Mechanics of Material

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Stresses and deflection in beams loaded about 1 principal axis
Lecture- 64

Expression to find shear center

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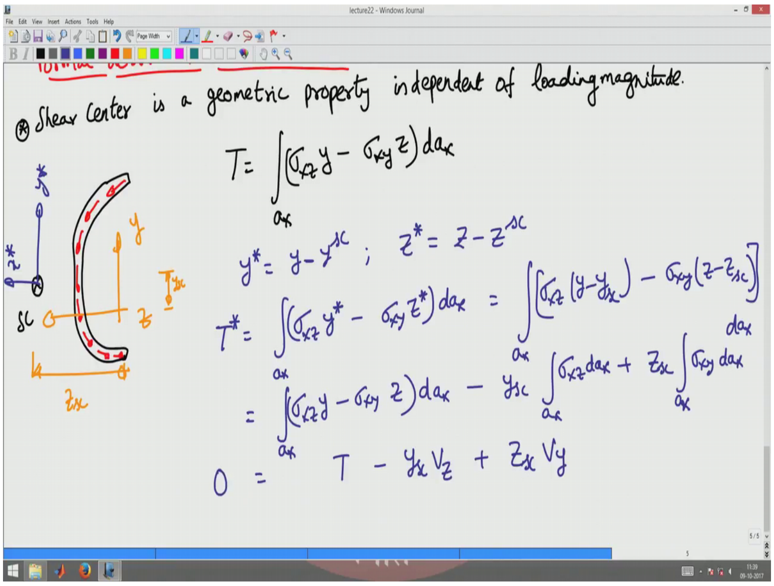

So, this is geometric property first point notice shear center is a geometric property independent of loading magnitude. So, let us say I have some arbitrary cross section, now in this cross section there will be both the shear sigma x y and sigma x z stresses coming in this cross section because this is inclined under this thin wall there will be a shear flow like this in this cross section.

If you recollect in our previous lecture when it started bending moment we define the torsional moment T as integral sigma x z y minus sigma x y z times d a x a x right. This was sort of definition of torsional moment about a chosen origin, now I want to take a torsional moment about of the shear center right shear center not about some origin, but I want to take the torsional moment about shear center the original coordinate system was y and z like this.

So, now I have to move this by an amount z s c and by an amount y s c that is a shear center with reference to the origin chosen to find these shear stresses. Then what happens so now, what happens when I incorporate this when I shift the origin, my y star now I have to compute the torsional moment about T star, or let me define a star coordinate system here this y star this z star, y star is related to y as y plus y s c no y minus y s c, now similarly z star is related through z as z minus z s c right.

So, now, the torsional moment about T star which is integral sigma x z y star minus sigma x y z star d a x a x would be integral sigma x z y minus y s c minus sigma x y z minus z s c into d a x a x. This I am rewrite it as integral sigma x z y minus sigma x y z d a x minus y s c integral sigma x z d a x a x a x, here plus z s c integral sigma x y d a x a x here.

Now what happens this is initial torsion moment minus y s c times this is nothing, but shear force v z this is nothing, but shear force v z plus z s c times the shear force v u y and what is the definition of shear center the torsion moment opposite shear center is 0. So, this has to be 0 now let us consider case wherein.

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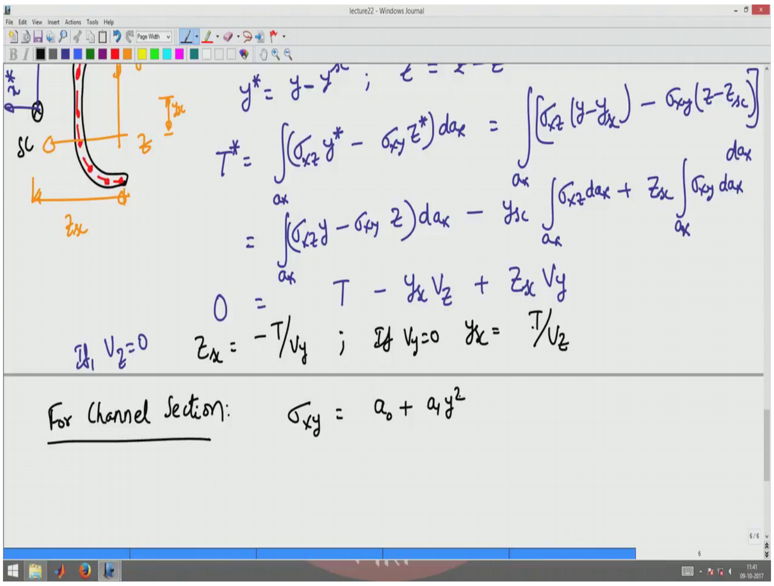

If Vz is 0, the case where we have been looking at all along then zsc is given by minus T divided by Vy, similarly if Vy is 0 then this is bending about direct direction alone we will see subsequently ysc would be given by T divided by Vz. Here, the torsion direction of T is anticlockwise moment positive only then from right hand thumb rule we will get the excess depositive x direction.

So, that is why you have minus T by Vy stress is at Hc. So, this how you find the shear center, so basically you find the torsion moment due to these stresses. Now let us ponder a little bit more on y sigma xy doesn't produce, any moment in this case any torsional moment in this case we found that for the channel section sigma xy was for channel section, sigma x y was of the form some a naught plus a 1 y square or minus a 1 y square right and I am write that as such.

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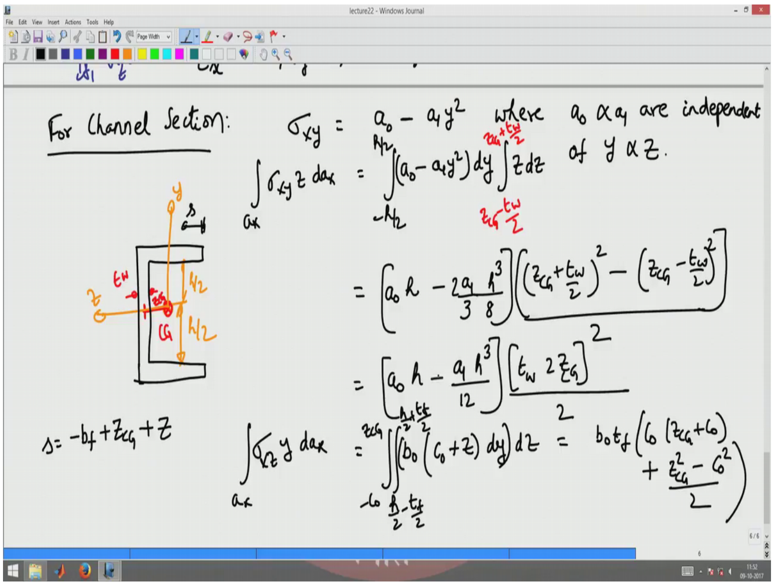

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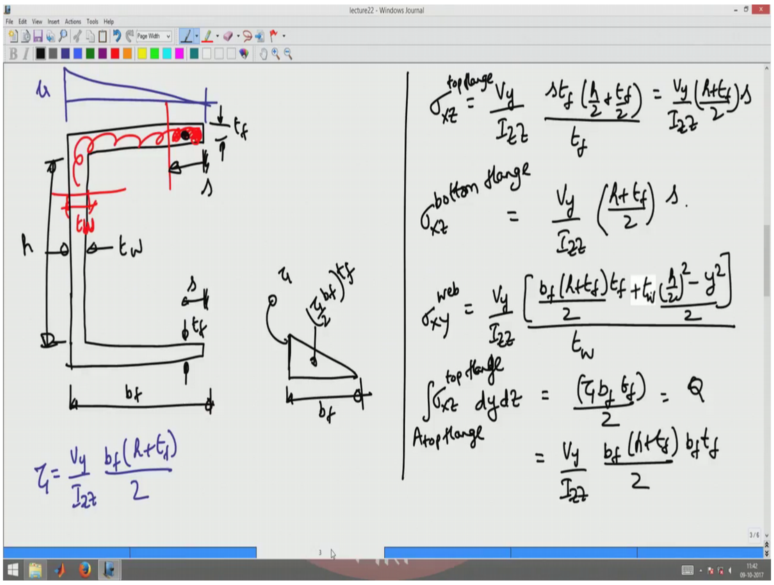

If you go back here we will find that if you go back here all this is some constant times constant minus some constant times square, y is a variable about which this stresses varies. So, basically this how sigma xy varies or sigma xz varies linearly with the independent variable which is s in this case.

So, coming back here I am write this as some function of minus a 1 times y square where a 0 a 1 are independent of independent of y and z the coordinate questions. Now what am I interested in, I am interested in finding integral sigma xy into z d a x, now for the channel section this would be integral a naught minus a 1 y square dy ok.

It varies from minus H by 2 to H by 2 into z, which varies from which is dz which varies from minus bf plus zcg to zcg, this is y this is z y varies from H by 2 to H by 2 and z varies from this is the cg this is a cg from the cross section it varies from this is tw the thickness of web is tw view the central line of the web is the web is at a is at zcg it will vary from zcg minus tw by 2, 2 is at zcg plus tw by 2.

So, now, what happens what if I integrate this I will get it as a 0 into H minus a 1 by 3 into 2 into H cube by 8 right, that is this part and the z part will be zcg minus plus tw by 2 whole square minus zcg minus tw by 2 the whole square by 2.

So, this will be a naught H minus a 1 H cube by 12 into by 2 a plus b would be tw a minus tw into 2 times zcg. So, that will be the moment produced due to sigma xz into z d a x, now integral sigma xz into y d a x as we computed ax sigma xz is a linear function in s which will be this second write it as some b naught into s, where s is measured from here that is s. So, how was that related to z that will be bf s would be would be minus bf plus zcg plus z.

So, that will be b naught into some constant let us say c naught plus z dydz dydz are in this ys from minus H by 2 plus tf by minus tf by 2 to H by 2 plus tf by 2 and z is from minus c naught to minus c naught to zcg. So, if I do this computation what I will get is I will get it as b naught into tf into c naught into zcg plus c naught plus z square by 2 which will be zcg square minus c naught square by 2 will be sigma xz into y. So, now I have to add these 2 get the net torsional moment.

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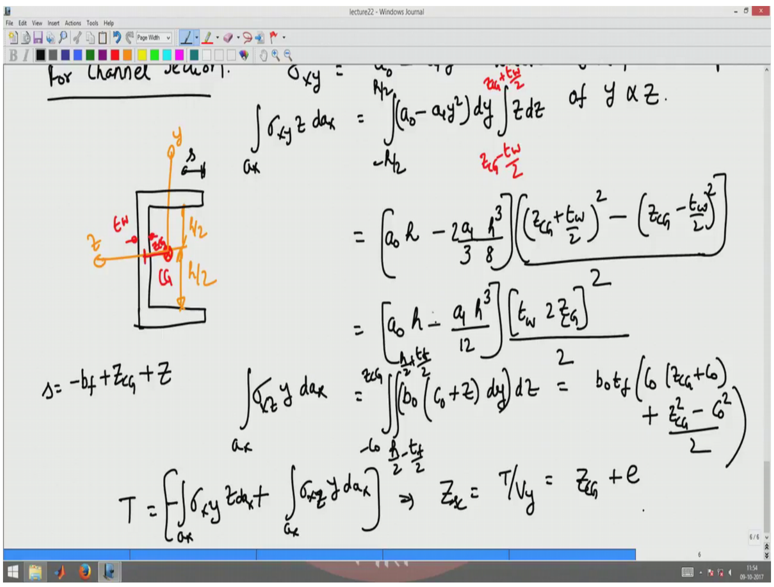

This will be integral sigma xy into z ax minus integral sigma xz into y d a x, let us subtract these to get the net torsional moment that I have to divide by that is the net torsional moment. So, from here I get zsc as T divided by Vy this expression will include will be in fact, zcg plus e where e is same as what we have computed before. So, you have to run through calculations to get through up to this step, now the final thing I want to show you is.

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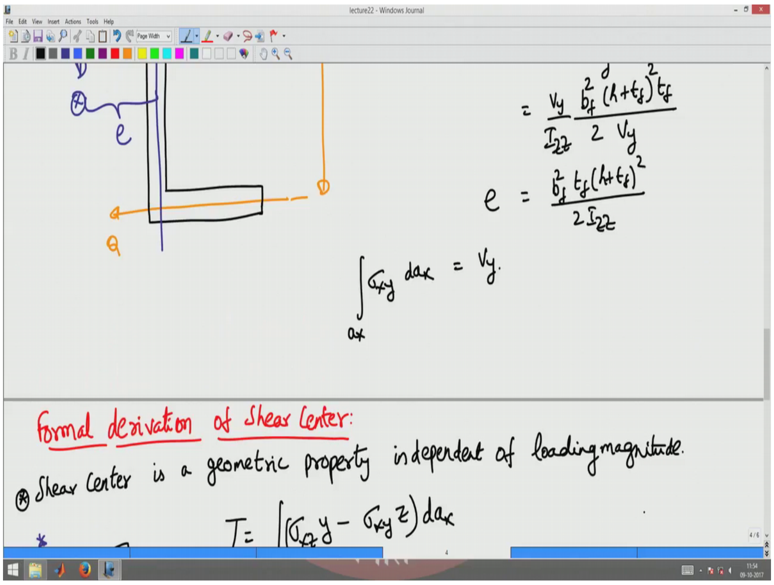

This integral sigma xy d a x is equal to Vy, I will try it for rectangular section and you can do it for other sections for rectangular section you found that.

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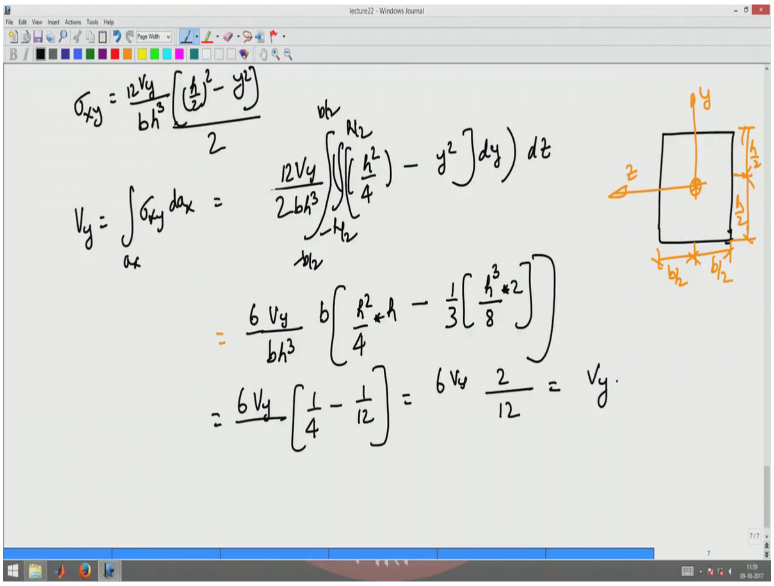

Sigma xy is Vy by bh cube 12 into h by 2 whole square minus y square divided by 2. So, now let us integrate this now Vy has to be integral sigma xy d a x ax.

So, this will be 12 Vy by 2 bh cube integrated over h square by 4 minus y square, dy integrated from minus h by 2 to h by 2 this entire thing integrated over dz from minus b by 2 to b by 2, because I am looking at a rectangular cross section with this as y and this as z.

So, this is a rectangular cross section of cg here wherein this distance is h by 2 and this distance is h by 2 and this width is b by 2 b by 2 each. So, this will be the 6 Vy by bh cube if I integrate dz I will get b into h square by 4 into h minus y cube by 3. So, it is 1 by 3 into h cube by 8 into 2.

So, what is this bb cancels h cube, h cube cancels this 6 into 1 by 4 minus 1 by 12 right. So, this is nothing, but Vy by 2 divided 12 into 2. So, that will be equal to Vy, so basically that is why integral sigma xy d a is equal to Vy we will stop here for today's lecture.

Thank you.