Mechanics of Material
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Stresses and deflection in homogeneous beams loaded about one principal axis
Lecture – 55
Deflected shape and rotation of cross section

Next, what we want to see is the deflection of the beam. I want to find delta y, next I am interested in finding delta y.

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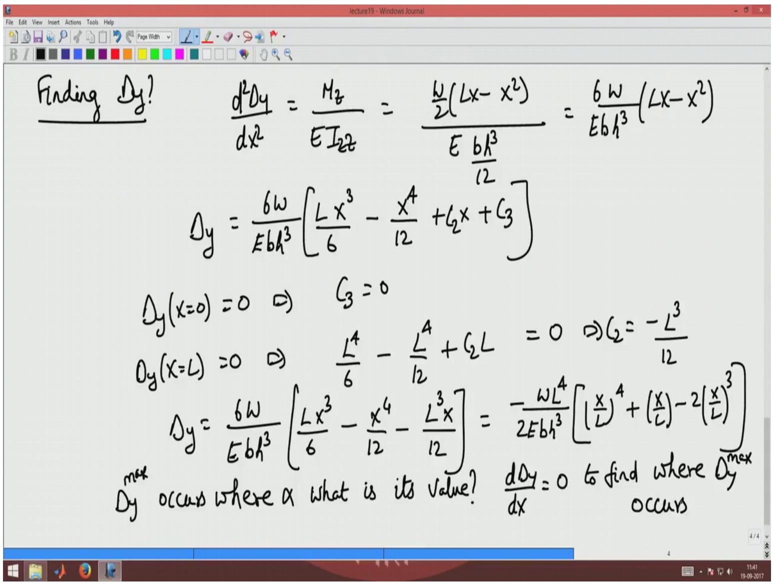

We add the governing equation d square delta y by d x square to be given by M z by E times I z z right from the bending equation. Now, M z we know is W by 2 into L x minus x square divided by E times for the rectangular section it is b h cube by 12.

So, this becomes W by 6 W E b h cube into L x minus x square. Now, I have to integrate this equation twice, which means delta y would be 6 W by E b h cube into L into x cube by 6 minus x power 4 by 12 plus C plus C 2 x plus C 3, that is what I will get if I integrate that equation twice.

From the boundary condition I know that delta y at x equal to 0, 0 delta y at x equal to L is 0. Hence, I use that boundary condition delta y at x equal to 0, 0 implies C 3 equal to 0 and similarly delta y at x equal to L equal to 0 would imply L power 4 by 6 minus L power 4 by 12 plus C 2 L has to be equal to 0 from where I get C 2 to be minus L cube by 12.

Now, then delta substituting these values and delta y expression, delta y becomes 6 times W by E b h cube into L x cube by 6 minus x power 4 by 12 minus L cube x by 12. This I can rewrite it as minus W by 2 E b h cube L power 4 x by L power 4 plus x by L minus 2 times x by L the whole cube.

So, now what am I interested in? I am not interested in just finding delta y, I want to find delta y max, occurs where and what is this value? Now, to find delta y max I have to set d delta y by d x must be equal to 0 to find where delta y max occurs.

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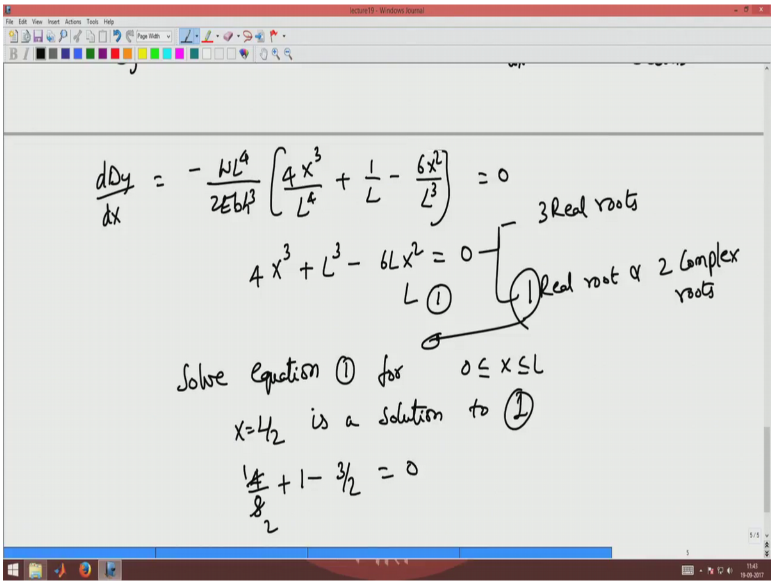

So, d delta y by d x would be minus W L power 4 by 2 E b h cube; the 4 x cube by L power 4 plus 1 by L minus 6 x by L cube, this has to be equal to 0 which means I get a cubic equation here, x cube 4 plus L cube minus 6 L into x equal to 0.

This cubic equation has various possibilities can result in 3 real roots or 1 real root and 2 complex roots, then I pick the real roots becomes the location, on the other hand if I add 3 real roots what will happen is there will be only 1 root which is lie in the domain of 0 to L because I am looking at a solution for or the this equation the domain 0 to L.

I want to solve see this is equation 1, solve equation 1 for x lying in the domain 0 to L. You can see now by inspection that x equal to L by 2 is a solution for solution to 1. Let us, substitute and find out x equal to L by 2 it will be 4 by 8 plus 1 minus 3 by 2, this is 1 by 2 and this is 0. Hence x equal to L by 2 is a solution to 1, we will use that.

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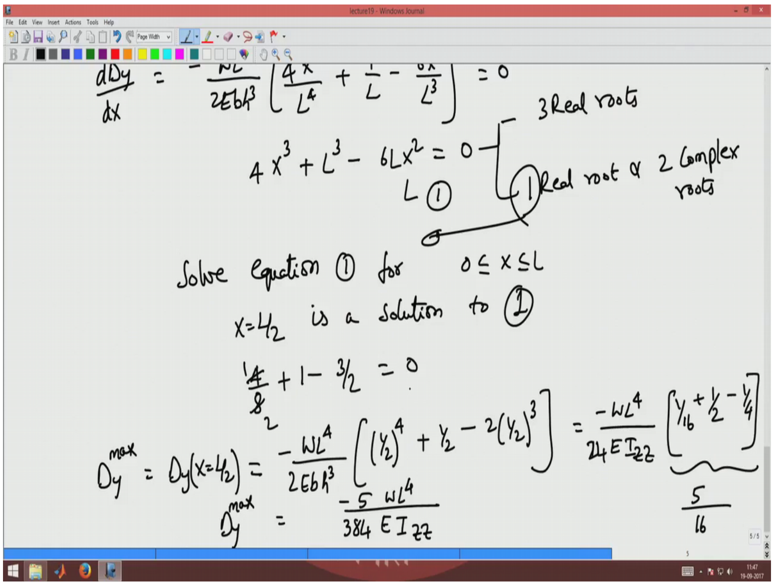

So, delta y max occurs at delta y at x equal to L by 2, this going back to a equation would be minus W L power 4 divided by 2 E b h cube into 4 times 1 by 2 whole power 4 plus 1 by 2 minus 2 times 1 by 2 the whole cube.

So, this will be equal to minus W L power 4, I reintroduce I z z. So, this will become 24 E I z z because it is by 12 multiplied by 12 into 1 by 16 plus 1 by 2 minus 1 by 4. So, this will be nothing but 5 by 384 W L power 4 minus E I z z, this factor we take LCM as 16 will be 5 by 16; 4 plus 2 6 minus 1 is 5 by 16. So, that is what it will be, so delta y max is this.

Now, why is delta y max negative? That is the question, we have to answer next. Delta y max is negative because you assume the pass wide has to be on the top, whereas the beam is deflecting at the bottom. It is moving in the negative y direction, that is why it is negative and the maximum deflection is 5 by 384 W L power 4 by E times I z z. So, delta y max is minus 5 by 384 W L power 4 by E times I z z, is negative because it is deforming in the opposite direction to the assume positive y direction.

Now, let us plot how the deformed shape looks like and how the slope of the deform shape looks like for the simply supported beam.

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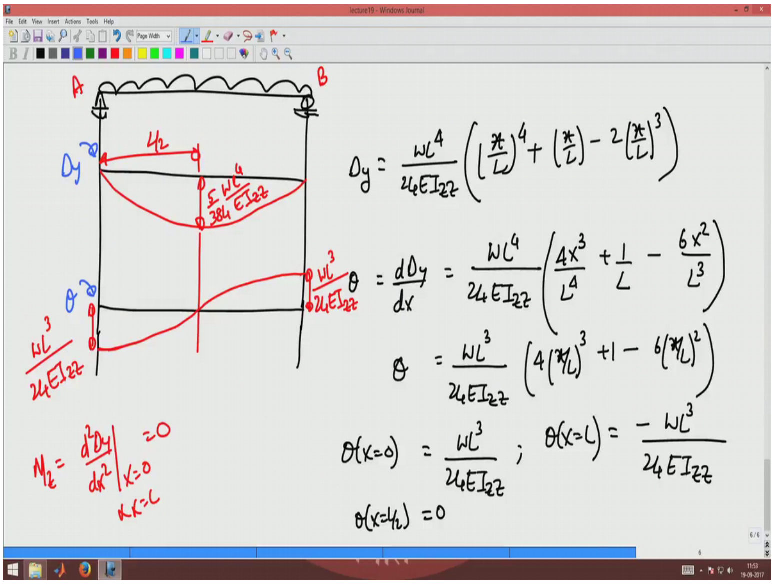

Subject to a u d L we found that delta y is given by W L power 4 by 24 E times I z z into x by L whole power 4 plus x by L minus 2 times x by L, the whole cube. So, it is a 4 third of polynomial and it is deform shape would look something like this; where the maximum bending moment will be 5 by 384 W L power 4 by E times I z z occurring at a distance of L by 2 and that is the location where the slope will go to 0 also.

Now, let us plot these slopes, theta is d delta y by d x that is W L power 4 by 24 E I z z into 4 x cube by L power 4 plus 1 by L minus 6 x square by L cube. So, this will be W L cube by 24 E I z z into 4 x by L whole cube plus 1 minus 6 x by L whole square.

Now, what is this theta? Theta at x equal to 0 is W L cube by 24 E I z z and theta at x equal to L would be minus W L cube by 24 E times I z z. Why is the sign of theta varying from x equal to 0 to x equal to L? That is the question.

Now, if I have to plot this cubic equation and you know that theta at x equal to L by 2 is 0, it is a continuous function. So, if there is a 0 the sign has to vary from positive negative, so this is a cubic equation. So, this curve would look like something like that and this value is W L cube by 24 E I z z and this value is also W L cube by 24 E I z z.

Now, here I have drawn it flat here because the next derivative of the bending moment here is 0 and bending moment M z is d square delta y by d x square that is a curvature.

So, d square delta y by d x square is at x equal to 0 and x equal to L 0, that is why the curve has 0 slope at this 2 ends.

At end C and B the slope of the slope curve is 0 because the bending moment is 0 at A and B, so that is how the theta slope will look like. So, this is delta of y, this is theta, the slope diagram and this is the deflection diagram.

Now, what is left? You have not still looked at the shear force. So, now, let us look at the shear force.

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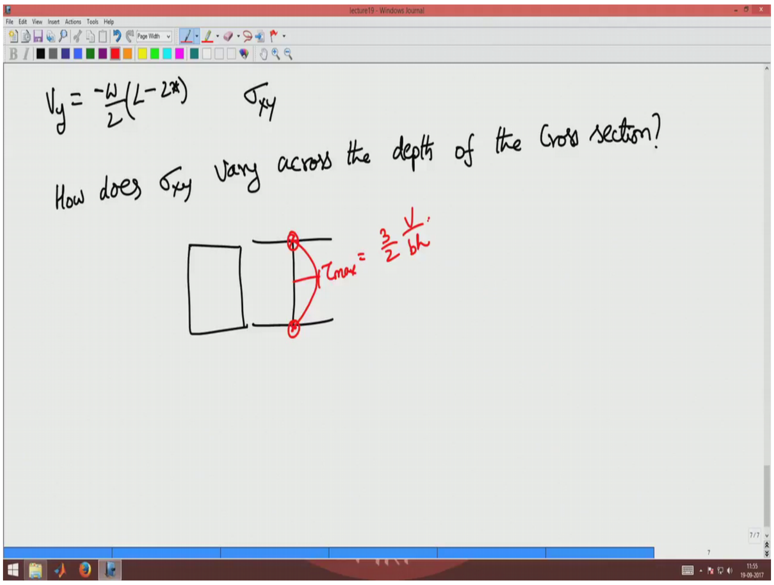

You just saw that V y, we saw that shear force was W by 2 into L minus 2 x; W by 2 into L minus 2 x is a shear force with a negative sign there, that is the shear force.

Now, what is the shear force going to produce? The shear force is going to be produced because there is a sigma x y stress. Now, how does this stress vary across the depth, this stress is produced by; the shear force is produced by the stress sigma x y. So, the question we have is, how does sigma x y vary across the depth of the cross section? Cross section is a question.

Clearly, you know that at top and bottom surfaces of the cross section, there is no compliment shear coming in because the why pin we said that only sigma y y stress acts instead has to be 0 at those 2 locations. So, it should have some variation along the depth to be given by, so if this is the cross section, you know that this 0 for sure at this 2 points and hence we will see in the next class that will be a parabolic variation like this with a maximum shear stress tau max being 3 by 2 V by b h, we will see this in the next class.

And, what we are seeing is, we are analyzing the simply supported beam subjected to a uniformly distributed load, we found the bending moment, the shear force, the deflection, the rotations and then we are yet to find how the shear stress varies across the depth, but we found out the bending normal stress where is across g depth, that is all for today.

Thank you.