

Mechanics of Material
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Stresses and deflection in beams loaded about 1 principal axis
Lecture- 64
Expression to find shear center

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Shear Center is a geometric property independent of loading magnitude.

$$T = \int_0^x (\sigma_{xz} y - \sigma_{xy} z) dx$$

$$y^* = y - y_{sc}; \quad z^* = z - z_{sc}$$

$$T^* = \int_0^x (\sigma_{xz} y^* - \sigma_{xy} z^*) dx = \int_0^x (\sigma_{xz} (y - y_{sc}) - \sigma_{xy} (z - z_{sc})) dx$$

$$= \int_0^x (\sigma_{xz} y - \sigma_{xy} z) dx - y_{sc} \int_0^x \sigma_{xz} dx + z_{sc} \int_0^x \sigma_{xy} dx$$

$$0 = T - y_{sc} V_z + z_{sc} V_y$$

So, this is geometric property first point notice shear center is a geometric property independent of loading magnitude. So, let us say I have some arbitrary cross section, now in this cross section there will be both the shear σ_{xy} and σ_{xz} stresses coming in this cross section because this is inclined under this thin wall there will be a shear flow like this in this cross section.

If you recollect in our previous lecture when it started bending moment we define the torsional moment T as integral $\sigma_{xz} y$ minus $\sigma_{xy} z$ times dx right. This was sort of definition of torsional moment about a chosen origin, now I want to take a torsional moment about of the shear center right shear center not about some origin, but I want to take the torsional moment about shear center the original coordinate system was y and z like this.

So, now I have to move this by an amount $z_s c$ and by an amount $y_s c$ that is a shear center with reference to the origin chosen to find these shear stresses. Then what happens so now, what happens when I incorporate this when I shift the origin, my y^* now I have to compute the torsional moment about T^* , or let me define a star coordinate system here this y^* this z^* , y^* is related to y as $y + y_s c$ no $y - y_s c$, now similarly z^* is related through z as $z - z_s c$ right.

So, now, the torsional moment about T^* which is $\int (\sigma_{xz} y^* - \sigma_{xy} z^*) dx$ would be $\int (\sigma_{xz} (y - y_s c) - \sigma_{xy} (z - z_s c)) dx$. This I am rewrite it as $\int (\sigma_{xz} y - \sigma_{xy} z) dx - y_s c \int \sigma_{xz} dx + z_s c \int \sigma_{xy} dx$ here.

Now what happens this is initial torsion moment minus $y_s c$ times this is nothing, but shear force V_z this is nothing, but shear force V_z plus $z_s c$ times the shear force V_y and what is the definition of shear center the torsion moment opposite shear center is 0. So, this has to be 0 now let us consider case wherein.

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The image shows a handwritten derivation on a whiteboard. On the left, there is a diagram of a channel section with a coordinate system. The origin is at the center of the web, with the x -axis along the length of the channel and the z -axis pointing downwards. The shear center is located at a distance z_x from the x -axis and y_x from the z -axis. The shear flow is shown as arrows along the top and bottom flanges.

The derivation starts with the expression for the torsional moment about the shear center T^* :

$$T^* = \int (\sigma_{xz} y^* - \sigma_{xy} z^*) dx = \int (\sigma_{xz} (y - y_x) - \sigma_{xy} (z - z_x)) dx$$

$$= \int (\sigma_{xz} y - \sigma_{xy} z) dx - y_x \int \sigma_{xz} dx + z_x \int \sigma_{xy} dx$$

Setting $T^* = 0$ and using the definitions of shear forces $V_z = \int \sigma_{xz} dx$ and $V_y = \int \sigma_{xy} dx$, we get:

$$0 = T - y_x V_z + z_x V_y$$

From this, the coordinates of the shear center are determined:

$$z_x = -T/V_y \quad ; \quad \text{if } V_y = 0 \quad y_x = T/V_z$$

For a channel section, the shear stress distribution is given by:

$$\sigma_{xy} = a_0 + a_1 y^2$$

If V_z is 0, the case where we have been looking at all along then $z_s c$ is given by minus T divided by V_y , similarly if V_y is 0 then this is bending about direct direction alone we will see subsequently $y_s c$ would be given by T divided by V_z . Here, the torsion direction

of T is anticlockwise moment positive only then from right hand thumb rule we will get the excess depositive x direction.

So, that is why you have minus T by Vy stress is at Hc. So, this how you find the shear center, so basically you find the torsion moment due to these stresses. Now let us ponder a little bit more on y sigma xy doesn't produce, any moment in this case any torsional moment in this case we found that for the channel section sigma xy was for channel section, sigma x y was of the form some a naught plus a 1 y square or minus a 1 y square right and I am write that as such.

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For channel section:

$\sigma_{xy} = a_0 - a_1 y^2$ where $a_0 \propto a_1$ are independent of y & z .

$\int \sigma_{xy} z \, dA_x = \int_{-h/2}^{h/2} (a_0 - a_1 y^2) dy \int z \, dz$

$= \left[a_0 h - \frac{2a_1 h^3}{8} \right] \left[\left(z_c + \frac{t_w}{2} \right)^2 - \left(z_c - \frac{t_w}{2} \right)^2 \right]$

$= \left[a_0 h - \frac{a_1 h^3}{12} \right] \left[t_w^2 z_c^2 \right]$

$\int \sigma_{xy} y \, dA_x = \int_{-b_0/2}^{b_0/2} \int_{-t_w/2}^{t_w/2} (b_0 + z) dy \, dz = b_0 t_w \left(b_0 \left(z_c + \frac{t_w}{2} \right) + \frac{z_c^2 - t_w^2}{2} \right)$

$\Delta = -b_1 + z_c + z$

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The image shows a whiteboard with handwritten diagrams and formulas. On the left, there is a diagram of a channel section with dimensions: height \$h\$, web thickness \$t_w\$, flange width \$b_f\$, and flange thickness \$t_f\$. A coordinate system is shown with \$y\$ and \$z\$ axes. A stress distribution diagram shows a linear variation of stress across the height \$h\$. On the right, there are several formulas for stress components:

$$\sigma_{xz}^{\text{top flange}} = \frac{V_y}{I_{zz}} \frac{t_f \left(\frac{h}{2} + \frac{t_f}{2} \right)}{t_f} = \frac{V_y (h+t_f)}{I_{zz}} s$$

$$\sigma_{xz}^{\text{bottom flange}} = \frac{V_y}{I_{zz}} \left(\frac{h+t_f}{2} \right) s$$

$$\sigma_{xy}^{\text{web}} = \frac{V_y}{I_{zz}} \left[\frac{b_f (h+t_f) t_f + t_w \left(\frac{h}{2} \right)^2 - y^2}{t_w} \right]$$

$$\int \sigma_{xz}^{\text{top flange}} dy dz = \frac{(7 b_f t_f)}{2} = Q$$

$$\sigma_{xz}^{\text{A top flange}} = \frac{V_y}{I_{zz}} \frac{b_f (h+t_f)}{2} b_f t_f$$

At the bottom left, there is a formula for the shear flow \$q\$:

$$q = \frac{V_y}{I_{zz}} \frac{b_f (h+t_f)}{2}$$

If you go back here we will find that if you go back here all this is some constant times constant minus some constant times square, \$y\$ is a variable about which this stresses varies. So, basically this how sigma \$xy\$ varies or sigma \$xz\$ varies linearly with the independent variable which is \$s\$ in this case.

So, coming back here I am write this as some function of minus a 1 times \$y\$ square where a 0 a 1 are independent of independent of \$y\$ and \$z\$ the coordinate questions. Now what am I interested in, I am interested in finding integral sigma \$xy\$ into \$z\$ d a \$x\$, now for the channel section this would be integral a naught minus a 1 \$y\$ square \$dy\$ ok.

It varies from minus \$H\$ by 2 to \$H\$ by 2 into \$z\$, which varies from which is \$dz\$ which varies from minus \$b_f\$ plus \$z_{cg}\$ to \$z_{cg}\$, this is \$y\$ this is \$z\$ \$y\$ varies from \$H\$ by 2 to \$H\$ by 2 and \$z\$ varies from this is the \$c_g\$ this is a \$c_g\$ from the cross section it varies from this is \$t_w\$ the thickness of web is \$t_w\$ view the central line of the web is the web is at a is at \$z_{cg}\$ it will vary from \$z_{cg}\$ minus \$t_w\$ by 2, 2 is at \$z_{cg}\$ plus \$t_w\$ by 2.

So, now, what happens what if I integrate this I will get it as a 0 into \$H\$ minus a 1 by 3 into 2 into \$H\$ cube by 8 right, that is this part and the \$z\$ part will be \$z_{cg}\$ minus plus \$t_w\$ by 2 whole square minus \$z_{cg}\$ minus \$t_w\$ by 2 the whole square by 2.

So, this will be a naught \$H\$ minus a 1 \$H\$ cube by 12 into by 2 a plus \$b\$ would be \$t_w\$ a minus \$t_w\$ into 2 times \$z_{cg}\$. So, that will be the moment produced due to sigma \$xz\$ into \$z\$ d

a x, now integral sigma xz into y d a x as we computed ax sigma xz is a linear function in s which will be this second write it as some b naught into s, where s is measured from here that is s. So, how was that related to z that will be bf s would be would be minus bf plus zcg plus z.

So, that will be b naught into some constant let us say c naught plus z dydz dydz are in this ys from minus H by 2 plus tf by minus tf by 2 to H by 2 plus tf by 2 and z is from minus c naught to minus c naught to zcg. So, if I do this computation what I will get is I will get it as b naught into tf into c naught into zcg plus c naught plus z square by 2 which will be zcg square minus c naught square by 2 will be sigma xz into y. So, now I have to add these 2 get the net torsional moment.

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for channel section.

$$I_y = \int_{-h/2}^{h/2} \int_{-a/2}^{a/2} (a_0 - a_1 y^2) dy dz$$

$$= \left[a_0 h - \frac{2a_1 h^3}{3} \right] \left[\frac{t_w^2 z_{cg}^2}{2} \right]$$

$$= \left[a_0 h - \frac{2a_1 h^3}{3} \right] \left[\frac{t_w^2 z_{cg}^2}{2} \right]$$

$$Q_y = \int_{-a/2}^{a/2} (b_0 + z) dy dz = b_0 t_f \left(b_0 (z_{cg} + b_0) + \frac{z_{cg}^2 - b_0^2}{2} \right)$$

$$T = \int_{-a/2}^{a/2} \int_{-h/2}^{h/2} \sigma_{xy} z dx + \int_{-a/2}^{a/2} \sigma_{xz} y dx \Rightarrow z_{xc} = T / V_y = z_{cg} + e$$

This will be integral sigma xy into z ax minus integral sigma xz into y d a x, let us subtract these to get the net torsional moment that I have to divide by that is the net torsional moment. So, from here I get zsc as T divided by Vy this expression will include will be in fact, zcg plus e where e is same as what we have computed before. So, you have to run through calculations to get through up to this step, now the final thing I want to show you is.

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$$e = \frac{V_y \frac{b^2 (h+t)^2 t}{2 I_{zz}}}{2 I_{zz} V_y} = \frac{b^2 t (h+t)^2}{2 I_{zz}}$$

$$\int \sigma_{xy} dx = V_y$$

Formal derivation of Shear Center:

Shear Center is a geometric property independent of loading magnitude.

$T = \int (\sigma_{xz} y - \sigma_{xy} z) dx$

This integral $\sigma_{xy} dx$ is equal to V_y , I will try it for rectangular section and you can do it for other sections for rectangular section you found that.

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$$\sigma_{xy} = \frac{12 V_y}{b h^3} \left[\left(\frac{h^2}{4} \right) - y^2 \right]$$

$$V_y = \int \sigma_{xy} dx = \frac{12 V_y}{2 b h^3} \int_{-h/2}^{h/2} \left[\left(\frac{h^2}{4} \right) - y^2 \right] dy dz$$

$$= \frac{6 V_y}{b h^3} b \left[\frac{h^2}{4} dz - \frac{1}{3} \left[\frac{h^3}{8} \cdot 2 \right] \right]$$

$$= \frac{6 V_y}{b h^3} \left[\frac{1}{4} - \frac{1}{12} \right] = 6 V_y \frac{2}{12} = V_y$$

σ_{xy} is V_y by $b h^3$ cube 12 into h by 2 whole square minus y^2 divided by 2. So, now let us integrate this now V_y has to be integral $\sigma_{xy} dx$.

So, this will be $12 V_y$ by $2 b h^3$ cube integrated over h square by 4 minus y^2 , dy integrated from $-h/2$ to $h/2$ this entire thing integrated over dz from $-b$

by 2 to $b/2$, because I am looking at a rectangular cross section with this as y and this as z .

So, this is a rectangular cross section of cg here wherein this distance is $h/2$ and this distance is $h/2$ and this width is $b/2$ $b/2$ each. So, this will be the $6 V_y$ by bh cube if I integrate dz I will get b into h^2 by 4 into $h - y^3$ by 3 . So, it is $1/3$ into h^3 by 8 into 2 .

So, what is this b cancels h^3 , h^3 cancels this 6 into $1/4$ minus $1/12$ right. So, this is nothing, but $V_y/2$ divided 12 into 2 . So, that will be equal to V_y , so basically that is why $\int \sigma_{xy} dA$ is equal to V_y we will stop here for today's lecture.

Thank you.