Mechanics of Material

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Displacement due to uniaxial loading, temperature and bending
Lecture – 51
Bending Equation

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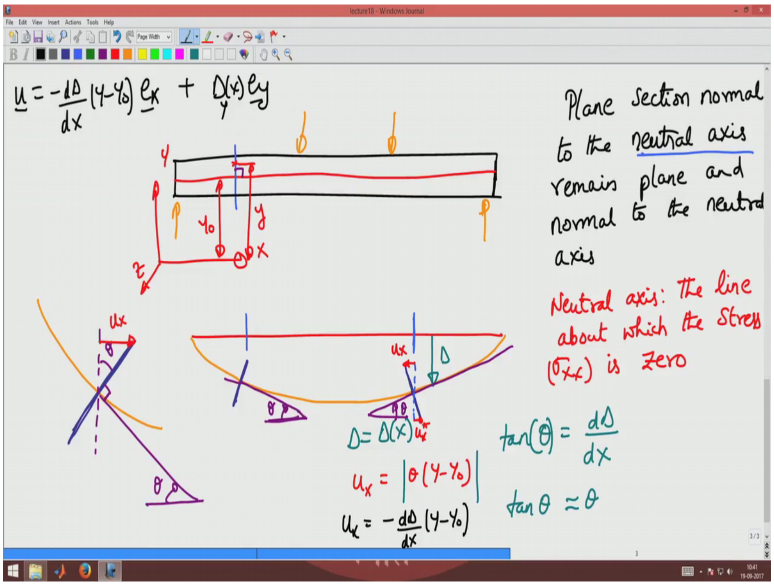

Now, let us proceed, we assume a displacement field, u to be minus d delta by d x delta y by d x to y minus y naught e x plus delta of y as a function of x e y ok.

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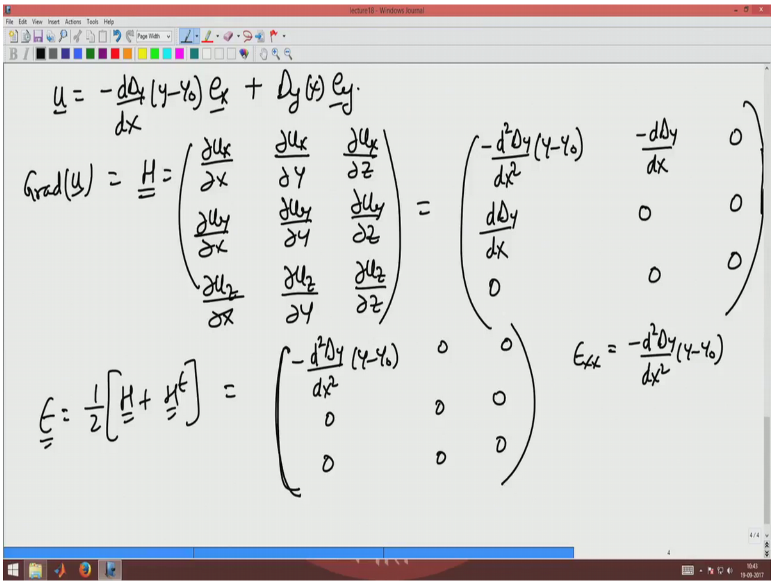

Now, I want to compute gradient of few. So, that I can compute the strain, gradient of u is H which is given by dou x by dou x, if you recollect we saw this 1 we are looking at strains dou x by dou y; dou x by dou z; dou u y by dou x; dou u y by dou y; dou u y by dou z; dou u z by dou x; dou u z by dou y; dou u z by dou z, this for the assume displacement filed would be minus d square delta y by d x square into y minus y naught minus d delta y by d x 0; d delta y by d x 0, 0, 0, 0, 0 ok.

Now, the linearized strain as we saw before going by 1 of H plus H transpose which in this case happens to be given by this matrix, y by d x square to y minus y naught 0, 0, 0, 0, 0, 0, 0, 0. Here, a few comments are to be made, the first comment is you have only epsilon x x strain as minus d square delta y by d x square into y minus y naught and there is no shear strain or other components of the normal strain in effect what you are saying is there is no pass on effect in this problem that is if the if you consider pass on effect where there is an actual strain will be a lateral contraction that is operating in the other 2 direction which are ignoring as a first approximation for the problem.

Basically what we add was we add only axial strain, we ignore the pass on effect which means you are ignoring the lateral contraction that will happen when there is a actual strain, that we have ignored as a first approximation.

The second thing is, since we assume plane instructions remain plane and normal to the neutral axis, there is no shear strain and there is no angle change in this (Refer Time:

03:18) system, there is no angle change in this (Refer Time: 03:21) system. So, since you have the plane being perpendicular even after deformation to a particular line which was initially it was perpendicular to; that is basically because of our assumption of the deformation field for the beams.

So, the strain is given by this expression, then we appeal to 1 dimensional consultation where in sigma x x would be related to epsilon x x through the relation E t and epsilon x x. This is 1 dimensional consultation because we are ignoring we pass on it is effects.

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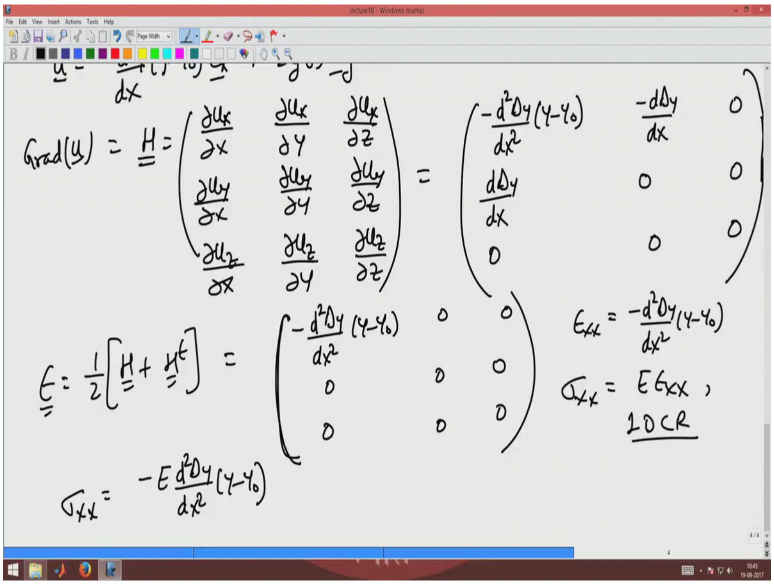

So, now then sigma x x is given by minus E times d square delta y by d x square into y minus y naught. Now, what we will do is we will go back and plug this expression for sigma x x the equilibrium equation that we are, that is we will plus sigma x x in a in this expression, in this expression, in this expression and then use this equilibrium equations this, this and this to find what is the variation of sigma x x, basically unknown function the unknown function here is d square delta by d x square. So, we want to estimate this, so we will see how to estimate this.

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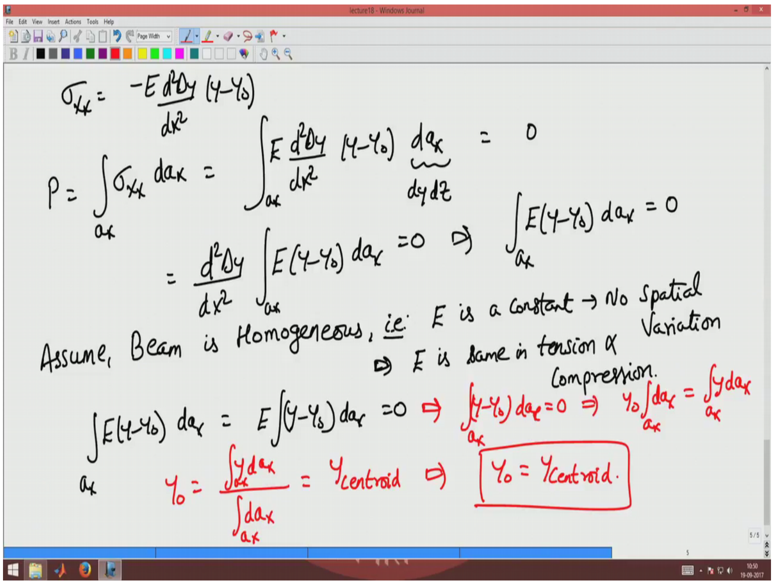

So, we saw that sigma x x is given by minus E times d square delta y by d x square into y minus y naught. Now, the actual force P is sigma x x into d a x; a x that will be E times d square delta y by d x square into y minus y naught, integrated over d a x this d a x is nothing but d y d z as you have seen many a times now, this as we equated to 0. Now, the integration with respect to y and z it is independent of x. So, I can pull out d square delta y by d x square outside the integration. So, this will be E times y minus y naught into d a x is equal to 0.

Since d square delta y by d x square cannot be 0, identically this will imply that integral E times y minus y naught d a x has to be equated to 0. Now, let us assume beam is homogeneous, that is E is a constant implying no special variation. So, I am not considering the case where the beam is made of 2 different materials for example, concrete and steel or wood and steel, wood and aluminum and so on. So, I am considering a case where E is the same throughout the beam and this also means that E is same in both tension and compression. This also means that E is same in tension and compression.

We will see why I am saying this shortly. Now, what happens now is, now this as I can pull out E since E is a constant. So, integral a of x E y minus y naught d a of x will give me now E times y minus y naught d a of x equal to 0. This, implies integral y minus y naught d a of x has to be 0. Since, we saw that E cannot be 0 because E has to be greater than 0 in material parameters when I estimated the restriction of material around is we saw that.

Now, this will imply y naught integral d a x must be equal to integral y times d a x or y naught is given by integral y times d a x divided by integral d a x. What is this integral y d a x divided by integral d a x is nothing but this centroid of the cross section, the y component of the centroid of the cross section; y of the centroid of the cross section. So, basically y naught is identified as this implies y naught is equal to y centroid of the cross section. Now, we have found what y naught is or we have found expression for y naught, next let us go ahead and find d square delta y by d x square.

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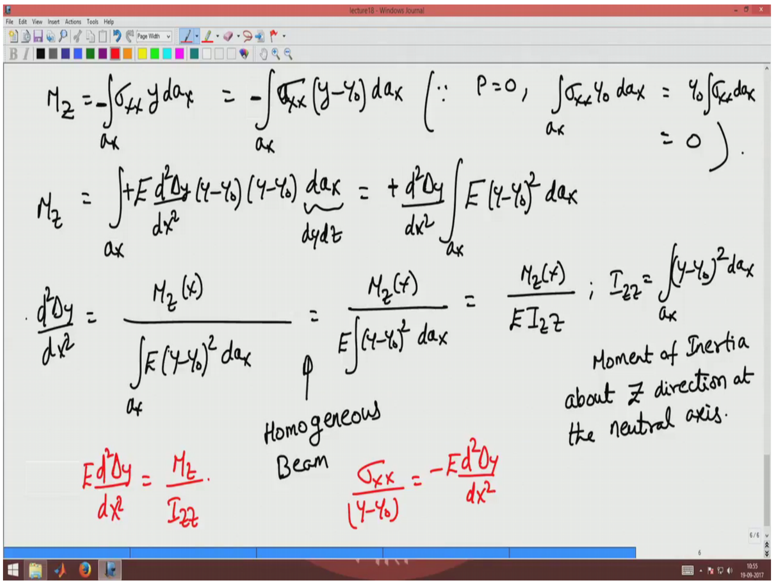

Now, I know that M z moment is given by integral sigma x x into y d a x a of x; this I am claiming the same as sigma x x into y minus y naught d a x a x, since P is 0, integral sigma x x into y naught d a x a x would be y naught into sigma x x d a x, so that is equal to 0 because just now we show that sigma x is d a x is 0. So, I can rewrite the moment equation as this. Now, I substitute for sigma x x from the previous equation that is it is minus E times d square delta y by d x square into y minus y naught into y minus y naught d a x.

So, this is nothing but again d a x is d y d z this is delta y is a function of x alone. So, I can move that outside the integral, this would be d square delta y by d x square; integral E times y minus y naught whole square d a x a x. In other words, this is nothing but from here I get d square y by d x square to be M z, so function of x divided by integral a of x; E times y minus y naught square d a x. We saw that if the beam is homogenous, E will be a constant and hence you get M z of x divided by E times y minus y naught whole square d a x, this is for a homogeneous beam assumption.

So now, what is this? This is nothing but M z of x divided by E times I z z, where I z z is given by integral y minus y naught whole square d a x a of x, this is moment of inertia about z axis and neutral axis, about zed neutral axis direction at the neutral axis, but I said direction of the neutral axis.

So, we get the equation minus E times d square delta y by d x square is M z by I z z. We saw before that sigma x x by y minus y naught is again minus E times d square delta y by d x square.

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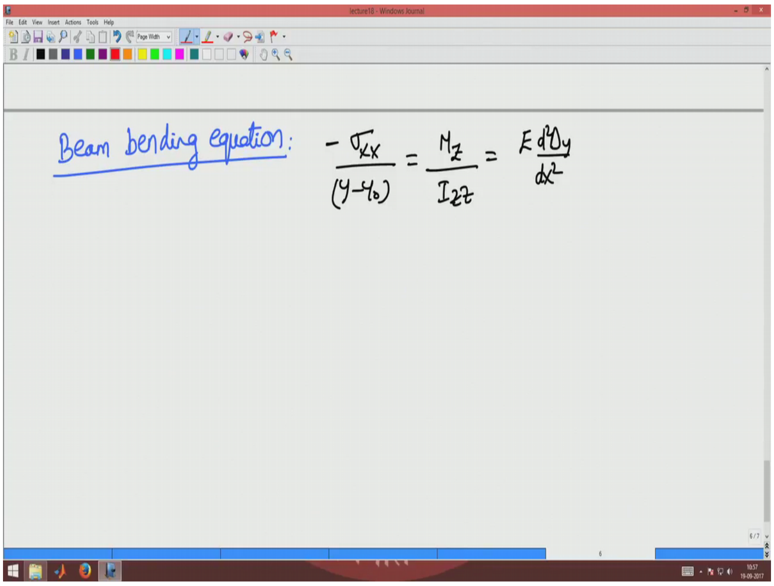

So, combining these 2 equations you get the beam bending equation, which is sigma x x by y minus y naught equal to M z by I z z; E into d x square. So, we have the beam bending equation given by this equation in here.