

**Mechanics of Material**  
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**Stresses and deflection in homogenous beams loaded about one principal axis**  
**Lecture – 57**  
**Finding centroid of a cross section**

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$$A_s = \left(\frac{h}{2} - y\right)b$$

$$y_s = y + \left(\frac{h}{2} - y\right)\frac{1}{2}$$

$$= \left(\frac{h}{2} + y\right)\frac{1}{2}$$

$$\sigma_{xy} = \frac{V_y y_s A_s}{I_{zz} b} = \frac{V_y}{I_{zz} b} \frac{1}{2} \left(\frac{h}{2} + y\right) \left(\frac{h}{2} - y\right) b$$

$$\sigma_{xy} = \frac{V_y}{I_{zz}} \frac{1}{2} \left(\frac{h^2}{4} - y^2\right) = \frac{V_y}{2bh^3} \left(\frac{h^2}{4} - y^2\right) = \frac{6V_y}{bh} \left(\frac{1}{4} - \frac{y^2}{h^2}\right)$$

$$\sigma_{xy} = \frac{6V_y}{bh} \left[\frac{1}{4} - \left(\frac{y}{h}\right)^2\right]$$

Maximum  $\sigma_{xy}$  occurs when  $y = 0$ .

$$\sigma_{xy}^{\max} = \frac{1}{4} \left(\frac{6V_y}{bh}\right) = \frac{3}{2} \frac{V_y}{bh} = \frac{3}{2} \sigma_{xy}^{\text{avg}}$$

Now, till now we have been analyzing the section with the rectangular cross section right. Now, for a change let us analyze a section with the I section. When the section is I and see what changes in our cal computations when a simply supported beam is like an I section.

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**Centroid of I-section:**

$$y_{cent} = \frac{\int y \, dA}{\int dA} = \frac{b_f t_f \left(\frac{t_f}{2}\right) + h t_w \left(\frac{h}{2} + t_f\right) + b_f t_f \left(h + t_f + \frac{t_f}{2}\right)}{h t_w + 2 b_f t_f}$$

$$= \frac{2 b_f t_f^2 + h t_w t_f + \frac{h^2 t_w}{2} + h b_f t_f}{h t_w + 2 b_f t_f}$$

$$= \left(\frac{h}{2} + t_f\right) \frac{[h t_w + 2 b_f t_f]}{(h t_w + 2 b_f t_f)} = \frac{h}{2} + t_f$$

$$z_{cent} = \frac{\int z \, dA}{\int dA} = \frac{b_f t_f \frac{b_f}{2} + h t_w \left(\frac{b_f}{2}\right) + b_f t_f \frac{b_f}{2}}{(2 b_f t_f + h t_w)}$$

$$z_{cent} = \frac{b_f}{2}$$

Still I am looking at a simply supported beam. Subjected to a uniformly distributed load  $W$  but the cross section is now I shape. Let us assume the following dimension for this section let us assume the web thickness is  $t_w$  and depth of web is  $h$ . Let us assume the flanges of width  $b$  and thickness  $t_f$  and thickness  $t_f$ . Let us assume this flanges also of thickness  $t_f$  and this flanges of width  $b$  same flange thickness and flange width.

Now, there are many questions that we have to answer what is the center of the cross section? What is a moment of inertia of this cross section? And so, on let us deal with each of them 1 by 1. As usual am I assuming this should be  $x$  this should be  $y$  and this should be the  $z$  axis and the ends for this this will be the  $y$  axis and this to be the  $z$  axis to start with. I do not know where the centroid is; I am assuming that I am at the bottom flange for the  $z$  axis and I am at this entire the web for the  $y$  axis this just an assumption assume the  $y$  and  $z$  axis to be anywhere you want to and correspondingly you will get the centroid value for those things.

Now, how do you find the centroid of the cross section. These things you should know already, hence I will go fast to find the why centroid is integral  $y \, dA$  divided by integral  $dA$  I then divide it into 3 zones one for this zone one for this web and the other zone for this top flange.

Basically, doing that I write it as  $b_f$  into  $t_f$  into the centroid of this would be  $t_f$  by 2 I know that from the previous analysis for the center of a rectangular section. This into  $t_f$

by 2 is for the bottom flange this is for bottom flange plus I have  $h$  into  $t$   $w$  into  $h$  by 2 plus  $tf$  this would be for the web plate plus  $bf$  into  $tf$  into  $h$  plus  $tf$  plus  $tf$  by 2 this is for a top plate. That is the centroid I am multiplying the area of the centroid of the respective sections divided by the area  $h$  into  $tw$  plus 2 times  $bf$  into  $tf$  the area of the bottom flange and the top flange.

This will tell us that now what will this give me this will give me  $bf$  into  $tf$  into  $2tf$  square into 2 plus  $h$  into  $tw$  into  $tf$  plus  $h$  square  $tw$  by 2 plus  $h$  into  $b$  of  $tf$  divided by  $h$  into  $tw$  plus 2  $bf$   $tf$ .  $H$  by 2 plus  $tf$  then I will get it as  $htw$  plus 2  $bf$   $tf$  divided by  $htw$  plus 2 times  $bf$   $tf$ . That will be  $h$  by 2 plus  $tf$ .

Similarly, now  $z$  centroid would be  $z$  centroid would be  $\int ax z \, dx$  by  $\int da \, x$   $ax$ . That is going to be  $bf$  into  $tf$  into  $z$  centroid would be  $bf$  by 2 this is for the bottom flange for the web it is going to be  $h$  into  $tw$  this is the area of the web into I have to give a position of this web the  $cg$  of this web is assumed as  $bf$  by 2 this is for the web plate plus for a top flange it is going to be  $bf$   $tf$  into  $bf$  by 2 for the top plate top flange divided by 2 times  $v$  of  $tf$  plus  $htw$ . It is easy to see that this will be nothing but  $bf$  by 2 which will be your  $z$  centroid. From this we find that by the centroid of this cross section is.