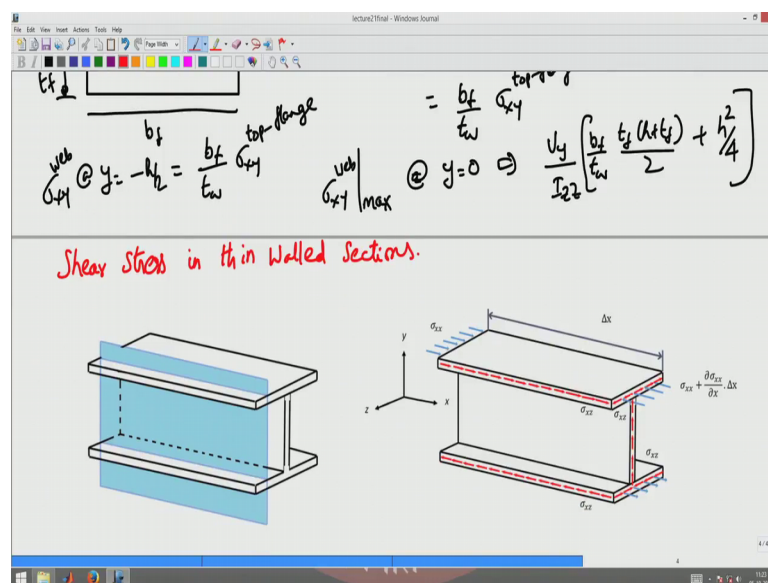


Mechanics of Material
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Stresses and deflection in homogeneous beams loaded about one principal axis
Lecture – 60
Horizontal shear stress in I section

Next we are going to look at shear stresses in thin wall sections.

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So, till now we have been looking at shear stresses in thick wall section. So, now we look at shear stress in thin walled sections. So, now let us look at this I section which is thin walled subjected to some bending moment, wearing bending moment along the axis of the beam.

Then what happens is, I have this I section which is subjected to varying bending moment along the axis of the beam. So, there will be shear force also in the sections.

Now, let me cut this I section using a plane as shown there, now if I cut that what happens is I expose the surface, now for this section to be in equilibrium which is of length delta x, I have a compressive stress of sigma x coming at this face and there is a stress of sigma x plus dou sigma x by dou x into delta x on this surface.

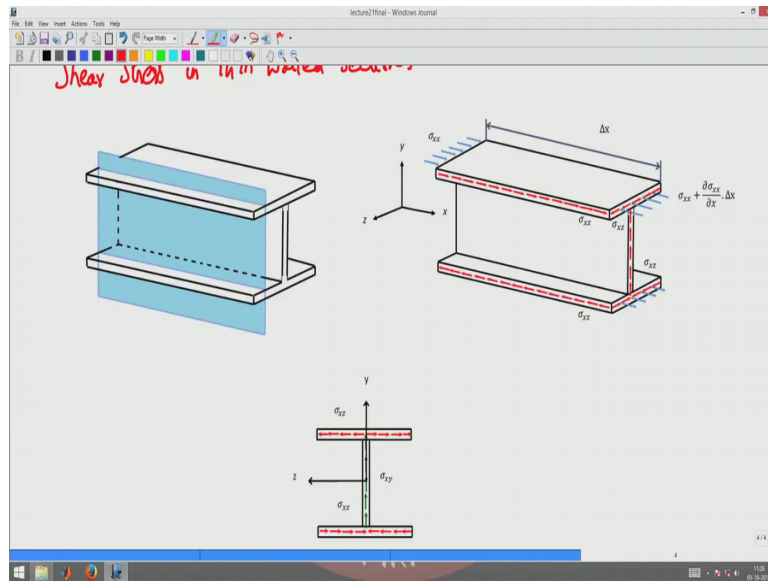
So, this stress is more than this stress, so there should be shear stress acting along the cut surface in this direction, and hence there should a complementary shear coming and acting on this flange in this direction on this side of the web. This is a complementary shear; this is a actual shear stress that is acting on this section to maintain equilibrium. Now to compute this shear stress, the expression remains the same is vq by ib ; except that now the b would be the thickness of this flange and your area that you have to consider should be taken from this end to the section point, where you are interested in finding the shear stress.

So, you have to take the entire flange thickness area, rather than sectioning the flange along plane which is cut along the parallel to the xz plane, now you are cutting a plane parallel to the yz plane.

That is you are cutting a plane like this and not horizontally or having a vertical plane of cut rather than horizontal plane of cut. Another point to understand here is at the bottom flange your tensile stress is coming in near. So, if tensile stress is more than the tensile stress acting on the other surface, that the shear stress should act like this along this direction rather than the direction that it was acting in the top flange.

And then the complementary shear would act like this, rather than the direction shown in the top flange and hence if you see together the σ_{xz} stress that is a flow; like from here It comes here transforms into σ_{xy} and then flows on to σ_{xz} there, similarly it comes here goes to σ_{xy} and becomes σ_{xz} there, so this is called as a shear flowing thin walled sections.

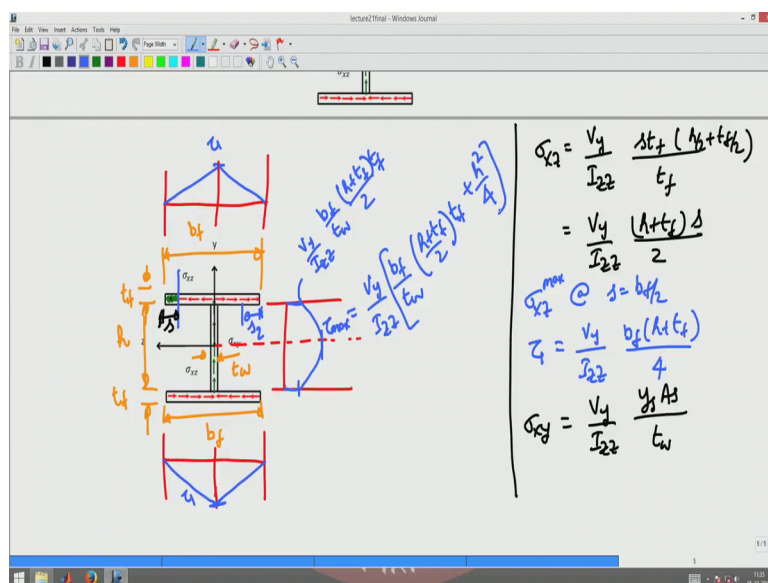
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Basically there is these shear stresses arise to maintain equilibrium of the sections when there is a varying bending moment. It is not necessary that always this varying bending moment should produce only sigma x y shear stress; you can produce even sigma x z shear stress as we see here.

While the expression to compute these shear stresses are the same. So, now let us go ahead and compute the shear stress distribution for this I section.

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As before let us assume that the thickness of the flange is t_f and the depth of the web is h , t_f , the thickness of the web is t_w , the width of the flange is b_f both top and bottom, the same with b_f . Now to compute this stress σ_x , I have to cut a plane, let me cut a plane like this ok.

Now, the area that I have to look up here is this area to compute the shear stress at that location. So, what is that area? That area would be let us assume that I am measuring s from this end, that is locating the section from the left extreme of the flange. So, I have σ_x given by V_y by I_{zz} by b , now is the thickness of the flange t_f and the area would be s into t_f into the centroid of that area, the centroid of this area is somewhere here that will be h by 2 plus t_f by 2 right. So, this becomes V_y by I_{zz} into h plus t_f into s by 2 .

So, now what happens when s is 0 the shear stress is 0 , as it should be because, I am not applying any shear on the outer surface of the flange and hence the variation of the shear stress along the flange is going to be something like this. This it varies from 0 it varies linearly up to their, when h is b_f by 2 , I will have the maximum shear stress which is σ_x max at s equal to b_f by 2 is τ_{01} which is V_y by I_{zz} into b_f into h plus t_f by 4 . So, that will be τ_{01} there and on this surface you can continue s because, the direction is changing.

So, what happens is this will decrease like this, again I have to take a s from this section move on here this will become s_2 , and I have to compute the variation like that.

Now, how do you assume this s is the question? You have to assume s such that it is for the along the same direction as the shear flow. Now same direction or opposite direction of the shear flow, but it cannot cross it cannot be continuous across the sections where the shear flow changes it is direction, it should be against the shear flow direction or in the shear flow direction; in both these cases I am assuming it to be against the shear flow direction.

But I cannot go from again the shear flow direction to a direction where it is along the shear flow direction. So, there will be the variation of τ_{xy} on the top flange, a similar expression will be obtained for the bottom flange also. So, the bottom flange also there will be a variation which is like this.

It is a same thing as we saw for the thick wall sections and this will be τ_{xy} , this value is going to be τ_{xy} . Now let us see what happens in the web, now we are interested in finding what is the variation in the web. The variation in the web is similar to what we had for a thick walled cross section. So, since it is similar to a thick wall cross section you will have σ_{xy} is that direction of that shear stress, that is again given by V_y by I_{zz} into y s into A s divided by the thickness of the web, because that is the b and when I am looking at the web I have to consider the entire flange area and a portion of the web area, that is similar to the expression that we obtained here for the web right.

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$$\tau_{xy}^{web} = \frac{V_y}{I_{zz}} \frac{b_f t_f \left(\frac{h}{2} + \frac{t_f}{2}\right) + t_w \left(\frac{h}{2} - y\right) \left(y + \frac{h}{2}\right)}{t_w}$$

$$\tau_{xy}^{web} = \frac{V_y}{I_{zz}} \left[\frac{b_f}{t_w} \left(\frac{h+t_f}{2}\right) t_f + \left(\frac{h}{2} - y\right) \right]$$
 when $y = h/2$

$$\tau_{xy}^{web} = \frac{V_y}{I_{zz}} \frac{b_f}{t_w} \frac{t_f (h+t_f)}{2}$$

$$= \frac{b_f}{t_w} \tau_{xy}^{top flange}$$

$$\tau_{xy}^{web} |_{max} @ y=0 = \frac{V_y}{I_{zz}} \left[\frac{b_f}{t_w} \frac{t_f (h+t_f)}{2} + \frac{h^2}{4} \right]$$

$$\tau_{xy}^{web} @ y = -h/2 = \frac{b_f}{t_w} \tau_{xy}^{top flange}$$

This same area I have to consider for the web, hence the equations will remain similar as what we obtained here.

So, let us go ahead and plot that resulting equation. So, what happens it at the web flange interface there is some shear stress which is non-zero, and then at the neutral axis location is where the maximum shear stress occurs and that maximum shear stress is same as what we had before; this will be nothing but as we saw here, that will be V_y by I_{zz} into $t_f b_f$ by t_w into t_f into h plus t_f by 2 is going to be I_{zz} into b_f by t_w , h plus t_f by 2 into t_f . And the maximum shear stress is going to be and this is τ_{max} that is going to be V_y by I_{zz} into b_f a t_w into h plus t_f by 2 into t_f plus h square by 4 .

So, that is going to be a τ_{max} same as before, so now let us understand where this variation in the shear stress profile is important.