Mechanics of Material
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Stresses and deflection in homogenous beams loaded about one principal axis
Lecture – 56
Expression to find shear stress

Welcome to twentieth lecture in mechanics of materials. The last class we saw the example problem of a simply supported beam subjected to uniformly distributed load. We want to analyze this simply supported beam.

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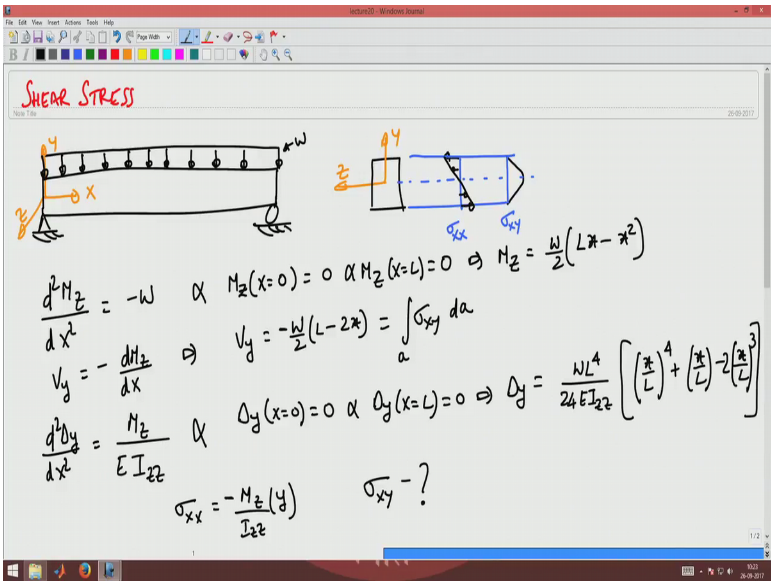

We solve the governing equation d square MZ by dX square equal to minus W with the boundary condition that the moment at the hinged end and the roller end has to be 0 to get the bending moment as W by 2 into LX minus X square.

And then we got the shear force from differentiating the bending moment with respect to X get this particular variation in near and then to get the deflected shape of the beam, we solve the governing equation got from the beam bending equation that that d square delta y by dX square is given by MZ by E times IZZ to get the with along with the condition that at the hinged end roller end the vertical deflections have to be 0 to get this expression for the displacement of the beam as the function of X. We plotted this and we plotted how does the angle change that is the slope of this function with respect to X.

What we know already from the bending equation is the bending normal stress sigma XX is related to the bending moment through this equation MZ by IZZ into y and the variation of this stress is a linear function of y. It varies linearly like this the top face in compression and bottom face in tension because MZ is negative you will get I left a negative sign here because and you will get this top face being compression bottom face being in tension from this equation in here.

Now, what we are interested is we are interested in finding what the shear stress is that is sigma Xy in particular we want to show that the shear stress varies parabolically that is maximum value being in the neutral axis location. We want to laid this shear stress distribution to this shear force because the shear force is what arises because of this shear stress.

If you recollect we have seen that the sigma x y d a is nothing, but Vy from the previous 2 lectures we saw this that v y is given by this expression there. Now, to understand why this sigma Xy arises? Let us look at a small demonstration.

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I have this 2 rubber blocks which are not connected to each other. Now, let me say I bend it by applying a non-uniform load, I am bending it by applying a non-uniform load, see what happens at the edges? Then see what happens at the edges? The edges are not flat the plane sections are not remaining plane, there is a shift in the white line and you can see that at this edge there is a zig zag nature of the plane. There the plane is not remaining smooth.

This because at this interface since it has not been glued together there is no mechanism to rest is the shear that comes at that interface and the shear stress is maximum at that interface because that will be the neutral axis of this cross section. Basically because of this thus the jaggedness in the cross section the assumption that we made in the unless is that a plane section remains plane are not holding good.

On the other hand, if I take a glued rubber block same size, but it has been glued here you can see I cannot separate them out. You can say I can not separate them out. Now, let us say what happens if I bend it let us say what happens if I bend it the straight line remain straight the edges remain parallel here, they are not going zig zag because the glue is resisting the shear stress that is developing at that interface and hence is able to do this and this one is much stiffer to bend compared to the other block which is much flexible.

Now we saw the importance of shear stress in the bending action and hence you want to be able to compute the shear stress; that we can design the structures appropriately for the shear stress if they have to resist the shear stress.

As we saw here if I have a non-uniform bending moment what happens is? The be the axial stress that develops at this face or at this face will be different, because the bending moment is varying from here to here. The bending moment is varying from here to here if I cut any one half of the beam this half was in pure compression, when I bend it together and hence what happens is this net total compressive force acting at this section should be balanced because the bending moment is very near for that it has to develop a shear stress at this interface.

The shear stress developed at this interface has to be counted by a complementary shear stress which we saw from moment equilibrium has to be same as this shear stress developed here should be in equilibrium for equilibrium moment equilibrium there should be a complementary shear stress developed by this face. That is what we are going to do we are going to do this calculation systematically now.

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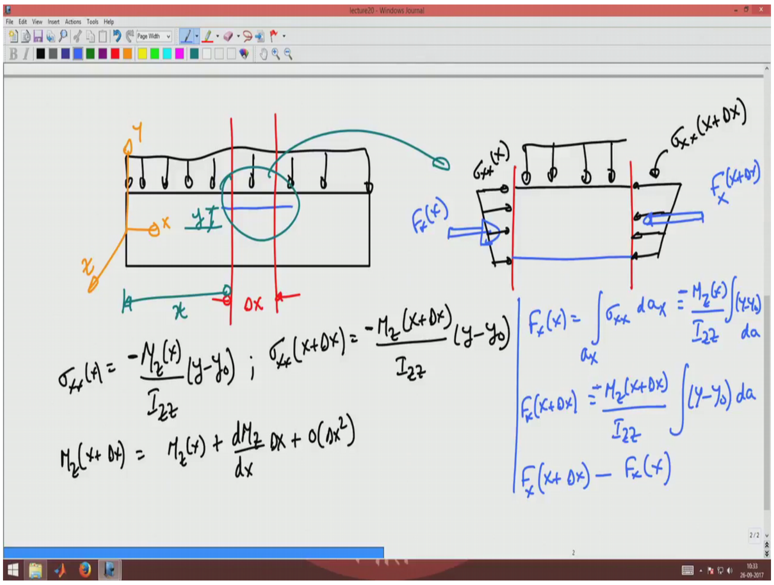

Basically, what we have is we have a beam subjected to some arbitrary loading and I am interested in segmenting this beam at 2 sections like this with a distance of delta X. A small sectioning along the axis of the beam if you recollect as usual we are assuming that this is a X axis this is the Y axis and this is the Z axis. Now, apart from sectioning along the vertical along the line parallel to the Y axis I want to section it along a line parallel to X axis also like this I want to section it like this let us assume that this distance is some Y arbitrary distanceY let us assume that this distance is some arbitrary distance X now I want rather free body diagram of this cut section.

This cut section I am drawing it here. Basically, what I have is I have that I have this vertical lines cut here, then I have the blue line cut here and there is some loading acting on top since it is some distance apart and I am assuming that the beam is bending like this there will be compression on the top face. Let us assume that this is sigma XX at X and similarly there will be some compressive stresses acting on this face which is sigma XX at X plus delta X

The magnitude of sigma is at X and X plus delta X will would not be same because bending moment is varying along the length of the beam therefore, sigma XX at X would be given by from our bending equation will be MZ at X by IZZ into Y minus Y naught and similarly sigma XX at X plus delta X would be given by MZ at X plus delta X divided by IZZ into Y minus Y naught.

Now, what happens I know I can write MZ at X plus delta X using Taylor series as MZ of X plus dMZ by dX delta of X plus higher order terms of delta X I left a negative sign here and a negative sign here. That is the coming from the bending equation you get the sigma XX at X is minus MZ by IZZ to Y minus Y naught and here it is X plus delta X by IZZ and Y minus Y naught. Now, substituting what I want to find next is I want to find the net compressive force acting on this face net compressive force acting on this face. That is, I want to find the net compressive force on this face and this face FX at X plus delta X and FX at X due to this cut

Now, what will FX be FX at X would be integral sigma XX into d a right multiplied by d a into a of X a of X. This is nothing, but MZ at X divided by IZZ which is a constant into integral Y minus Y naught d a similarly FX at X plus delta X would be MZ at X plus delta X this is the function of X independent of da X, just integration with respect to Y and Z. This will be IZZ integral Y minus Y naught da.

That will be f of X plus delta X what I am interested is there a difference between these 2 is what I am interested in. I want to find F of X at X plus delta X minus F of X at X I left a negative sign at both these instances.

I am interested in finding this. Substituting for the bending moment what I get is let us say this is delta FX and now I am interested in finding delta FX.

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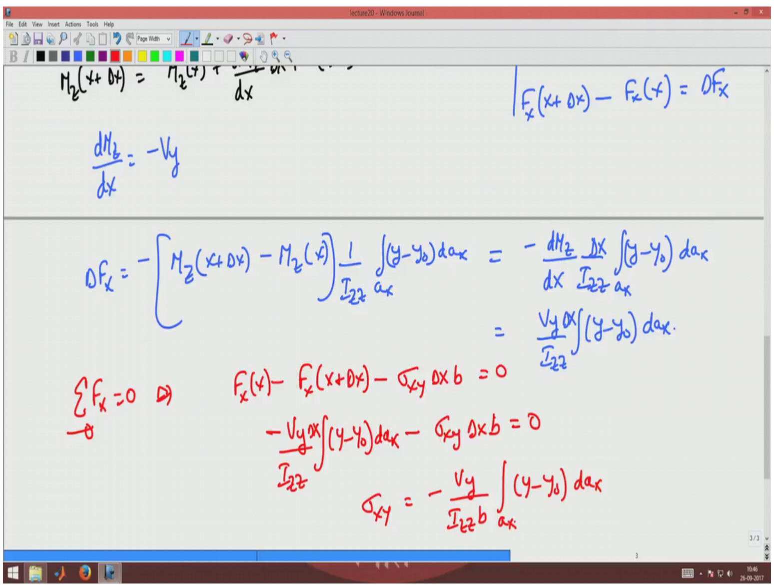

Delta FX would be from this equations minus MZ at X plus delta X minus MZ at X to 1 by IZZ integral y minus y naught da XAX.

From the expression for MZ at X plus delta X given here from the expression for MZ plus X plus delta X given here this can be written as minus dMZ by d X 1 by IZZ integral y minus y naught da XAX.

Now, from our equation for the d MZ by dX we find that from the shear force relating the bending moment to the shear force this is the governing equation for that from this we can write this as Vy by IZZ integral y minus y naught da X.

Now, that is delta fX now basically now writing the horizontal force equilibrium what we find is there is the sigma there is the sigma XZ acting horizontally like this the complementary shear stress would act vertically like that on that face. Basically, now since the fa horizontal force equilibrium demand that this delta FX which is 0 should be balanced by this shear stress.

This shear stress is acting over a area whose length is delta X and whose width is into the ee into this plane which will be the B. Basically if I have cross section like this the shear stress is acting on this plane like this and hence it is width would be this distance B times this distance delta X this do distance B times this distance deltaX

Now writing the horizontal force equilibrium I get it as summation FX equal to 0 would imply that F of X at X minus F of X at X plus delta X minus sigma XY into delta X into B must be equal to 0.

From our expression for delta X we find that this is nothing, but minus Vy by IZZ into integral Y minus Y naught da X minus sigma XY delta XB must be equal to 0. From here I get sigma XY to be minus Vy by IZZ into B. I left a delta X here delta x there. Now, this will be delta X here. This will be minus Vy by IZZ into B into integral y minus y naught DAX AX.

Now, let us understand what this term integral y minus y naught dAX is, say I have a cross section.

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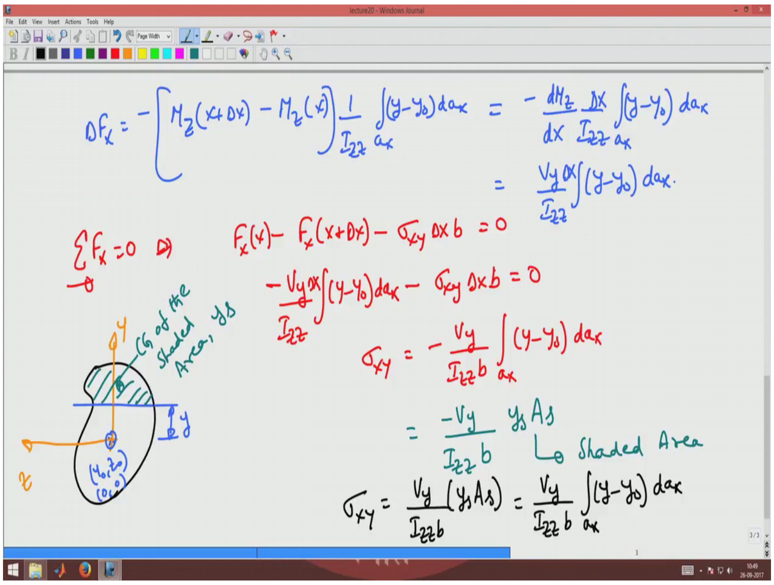

I have some arbitrary cross section I segment it at some distance y the coordinate system is y and Z a segment it at some distance segmentary at some distance y a segmentary at some distance y and y naught is this location of the neutral axis which I am assuming that I am at the neutral axis location. Y naught is the center this is y naught that this y naught comma Z naught or I am taking that is a origin. This will be 0 comma 0.

That is why. This is nothing, but the centroid of this area it is this hatched areas CG. CG of the shaded area is say y s then this integral would be minus Vy by IZZ into B into y s into As where y s is the shaded area As is the shaded area and ys is the center of the shaded area.

Now, why is this negative sign there this negative sign tells us that the sigma XY is a reaction to the applied shear force the shear force is r s or a section. And here we saw that dash in does not matter whether it shears like this or whether it shears like this essentially the conditions remain the same and hence you can draw the negative sign.

In shade dash it does not matter whether it shears like this or whether it shears like this the criteria remains the same and hence you can draw the negative sign and in general right sigma XY as Vy by IZZ into B into ys to as which is a centroid of the shaded area and as is the area of the shaded area or this is Vy by IZZ into B into integral y minus y naught BAXAX.

That is the expression to find the sigma XY. Now let us move ahead and compute sigma XY for a rectangular section.

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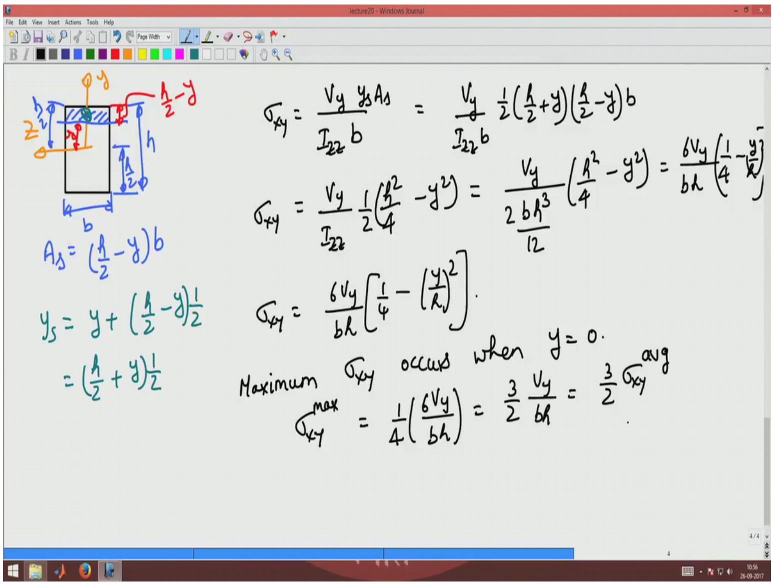

Let us say I have a rectangular section, now let us say that is y and that is Z that is Z that is y and this is Z. Now I have to compute get the section somewhere in between and get the area of this section area above that section point and I have to find the CG of that area.

What is As? As if this width is b depth is h the centroid would be at a distance h by 2. The area of the shaded region would be h by 2 minus y into b. Where, this distance is h by 2, this distance is y and hence this distance would be h by 2 minus y. Times the width of the cross section gives us the area of the shaded region.

Now, I want to find ys I want to find ys, that is the centroid of this shaded region the centroid would be y plus half of the thickness of that section which will be h by 2 minus y into one half. This will be h by 2 plus y into 1 half

Now what is the shear stress sigma XY from a previous expression is Vy by IZZ into b where b is the width of the section at the location where you are finding the shear stress into y s into a s. This is going to be Vy into Vy by IZZ into b into y s is 1 by 2 h by 2 plus y into As is h by 2 minus y into b that is sigma x y is Vy by IZZ into b into h square by 4 minus y square into 1 by 2. Now, let us substitute for IZZ this will be Vy by b h cube into h square by 4 minus y square 2 by 12. Let us substitute for IZZ it will now be this expression n here. This will be 6 Vy by b h into 1 by 4 minus y by h the whole square.

You got the expression for sigma XZ as 6 Vy by b h into 1 by 4 minus y by 4 minus y by h the whole square. Now, where will the maximum of this occur the maximum of this from differentiating this with respect to y and equating to 0. You can see that max maximum of sigma XY occurs when y is equal to 0. That is the neutral axis location. Because you have chosen the coordinate system to be coinciding with the neutral axis 0 and this maximum sigma XY maximum is 1 by 4 into 6 Vy by b h that is 3 by 2 Vy by b h that is 3 by 2 sigma XY average Vy is the shear force. B by b h gives you the average shear stress acting on the section. Now, you find that the maximum shear stresses nearly 50 percent more than the average shear stress

That is why you have to do a rigorous analysis to find what the stresses are. Now, we find that maximum shear stress is 3 by 2 times sigma x y average this 50 percent is for this section for different sections there will be different percentages of the sigma average that will occur.