Mechanics of Material

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Stresses and deflection in homogeneous beams loaded about one principal axis
Lecture – 53
Shear force and bending moment diagram

Welcome to the 19th lecture of mechanics of materials, in the last lecture we derived the governing equations for a beam bending problem.

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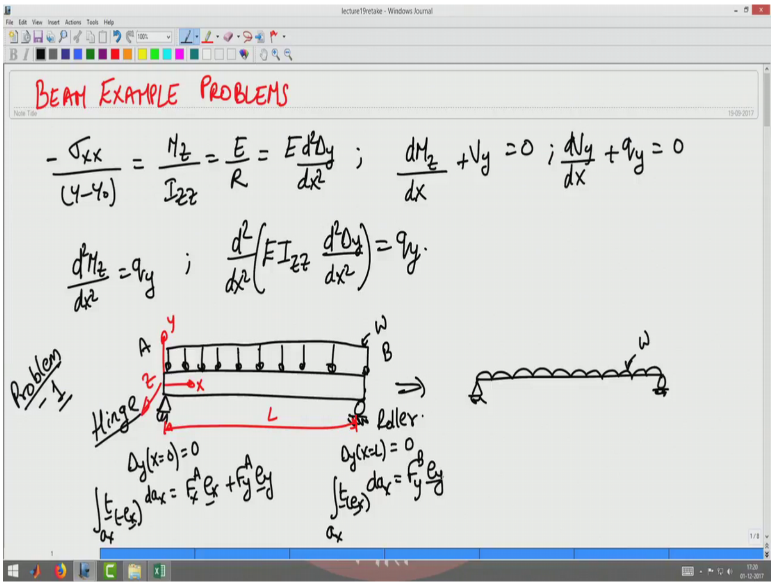

The equation we derived was minus sigma axis by y minus y naught equal to M z by I z z equal to E by R to E d square delta y by d x square, where R is the radius of curvature and then we also saw in the last but previous lecture that d M z by d x plus V y has to be 0, d V y plus by d x plus q y has to be 0.

Combining these 2 equations, we obtain the equation d square M z by d x square equal to q y by d x square equal to q y and we combined all these equations to obtain a governing equation terms of the applied loading q y and the unknown displacement delta y of x. Now, let us apply this problem; apply this equation to solve some problems. The first problem that I am going to look at is, a simply supported beam, subjected to uniformly distributed load W, which can be; this same problem can be represented in 1 dimensional form as the following. Many of you would be familiar with this, this is of point to W, what this support tells us is this is a hinged support.

So, delta y if I measure my cone system from here, say this x, this is y and this is z and this length is L, then what this hinged support says is, at x equal to 0 delta y has to be 0 it is free to rotate, since a simply supported can rotate freely, but it cannot resist any stresses, since there is no continuity at the hinged end, it cannot resist any stresses.

So, basically that being a hinged, it will resist the vertical deflection, it will resist the horizontal deflection and hence there will be the traction is acting on that piece of e x should be F x e x plus integrated this over the area d A would be F of x e x plus F of y e y, there is no shear coming on the other direction. So, there will be traction starting on that surface.

Similarly, this being a roller end delta y at x equal to L has to be 0 and it traction that is acting on this end e of x d a x would be just this will be of B of y e y, say this is A and this is B. So, that will be the boundary condition that we are going to apply.

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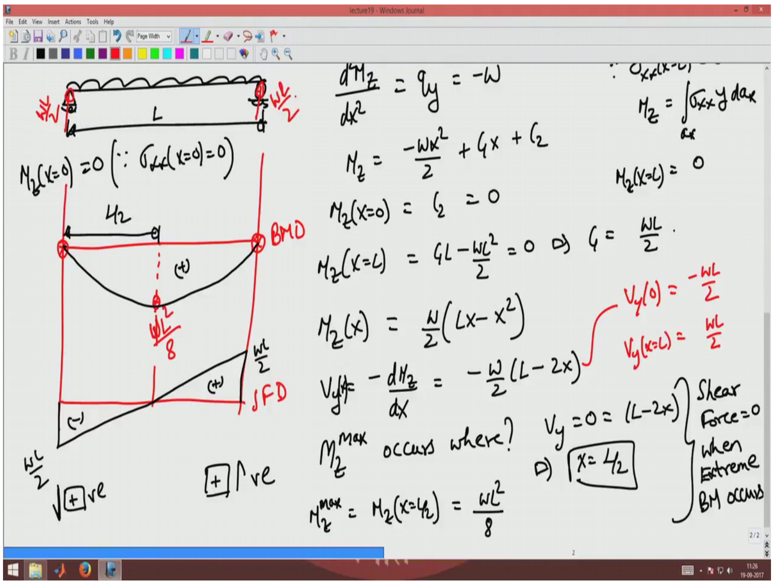

Now, let us go about solving this problem, I will use the 1 dimensional approximation from now on, I have this u d L this should be over a length L. So, I have the equation d square M z by d x square is equal to q y, which in this case since the starting vertically downwards be minus W, now then if I integrated twice I will get M z to be minus W x square by 2 plus C 1 x plus C 2.

Now, let us look at x equal to L end, there is no horizontal force that is developed and hence since sigma x x has to be 0 because of no continuity or no provision for horizontal stress to develop, you will find that since sigma x x at x equal to L has to be 0 M z which is integral sigma x x into y d a x a x has to be 0 at M z at x equal to L.

Now, let us look at this end; hinged end there is no horizontal force applied throughout in the problem. So, F x has to be 0 and there is no continuity of the member at the A end again, since there is no continuity of the member at the A end, there is nothing to support and axial stress sigma x x, if I cut there I expose sigma x x there is nothing on the other side, let us say sigma x x is 0 support does not provide the sigma x x and hence this end also M z at x equal to 0 has to be 0, since again sigma x x at x equal to 0 end is 0.

So, I use this condition that M z at x equal to 0 is C 2 and since this is 0, C 2 has to be 0 and then I use the condition that M z at x equal to L has to be C 1 L minus W L square by 2 has to be 0 from here I get C 1 to be W L by 2. Hence, M z moment has a function of x is given by W by 2 into L x minus x square. Now, from the governing equation again, once I know my moment again I estimate the shear force V y from this equation. So, I will do next, so V y is minus d M z by d x, which is minus W by 2 L minus 2 x, so that is V y of x.

Now, I am interested in finding where bending moment M z max occurs at, occurs where is the question, to maximize the function I have to set the derive to do 0, the maximum or minimum of a function occur where derived where the function goes to 0 and hence V y equal to 0 is L minus 2 x and that will be x is L by 2, this L by 2. So, maximum M z max will occur where M z is at x equal to L by 2 and that will be W L square by 8.

L B L square by 8 there is a maximum bending moment and this shows that shear force is 0, when maximum extreme bending moment occurs. So, let us now first plot these things on this figure that is I am drawing the shear force and bending moment diagrams; the bending moment diagram is a parabolic equation. It is maximum at L by 2 and at x equal to 0 and x equal to L it is 0. So, I know that it is 0 here, 0 here and there is maximum at L by 2. Now, the question on which side I have to plot?

Usually, you take the positive bending moment to be plotted on the top and negative bending moment to be plotted at the bottom, but we will follow the convention that the bending moment diagram, the line will coincide with the line where the tension occurs. The beam bends like this, you can see that the top surface is in compression and the bottom is in tension because the length at the bottom has to increase, length of the top has to decrease for it to when it curves like this. Hence, what happens is, the tension occurs at the bottom. So, we will plot the positive bending moment at the bottom and negative bending moments at the top.

So, at L by 2, you have W L square by 8, this occurs at L by 2. Now, it is a parabolic equation, so I start a linear equation, so I join this like this, it becomes a parabolic curve like that.

Now, let us plot the shear force, this is the bending moment diagram, what I am plotting now is the shear force diagram. What happens for the shear force at x equal to 0 from here V y at 0 is minus W L by 2 and V y at x equal to L is W L by 2. Now, what is the meaning of this? The shear force has to sign, which corresponds to the sign of the shear stress. So, we will plot it with the sign as such and shear force at L by 2 is 0. So, the shear force variation would be something like this, it is a linear curve. So, it is negative here, positive here and this is a positive bending moment W L by 2 and that is W L by 2.

Let us understand why this is negative and this is positive. So, if I take a section here, the positive shear force should act like this; this is a positive shear force. This negative meaning it is acting upward there, which is right because I am applying a downward force, so the resultant will be an upward acting force. On this phase, this is a positive shear and the reaction forces acting upward there. So, that is why there is sign of the shear force changes from the left end to the right end. What it means is, from force equilibrium you can see that this reaction force should be W into L by 2 and this reaction force should be W into L by 2, both acting vertically upwards.

Since the beam is symmetric, we expect the reaction forces should be symmetric; same. Now, you have found the bending moment and shear forces, next let us find the stresses and displacements.