# Mechanics of Material

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## Stresses and deflection in homogeneous beams loaded about one principal axis
## Lecture – 58
## Parallel axis theorem and its application

Now, for the center of the cross section is we have found.

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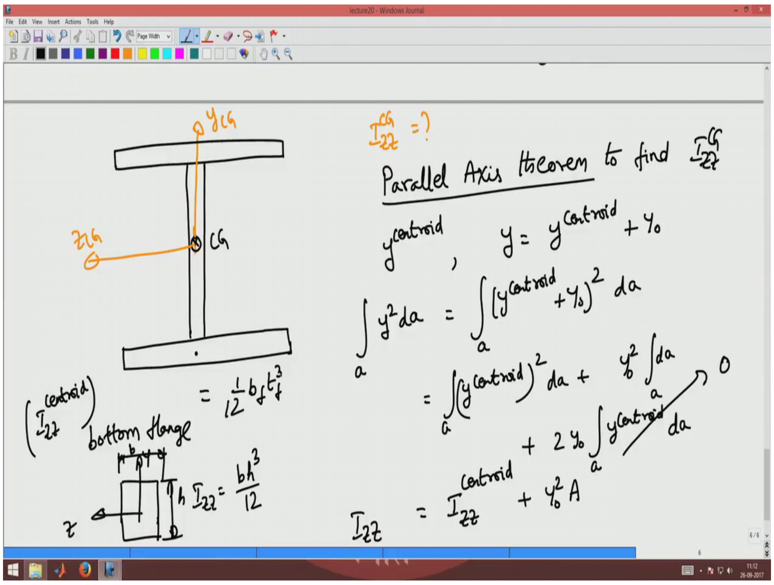


Now, the center of the cross section is this that is the C G and hence your axis would be this will be y and z axis that be a y and z axis about the C G. Now I am interested in finding I z z moment of inertia I z z about the center of cross section is what I have to find. Here what we you make use of is was called as the parallel axis theorem to find the moment of inertia. Here we are going to use parallel axis theorem to find I z z about the centroidal axis ok.

What does a parallel axis theorem state? Basically if y is measured about the centroid of the cross section, then if I have y which is y centroid plus some distance y naught integral y square d a, where a is the area of the cross section is given by y centroid plus y naught square d a right. Now y center plus y naught the whole square d a, this will be nothing, but integral y centroid squared d a plus y naught is a constant. So, it is y naught square integral d a area of the cross section plus 2 times y naught integral y centroid d a area of the cross section ok.

So, this is going to be nothing, but I z z above the centroid plus y naught square into area of cross section, y centroid integrated over d a your axis about the centroidal axis. So, this term will evaluate to be 0. So, you get this term will evaluate to be 0. So, I z z about some axises, I z z about central axis plus eight y naught square ok.

So, we will use this theorem now to find I z z of this I section. Now I know that the C G of the bottom flange is at this point. So, and I know that I z z about I z z of the centroid of the bottom flange is 1by 12 b f into t f cube. We saw that for a rectangular section this comes from a rectangular section this is z and this is y of a centroid and if this is h and this with this b, I z z is b h cube by 12 we saw this in the last lecture.

From here h separate for the bottom flange or in the thickness now is t f and the width is b f. So, from there I get I z z of the bottom flange should be that, and now I z z about the C G of the cross section for the bottom flange is then I z z well as C G of the cross section for the bottom flange is b h q by 12, b t f q by 12.

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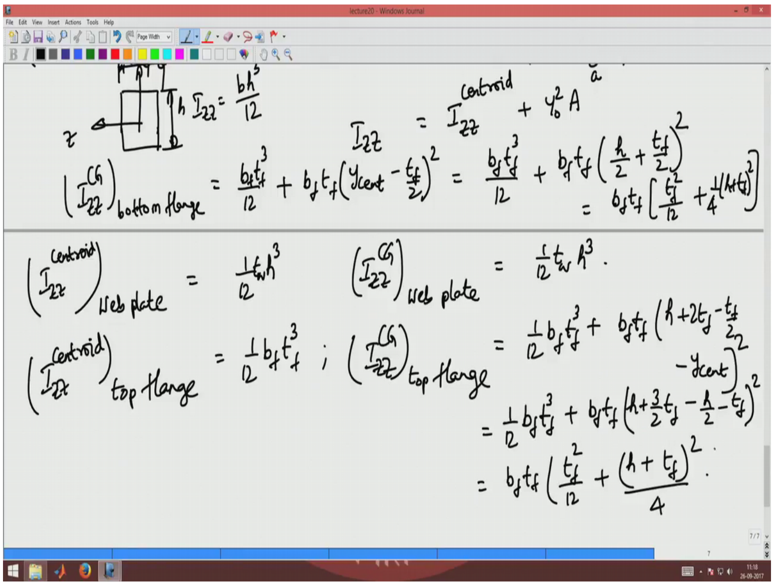

That is that is about the central axis plus the area of the bottom flange which is b f into t f into y naught square would be, your what we found before. The centroidal axises h by 2 plus t f we found that the centroid y centroid of the I section is h by 2 plus t f. So, now, what happens is this becomes we measured the center from the bottom fiber of the bottom flange. So, this will be y centroid minus t f by 2 the whole square ok.

So, that will be b f t f cube by 12, plus b f t f into h by 2, plus t f by 2 the whole square ok.

So, this by simplifying we will get it as b f t f into t f square by 12 plus 1 by 4 h plus t f whole square, that is for the bottom flange. Now let us do it for the web I z z about the centroid for the web plate would be 1 by 12 into t w, which is the width of the plate into h cube which is the depth of the web plate ok.

Now, the C G is coinciding with the C G of the web plate C G of the cross section is coinciding with the C G of the web plate and hence I z z about C G of the cross section for the web plate reminds 1 by 12 t w into h cube ok.

Now, I have to do the same thing for the top flate top flange, the I z z of the centroid of the top flange would be 1 by 12 b f into t f cube. Just wake out was for the bottom flange and I z z about the C G of the cross section for the top flange would be 1 by 12 b f t f cube plus area of top flange is b f t f plus h plus t f h plus 2 t f basically what we are doing here is, now we want this distance you know that this distance is y centroid.

Now, you want to find and this distance is t f by 2 and you are interested in finding out what this distance is. This distance would be h plus 2 f 2 t f minus t f by 2 minus y of centroid. So, that will evaluate to be 1 by 12 b f t f cube, plus b f t f into h plus three by 2 t f minus h by 2, minus t f b 2 minus t f ok.

So, that is going to be nothing, but same as what we got for the bottom flange, which is b f t f into t f square by 12 plus h squared here squared there h plus t f by h plus t f the whole square by 4 ok.

So, now I have what I have to do for the entire cross section is, I z z for about a C G for the cross section is.

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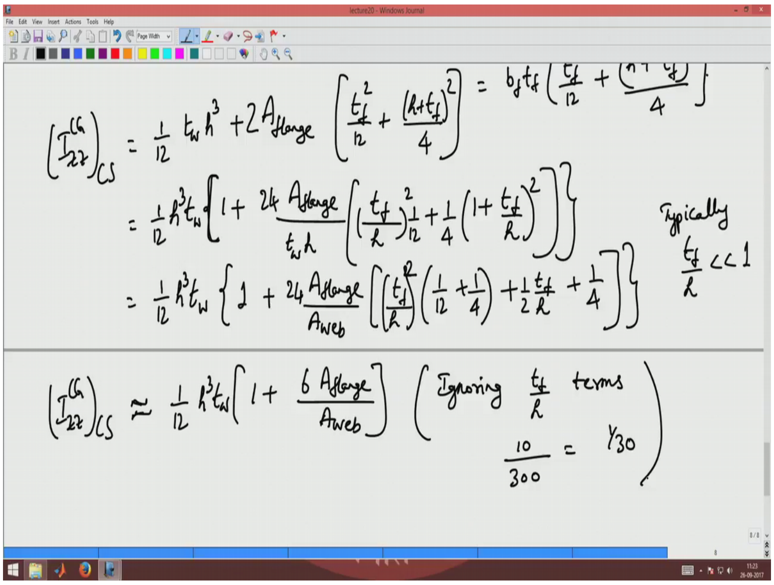


I have to add all this three moment of inertias of the three plates, which will be 1 by 12 t w into h cube, plus b f into t f is area of flange into p f square by 12 plus h plus t f square by 4 into 2 because I add the same thing for the top and the bottom flange. Let us simply this further this will be 1 by 12 h cube to t w, plus I am pulling that out. So, it will be 1 plus 2 times area of flange divided by t w into h will be into 24 into t f by h the whole square plus into 1 by 12 into 1 by 4 into h into 1 plus t f by h the whole square ok.

Now, this is nothing, but 1 by 12 into h cube t w into 1 plus 24 times area of flange by area of web into if I expand this, I will get t f square by h the whole square into 1 by 12 plus 1 by 4 plus t f by h into 1 by 2 plus 1 by 4.

Typically the thickness of the flange typically t f by h would be much lesser than 1 and hence I ignore the higher order terms of t f by h to get the final expression, I approximate I z z of the centroid of the cross section of the centroid of the cross section to be 1 by 12 h cube t w, into 1 plus 6 times area of flange by area of web. Ignoring this is an approximate expression ignoring p f by h terms.

Because typically the flange will be of 10 mm width or the depth of the cross section would be 300 mm. So, this is 1 by 30. So, I am ignoring 1 by 30 in comparison to one. So, that is what I have done ok.

So, this is the moment of inertia of the I shaped cross section. Now what changes all the computation that we did in the last class, and in this class remain the same except that I have to use this I z z in the expression for computing delta y.

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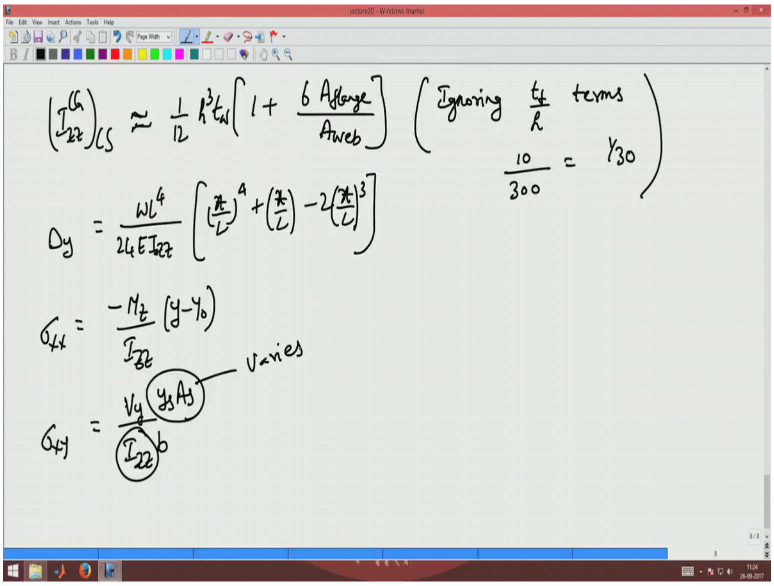


Delta y as we saw in the beginning of the class would be w l power 4 by 24 E times I z z into x by L whole power 4 plus x by L into minus 2 times x by L the whole cube ok.

So, basically delta y remain the same except that I have to use this modified expression for the moment of inertia, similarly sigma x x stress remains the same except that I have to use this moment of inertia instead of the moment of inertia for the rectangular section.

Now, but sigma x y will vary with the depth of the cross section in a different manner; because that is given by V y by I z z into b into y s into A s. This variation not only the I z z varies even y s into a s varies. So, in the next class what we will do is, we will see how the shear stress varies in a I section that depends upon the later thickness of the web and the later thickness of the flange also. So, we will see how this shear stress varies for a, I section in the next class we conclude here for today's lecture.

Thank you.