Mechanics of Material

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Displacement due to uniaxial loading, temperature and bending
Lecture – 52
Radius of Curvature

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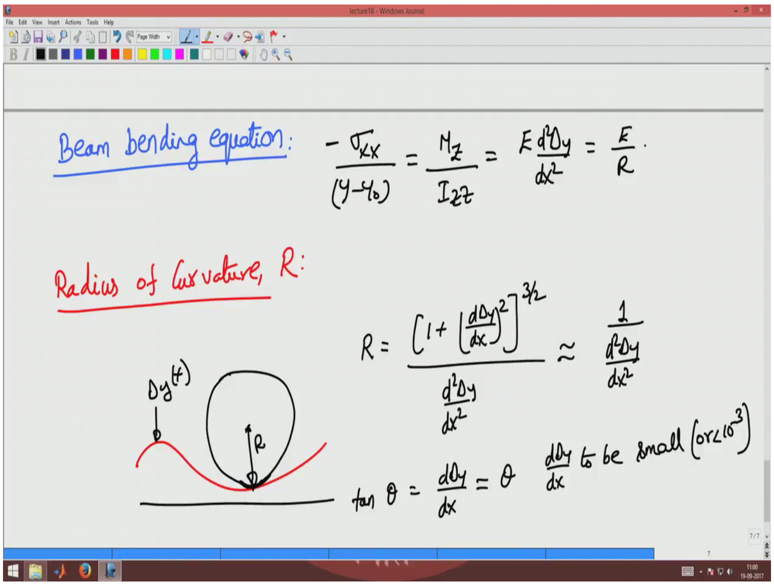

Now, let us understand what radius of curvature is, first. Next what we are trying to do is we are trying to understand what is the meaning of radius of curvature.

Curvature R, now say I have curved like this, now what we are interested is, we are interested in finding at a given point what is a circle? What is the radius of the circle? R, such that locally you coincide with the curve at that point, the radius of circle, whose curvature would be same as the curvature of the curve at that point locally. So, R from identical geometry is given by 1 plus d delta y by d x whole square whole power 3 by 2 divided by d square delta by d x square, where delta y is the curve; the equation of the curve as a function of x, this x which deformed into like that, delta y is the equation of the curve.

So, now what happens, we approximated theta; tan theta which was d delta y by d x as being approximately equal to theta because we assumed d delta y by d x to be small or of order 10 power minus 3 or lesser. Hence, what we can do is, compare to 1 here, this d delta y by d x whole square will be small. So, I can approximate this as 1 by d square delta y by d x square. Now, rewriting this, then this equation becomes E by R, that is a bending equation.

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So, if I take a wire and wound it all around this pen, the rays of curvature of that wire would be the radius of the spin of that actual location, the rays of curvature of the pen; radius of curvature of the wound wire which is bent around this pen would be the radius of this pen locally.

So, I will use that radius of curvature here to estimate, what my bending stresses are directly in that case, so basically this is c by R.

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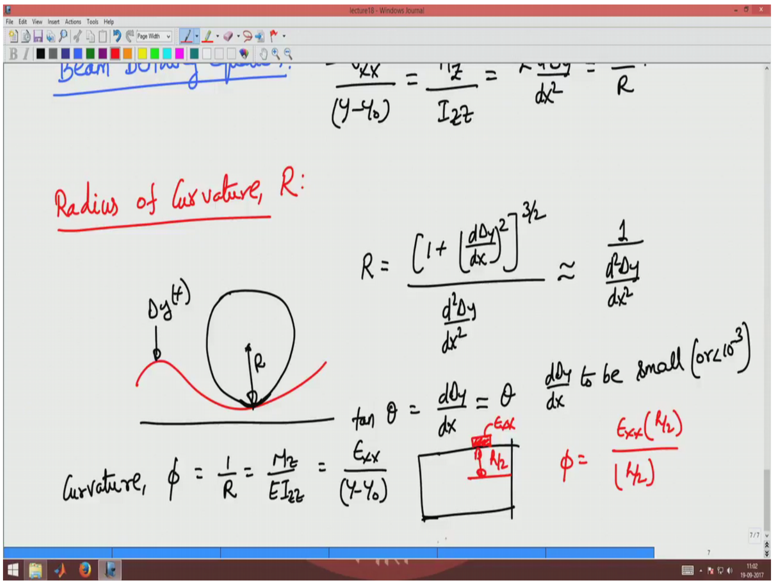

Now, there is 1 more quantity called as curvature, which is denoted by phi, which is nothing but 1 by R which is M by E I z z, which is nothing but epsilon x x by y minus y naught, that is if you measure the strain at a particular location the actual strain at a particular location and divided by the distance from the neutral axis that will give you a measure of curvature.

That is for our cross section like this of a beam and I am measuring the strain; the actual strain at this location, I am measuring epsilon x x at this location and at this location from the neutral axis at a distance h by 2, then curvature is given by the strain at h by 2 divided by h by 2, that is a measure of curvature of the beam.

In other words, there will be nothing but the moment; bending moment divided by E times I z z. So, to summarize what you have seen in today’s lecture is 2 things, you have found the bending equation for the stresses, the bending equation is given by minus sigma y minus y naught equal to M z by I z z equal to E by R equal to E phi equal to E d square delta by d x square delta y.

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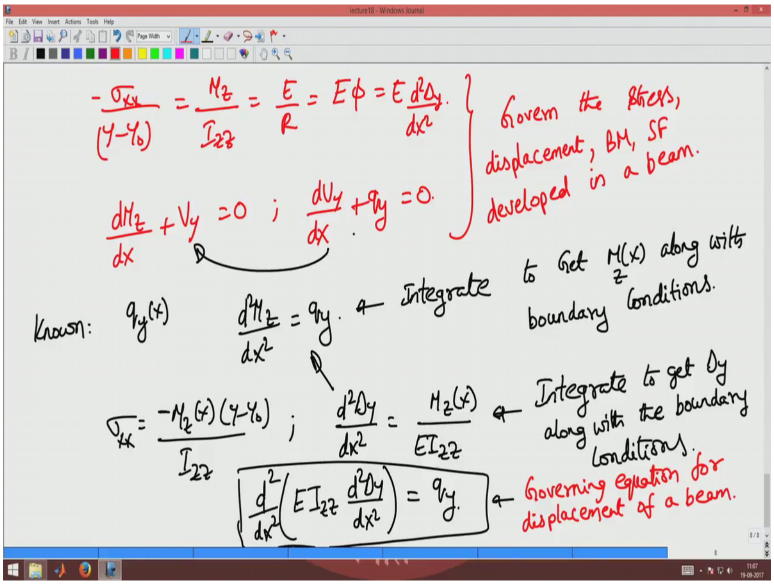

So, and then we are 2 more governing equations d M z by d x plus V y equal to 0 and d V y by d x plus q y equal to 0. So, now, these equations govern the stress displacement bending moment and shear force developed in a beam. Now, how do you apply this for a given loading q of y? The input known quantity is q of y as a function of x is a known quantity, now you take this and then you substitute you will get by substituting for V y in here, you get d square M z by d x square plus is equal to q y, you integrate this; integrate to get M of x M z of x along with boundary conditions.

Once, you get M z of x, you go to the first equation; you go to this equation and what you do is you get the stress as M z of x divided by I z z into y minus y naught with a negative sign that gives you sigma x x stress, then you use the final equation to get d square delta y by d x square to be given by M z divided by E times I z z of x. Now, you integrate this, integrate to get delta y along with the boundary conditions.

Now, combining these 2 equations what I get is, I get d square by d x square to E times I z z d square delta y by d x square equal to q y. So, this is the governing equation for displacement of a beam that is the governing equation for the displacement of the beam. Your finite element codes x app, E tabs, abacus, anses already solves this equation to get the response of a beam.

So, in the next class what we will do is, we will take a specific examples and work out details on how to solve this problem and how to get the stress, strain, your displacement, your shear force and all the response quantities for a beam.

Thank you.