Mechanics of Material

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Stresses and deflection in homogeneous beams loaded about one principal axis
Lecture – 59
Vertical shear stress in I section

Welcome to the 21st lecture in mechanics of material. The last lecture we saw how to compute the shear stresses in shear stresses due to bending. In particular we saw that the sigma x y shear stress could be computed from the expression, V y equal to sigma x y equal to V y into Q divided by I z z into b ok.

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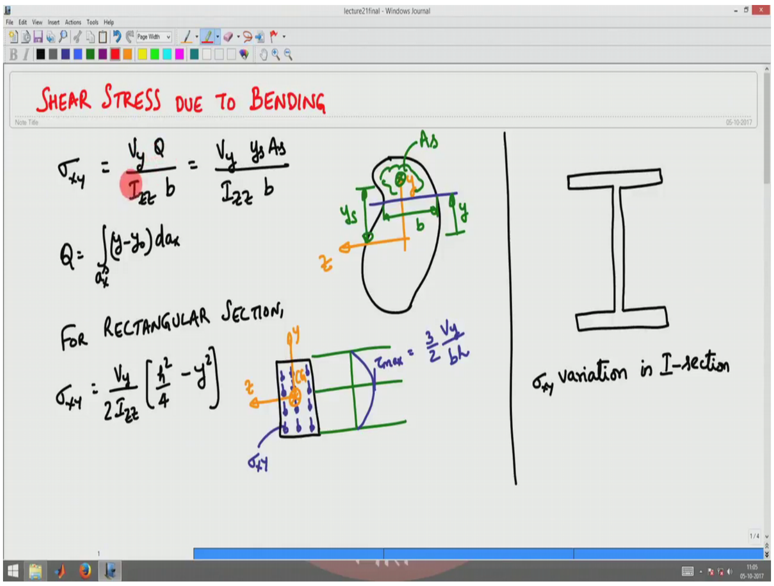

But Q is nothing, but the first moment about the neutral axis. It is in particular given by y minus y naught into d a. So, that we can write Q as y s, where y s is the centroid of the section area up to which we are interested in finding the shear stress and A s is the area of the cross section up to the point where we are interested in finding the shear stress.

In particular if you are interested in finding the shear stress of this blue cut, you have to consider the area above the blue cut from for A s; and y s would be centroid of that area measured from the neutral axis location, where this is at the neutral axis location as origin of this coordinate systems at the neutral axis location. So, now, using this expression we found that for rectangular cross section, the sigma x y will vary parabolically given by the expression here and the maximum shear stress is 1.5 times the average shear stress in a rectangular section that is what we found.

Now, let us go ahead and find what is the shear stress distribution in a I section. Here there are two issues; one is whether there will be a sigma x y shear stress that will develop or sigma x z shear stress that will develop in the I section. This depends upon the relative thickness of the flange to the relative thickness of the web ok.

So, basically now let us assume that the flange is thick enough that there can be a variation of sigma x y along the depth of the flange ok.

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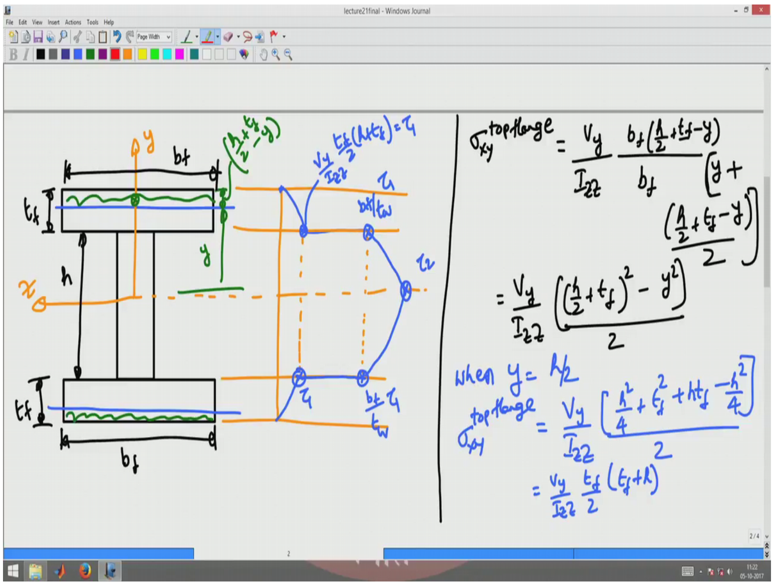

So, basically now what we are saying is we are considering this I section and we are interested in finding what will be the shear stress distribution on this I section ok.

Let us assume that the flange is of width b f of thickness t f, and this I to be h, again this is of length t f and the flange width is top flange is also of same width b f. Now if I am interested in finding the shear stress at some location here; shear stress at some location given by that blue line cut, then what I should do is I have to consider the area which is this area and the C G of that area which will be somewhere here. So, let us assume that this section is at a distance y from the neutral axis location. So, now, then this distance would be h by 2 plus t f minus y, because h plus h by 2 plus t f would be the distance from here to the top of the top flange. So, that will be that distance.

So, your expression for sigma x y in the top flange would be V y by I z z into here the cross section width is b f, because the blue line that cuts the cross section is of length b f. So, the denominator is b f, the centroid of that cross sectional the shaded portion is given by b f into h by 2 plus t f minus y that is the area into the centroid would be y plus h by two plus t f minus y by 2 right, because that is the half of the distance as the centroid of that rectangular region ok.

Half of the depth of the rectangular region is the centroid of the rectangular region. So, what does this boil down to? This boil down to V y by I z z. b f b f cancels. So, I will be left with h by 2 plus t f whole square minus y square by 2 ok.

So, what happens when y is equal to h by 2 to? H by 2 plus t f that is at the top flange top surface of the top flange you can see that the sigma x y goes to 0 as required, and it has a parabolic variation through the thickness of the flange. So, if I were to plot the variation of this stress along the depth of the cross section, it will vary from 0 here to some value here at the bottom flange bottom surface of the top flange, that will be y is equal to h by 2, when y is equal to h by 2 sigma x y of the top flange is V y by I z z into h square by 4 plus t f square plus h into t f minus h square by 4 by 2. So, that will be V y by I z z into t f by 2 to t f plus h ok.

So, that is the shear stress at the top of the flange, bottom surface of the top flange. So, this value is going to be V y by I z z into t f by 2 t h plus t f. Now what happens when I move on to the web?

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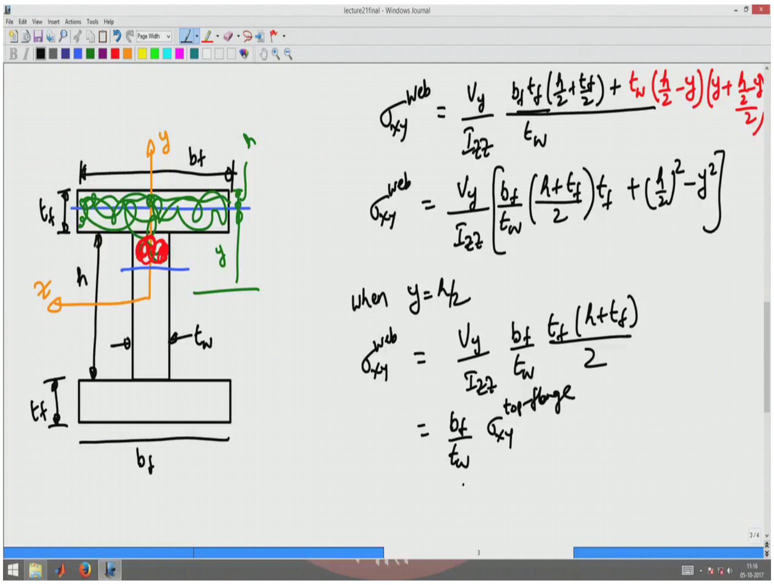

When I move on to the web, I have to take a section somewhere here. So, now, I have to find the area; that I have to consider is this centroid area the center area I have to now consider for my computation of the a s and the center of the area y s.

Now, let us find what is the expression for sigma x y of the web region. So, that will be V y by I z z into now the width of this cut is t w. Let us assume that the web is of thickness t w. So, the denominator is t w, now area consist of two areas one is the flange area which is b f into t f into its centroid, that centroid would be h by 2 plus t f by 2 that will be the centroid of that flange plus the centroid of this region and area of this region is what I have to consider. So, that will be t w into h by 2 minus y, and the centroid of that region would be y plus h by 2 minus y by 2 ok.

So, this gives me the expression for sigma x y of the web to be b y by I z z into b f by t w into h plus t f by 2 into t f plus h by 2 the whole square minus y square ok.

So, what happens when y is equal to h by 2? When y is equal to h by 2 sigma x y of web would be V y by I z z into b f by t w into t f into h plus t f by 2.

You find that this is nothing, but b f by t w or sigma x y of top flange at the same interval same section point. So, basically what happens is b f is greater than t w. So, this ratio is greater than 1 and hence what will happen is there will be a jump in the shear stress ok.

So, basically there is a jump in the shear stress at that location. So, basically you will find that the shear stress increases from this value to some value b f by t w times if this is tau 1 this will be tau 1 ok.

Now, where does the maximum shear stress occur in the web? Maximum shear stress occurs in the web at y equal to 0 that is at the neutral axis location. So, basically what we find is sigma x y web maximum value of that would be at y equal to 0, that will be V y by I z z into b f by t w t f h plus t f by 2 plus h square by 4 ok.

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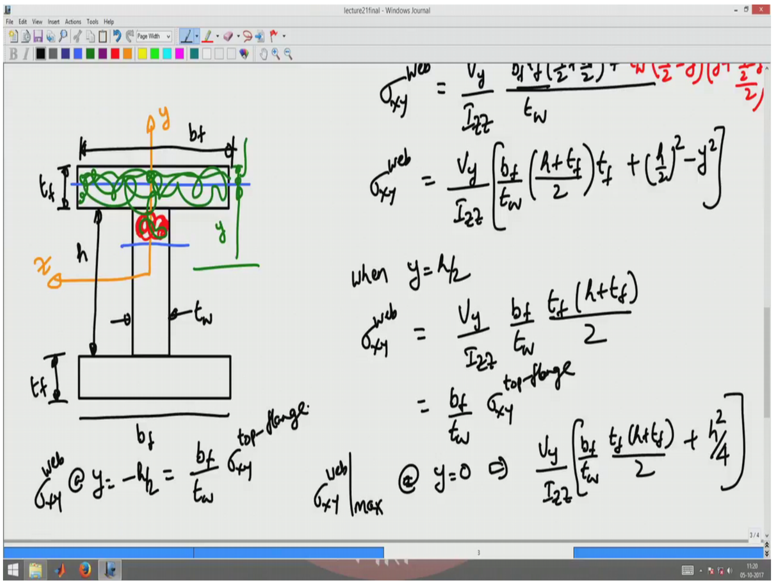

So, that is the maximum value of the shear stress in the web. So, what happens is there is an additional term of h square by 4 to whatever was contributing from the other flange web interface. So, this increases from here to some value here let us say that is tau 2. So, the variation is like this and you can see from symmetry we will get it back like that and this is something there, you will see now I am drawing this bottom portion of it. This will be tau one this point would be b f by t w into tau ok.

Now, go back to the web and find what is the sigma x y of web at y is equal to minus h by 2. You will find that it will be same as this one was it will be same as b f by t w into sigma x y of top flange ok.

Now, the bottom flange you have to do the same thing as we did for the top flange except that y is negative here. So, you will get a similar expression for the bottom flange also for sigma x y. There is nothing like top or bottom here I could have divide done a section near like this and consider this area and done the same thing with respect to computation of sigma x y for the bottom flange ok.

So, these two are having the same dimensions and hence you will get a stress distribution something like this given in this figure here. You will get a stress distribution something like this for the I section. The point note here is at this section that is at the bottom of the top flange or top of the bottom flange there is a shear stress coming in near. You know very well that you are not applying a shear stress at this junction ok

There is a possibility of shear stress arising at the web flange interface because continuity of the material, but there is no shear stress applied the free surface of the bottom of the top flange. So, how will this tau 1 be resisted by the flange it is a question. The flange is of sufficient thickness, then you can and the web is of sufficient thickness that the overhanging portion of the flange is small, then you can assume that the tau 1 will be resisted by the flange ok.

On the other hand if I add a very think wall section like (Refer Time: 15:27) seal sections for I section. So, then what will happens is the, flange thickness will be like 10 mm or 12 mm or 20 mm at most. In those cases it cannot resist this tau 1. So, there should be a different mechanism by which there is a equilibrium satisfied by the sections.