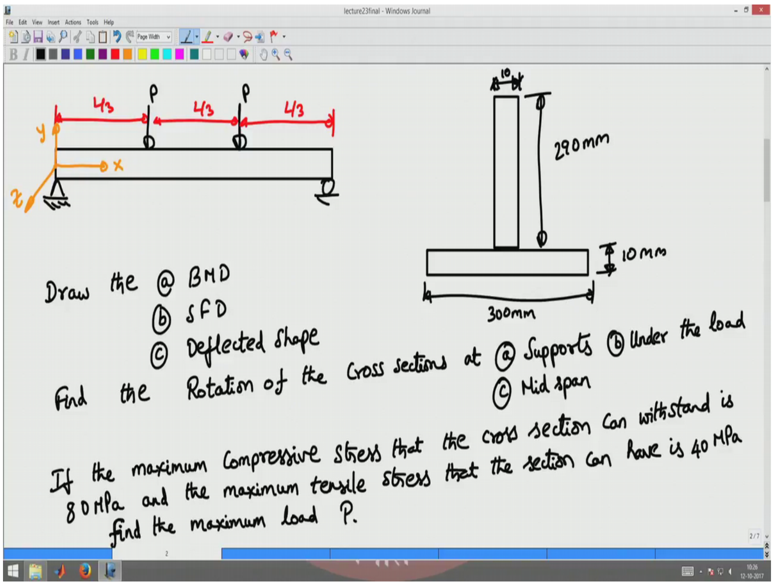

Apart from finding what is the rotation of the cross section at supports under the load and mid span. Also I assume that this beam as a cross section how in the form of a t with dimensions with the width of the flange being 300 mm, the thickness of this flange being 10 mm, the thickness of the web being again 10 mm and the depth of the web being the t being 290 mm. So, with this as the dimensions of the cross section we are interested in finding what will be the maximum load P that can be applied.

So, that the maximum compressive stress does not exceed 80 mpa and tensile stress does not exceed 40 mpa mega Pascal's. So, let us go ahead and first find the variation of the shear force.

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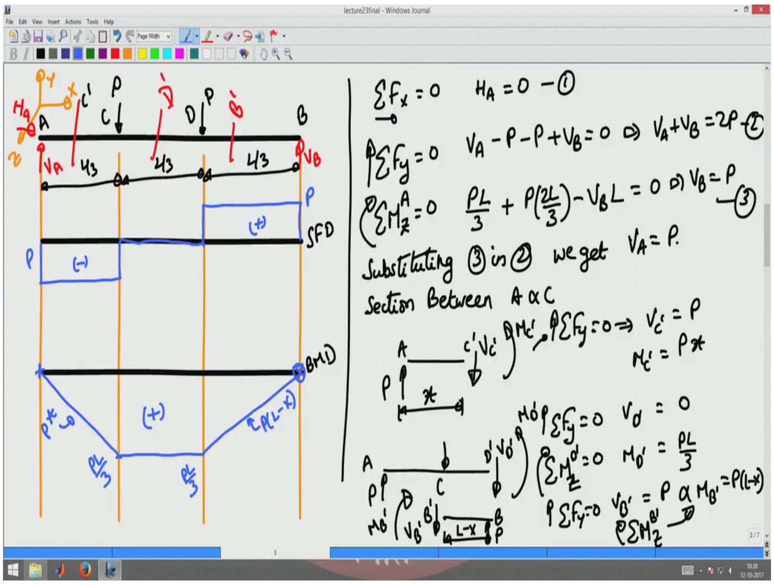

This is simply supported beam subjected to two point loads P and P. The support reactions are going to be vertical reaction force at A, horizontal reaction force at A and then vertical reaction force at B because B is a roller the roller B and horizontal reaction force at B ok.

Now, we will write the coordinate system that we have assumed for the beam is x y and z. So, what I will do is I will first write the force equilibrium along the x direction to be 0 that is horizontal direction to be 0, this will tell me since there is no other horizontal force other than H A, H A has to be 0. This is a simple problem, but you have to follow this systematic procedure to solve any complicated problem also. So, I am illustrating the ideas in a systematic manner using the simple problem.

So, now, next equation we have is F of y the vertical force equilibrium I am taking the upper (Refer Time: 04:24) trigonometrical force as positive. So, that will give me V A minus P minus P plus V B equal to 0 this will give me V A plus V B is equal to 2 P. So, to name the location this is A, this is B, the point of action of the loads are C and D respectively.

Now, the other equation that I have is the moment equilibrium equation is a plane problem. So, I will have only 3 equations. So, the next equation I have is M Z moment has to be 0 I am taking anticlockwise moment as positive and I am taking this moment equilibrium about the point A, moment equilibrium about point A.

So, what will happen the point load acting at C produces a anticlockwise moment which is P times the lever arm or the distance from A to C, which is L by 3, C to D is also L by 3 and D to B is also L by 3 that is a problem definition. So, you have P into L by 3 plus the point load at D produces a anticlockwise moment. Now, the lever arm for point load at D is L by 3 plus L by 3 that will be 2 L by 3 plus the shear plus the reaction force at B produces a and clockwise moment which is minus V B into , the distance of B from the point , point A is L. So, this has to be equal to 0. From here we get that V B has to be P.

Now, substitute this V B is equal to P in the second equation say this is the equation 1, this is equation 2, equation 3, substituting 3 in 2 we get V A also to be equal to P ok. Now, take a section between A and C, A and C I take a section here. So, what happens? If I cut there the shear force and the bending moment at the cut section stands exposed. So, basically I will have let us say this one was C prime, A C prime at a as a reaction force P. So, the force and moment equilibrium of this cut section will tell me the shear force V C prime and the moment M C prime is got from the vertical force equilibrium of this section, sigma F y equal to 0 of this section tells me that V C prime must be must be equal to P and M C prime must be equal to P into this distance, this distance is x, P into x.

So, the first line here is I want to draw the shear force diagram and then this is a bending moment diagram in the second line. So, the shear force is P is acting vertically downward in a phase which has the positive normal pointing along the positive direction of the coordinate system. The positive normal at C is pointing along the positive direction of the coordinate system, but the shear force is acting in the negative direction of y and hence this shear force is a negative shear force ok.

So, I have P here negative shear force P, P, just negative shear force there. Now, the dash of moment is clockwise the positive is anticlockwise in the positive normal direction. So, that is a positive moment. This positive moment will produce a tension at the bottom fibre of the beam because the beam is bending like this there will be tension in the bottom fibre and compression in the top fibre, we want to draw the bending moment diagram in the tension side meaning the line that we draw will remain the tension side of the beam.

So, basically the tension occurs at the bottom. So, even though this is a positive moment I draw it in the bottom of this beam. So, basically it is a linear function when x equal to 0 the bending moment is 0. So, I have it here, that is required because this is a ink jet A. So, basically I have bending moment varying like this P into where this moment is P L by 3, because x is L by 3 there and this variation is given by P into x.

So, basically we got the variation of the bending moment in for A section C prime between A and C. Now, let us do the same thing for a section D prime between C and D. Now, what happens the free bar diagram for this cut section would be vertical force P acting at A, downward force acting at C and the D prime where there will be a shear force V D prime possibly acting there and the moment M D prime acting there.

Now, again vertical force equilibrium will tell me that F y equal to 0 will tell me that V D prime is 0 and moment equilibrium about D prime of the z moment taking this as 0 will tell me that M D prime must be equal to P L by 3, M D prime will be equal to P L by 3. So, the shear force is 0 which means that is easy to draw the line remain on the beam that is 0.

The bending moment is also caution in this section which is P L 3. So, it will be like this constant bending moment P L by 3 which is evident because the shear force is 0 in this section. So, the bending moment cannot vary.

Now, coming to the section between D and B, again I take a section B prime which is between D and B and if I look at the other half from B to B prime I will get B to B prime there is this P and this distance between B to B prime will be L minus x, where x is measured from A. So, the length of the beam is L that is the distance from A to B is L and hence the distance of B prime will be L minus x. This you can see is a mirror image of this A C prime cross section. So, I will have a shear force V B prime acting vertically down and the moment which is a clockwise moment M B prime acting there similar to as above you will write the vertical force equilibrium to get V B prime as P and M B prime as P into L minus x that comes from the moment equilibrium M Z about B prime taking anticlockwise moment as positive ok.

So, this shear force is a positive shear force because in the positive normal direction at B the shear force is acting vertically upwards, at B ratio therefore, starting vertically upwards, but is now positive y direction for positive x direction which coincides with the normal direction of the cut at B. So, this is a positive shear force. So, I will have this shear force going up now by P and remaining constant as P there. This is a positive shear force with P there.

On the other hand bending moment at x equal to L is 0. So, this is 0 and it varies as P into L minus x. It is a linear variation from here to there. When x equal 2 L by 3 you can see that the bending moment is L by 3, P L by 3. So, there is from P L by 3 to 0 as P into L minus x that is the variation of the bending moment here.

This variation of bending moment is positive even though you are plotting it below the beam axis this is because you want to plot a bending moment along the fibre which is in tension. So, the bottom fibre is in tension now, so you are plotting the bending moment diagram in the tension side which is below the beam. So, now, we have got the shear force and a bending moment diagram, next we want to get the deflected shape.