Mechanics of Material
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Stresses and deflection in beams loaded about one principal axis

Lecture- 63

Shear center of Channel section

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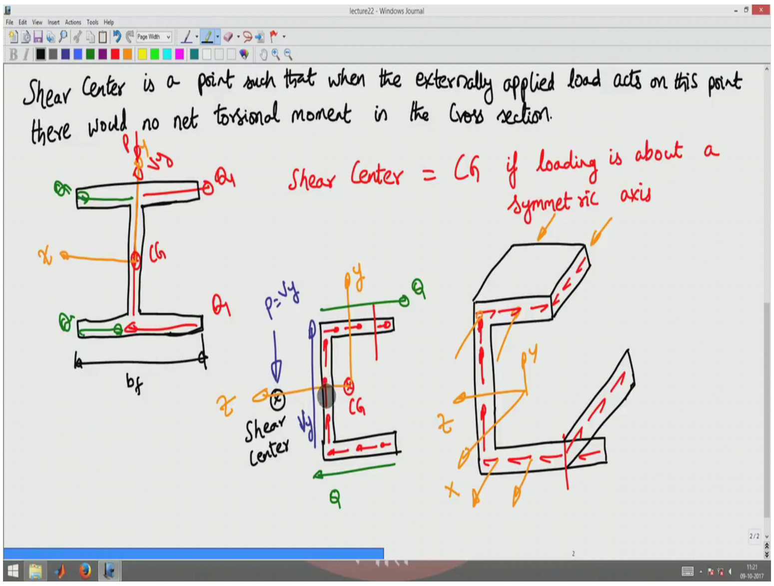

Now, let us proceed with the channel section or the C section here, how will the shear force how will the shear vary to the bending in m z direction? The shear stress is going to vary something like this. Let us make a cut here, make a cut there. If I make a cut there, and mz pass mz moment produces compressive force on the top flange. There is that compressive stress coming on the top flange, which is computed by mz by I z z into y minus y naught.

Let us assume that the this is y, this is z and this is positive x. Basically this stress here is going to be more than this stress here is going to be more than the stress there and against to balance that there should be a shear force or shear stress acting in this direction on this face. And hence a complementary shear will act like this, on that face. Similarly, if I cut at the bottom here the shear stress should act like this because there a tensor stress is acting on the bottom face there is this tensile stress acting at the bottom face. This is more and end in a section made here. That should be the shear stresses acting like this.

Because, the indicated force tensile force is more along the partial x direction for a m z model which is positive.

It has to way like this and in the web as usual the shear stresses has to vary like this. There is the sigma x y shear is a regular shear. In a channel section if you see the distribution of shear stress is something like this. They vary some shear stresses something like this. In a channel section the shear stress varies like this. Now, what happens there is the horizontal force equilibrium will demand that, this force Q should be balanced by this force Q and this vertical shear this vertical shear force Vy acting in the web, that lies up to Vy as we saw in the last class.

That should be balanced by the externally applied load P whose will be Vy. Now, what happens these Q produce this Q produce a clock wise moment torsion moment, the moment is acting along the CG or the cross section. It produces a clock wise moment that should be balance by the eccentrically applied load P to the shear force QVy acting in the cross section.

This produces a anticlockwise moment, what we are interested? Is we are interested in balancing this Vy times e with the Q times the h which is the lever arm there. We have to now compute this Q and we have to relate it to the distance e, which is from the shear center to the center of the web or the cross section.

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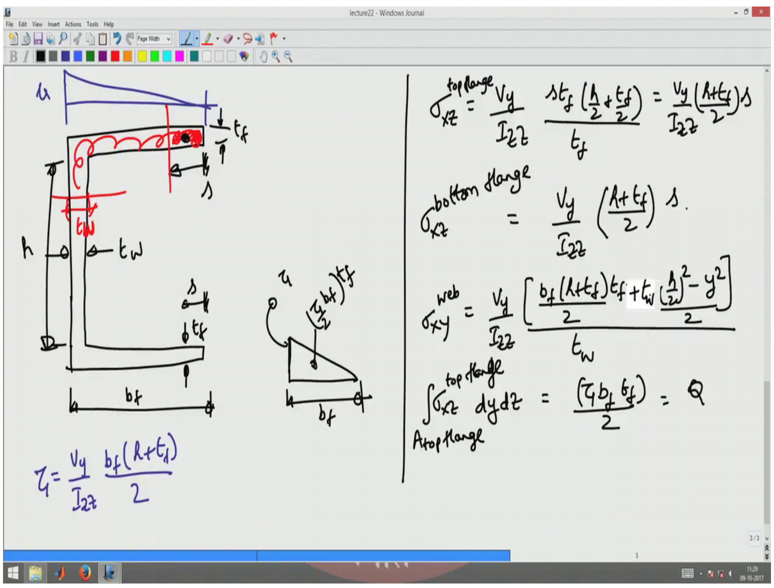

Let us go about doing that, I have a channel section. Now, I have to choose a s starting from here, we said that the s can be in the direction of the shear flow or opposite to the direction of shear flow, but cannot cross from opposite to in the direction of shear flow. So; that means, my shear s from here. Then for a section taken here I have to find y times the area indicated by that red shaded region. Sigma XZ would be Vy by I z z into let us assume that the flange is of are of thickness tf this flange is of thickness tf, web is of thickness uniform thickness TW is of uniform thickness TW the width of the flange is bf and the depth of the web is h.

Now, the area of the shaded region would be s times tf into the CG of that section, would be h by 2 plus tf by 2. That is going to be divided by the thickness of this cut which is tf. This is nothing but Vy by I z z into h plus tf by 2 into s. There is sigma XZ, in the top flange the bottom flange also I can measure as from here and it will have a similar variation. This is for a top flange, sigma x z for the bottom flange would have a similar variation for a same reasons that we said for top flange, h plus tf by 2 into s.

Now, in the web region as you know there is a sigma x y stress produced, sigma x y in the web. Just like what we did for I section and rectangular section is going to be Vy by I z z into bf into h plus tf by 2, into tf plus h by 2 the whole square minus y square. For a web area divide by TW. Basically, what I am doing here is I am cutting it here this thickness is TW this is TW. And I am finding this entire area now that entire area I have to find which I decompose into a flange area. The flange area gives me this component of the shear stress and the web area gives me this component of the shear stress. Web area times the center of that web area, give me this component of the shear stress. By the way I left a 2 here by 2.

That is on the web, now what I want to do is I want to compute integral sigma XZ top flange into dy dZ for area of top flange. From here, you see that the stress varies linearly from here you see that the stress is varying linearly. And hence, I can directly find this variation. I know that the stress is varying linearly, that is a linear variation. This being tau1 a tau1 is tau1 would be Vy by I z z into bf into h plus tf by 2. Because, I substitute for s bf in this expression I substitute for s bf in this expression because s is bf because this width is bf. I get that as tau1.

Now, this integral I can evaluate it as tau1 into bf into tf by 2. Because, is linearly varying the sum of the loads total load would be 1 half of the height of the triangle region times the length of the base. If I have load varying like this where here the force is this force magnitude was to be tau 1 bf into tf sorry if the stress were to be tau 1 there, the stress where to tau1 there, then the net stress acting one this region would be and if this width were bf. The net load acting on this area would be tau 1 by 2 to bf into tf. That is the cross-sectional dimension of the flange.

That is what this one is. Found essentially your Q.

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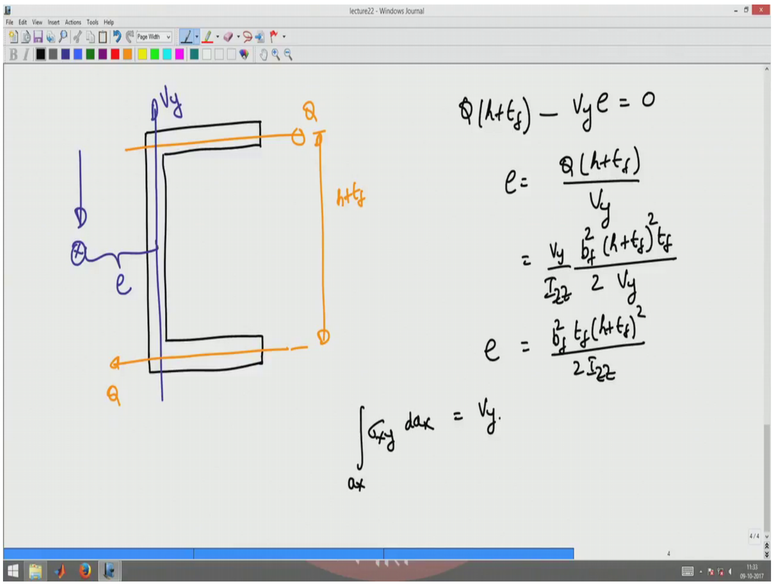

Now why do I want, I know that the channel section there is this net force Q acting like that. And net force Q acting like this and this distance this what I am interested in. That distance would be h plus tf. And there is this shear stress sigma XY which will if I integrate I will get it as Vy there Vy and I have this externally applied load acting at some distance e from here.

I have to find what is that distance, writing a moment balance for the torsional moment, I will get it as Q times h plus tf which is a clockwise moment. Minus Vy times e must be equal to 0. From here, I get e to be Q times h plus tf by Vy. Where Q is what we go in the previous expression sigma1 Q is Vy by I z z into bf into h plus tf by 2, into bf tf. Now, there is Q, I have Vy Vy cancelled. Vy by I z z into bf into h plus tf bf square into tf square by 2 times Vy.

Now, this is nothing but bf square tf h plus tf whole square by 2 times I z z that will be your point about the shear to appraisal load. That there is no net torsional moment coming in the cross section. Now, let us do systematically and how do I know that? Instead of doing it adhocly by integrating sigma XZ over the cross-sectional area finding Q and doing this way let us see how it gels in with our overall approach.

Before going there, I would like to show that integral sigma x y web this I pointed out in the last class also I am going to point it out in this class also sigma xy dax would be nothing but Vy. Integrated over the ax in this case, it is integrated over the web. That will be nothing but Vy we have to do the calculation check that it is Vy. If you are getting something different then, it seems that it means that some calculation mistake has been done. Somewhere now, though I give a formal derivation for finding this shear center. It is only for illustrative purposes in this course. In advance course you will be dealing with these equations in more detail.