Mechanics of Material
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Displacement due to uniaxial loading, temperature and bending
Lecture - 50
Displacement Field

Welcome to the 18th lecture in mechanics of materials, in the last lecture we started looking at how a beam will deform and what are the stresses that the beam develops, in particular we are looking at the stresses that the beam would have.

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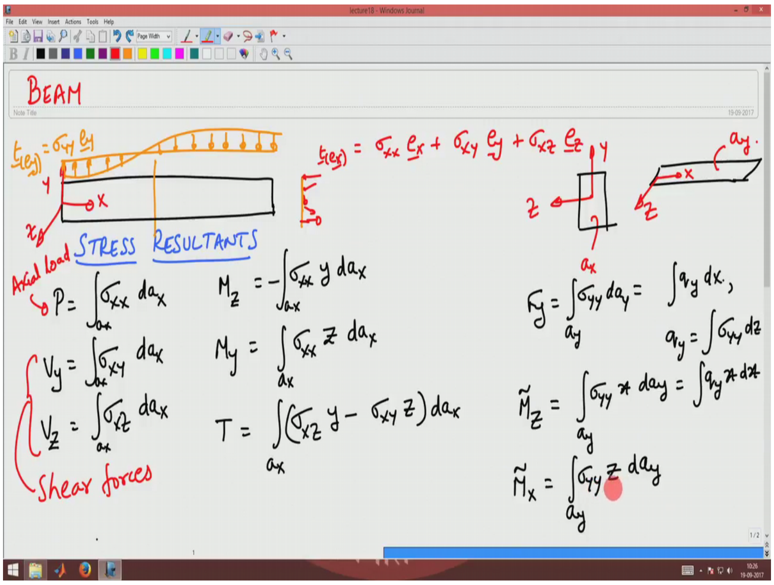

In particular what we did was, we exposed a cut surface or we introduced a cut surface in the beam and we exposed the traction vector that is acting on that beam, the cut surface was essentially a cut surface along the x direction with the normalize the x direction as shown here and then we wrote what are the stress resultants and we wrote the global movements and forces are acting on the beam on various cut surfaces and the boundary of the body.

In particular, we saw that, we said that these are acting only on the top surface of the beam that is on the positive e y direction normal along there is some stresses acting on the beam that too there was only a sigma y y component of the stress acting on that surface that is apply loading and then when you introduce a cut in the beam, somewhere

here, we saw that the traction is acting on the beam, we saw that the traction that is acting on the beam will change it is magnitude as well as the direction, as shown here.

In general that traction was given by an expression like this, which is a usual expression you get when you have e x as e normal to the cut. Now, this traction starting over an area given by ax, wherein it is I assumed here to be a rectangular cross section with y and z over into like that. That is, it is acting on a cut surface here for example, if you take this as the beam, this cross section area is where the sigma x x acts, sigma x y acts and sigma x z acts.

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This a beam where loading it like this, loading on a top phase. So, basically what is happening is the beam is bending like this. So, what are the stress resultants for that, the stress resultants are the net force and net moment that will act at the cut surface because of this traction. The net force that is acting in the axial direction is given by P which is a sigma x is d a x, the shear forces V y and V z are the shear forces, these are the shear forces this is the axial load.

So, basically the shear forces V y in a cut surface acts like this and the shear force V z in the cut surface acts like this, the actual force P acts in this direction. So, those are the stress resultants; those are stress force resultants. Similarly, you have stress moment resultants, which is the torsional moment T which is given by this expression in here and then you have the moment M z acting along this direction given by this expression here, the moment M c given by this expression here, which is a moment. So, basically this is y, this is x and this is z. So, M c moment will produce a deformation something like this, this is a M z moment deformation.

M y moment on the other hand will produce a moment like this, you can see, it deforms in the plane. M z moment produces a moment like this wherein the deformation would be like this. m z is like this, M y produces a moment which is like this in the plane, the moment M y produces in plane deformations and then due to the attraction force acting on the boundary of the body, the y direction wherein that area is given by a y, it spans in the x z plane, there the net force F c f is given by this expression, where in we introduce unassembled q y, this is the integration of this.

Sigma y y over the width of the cross section that is over this width you are integrating what are stress that is coming in and your net force per unit length is given by q y here, correspondingly that will produce a moment M z which is q y to M 6 and M x which is sigma y y times z into a y.

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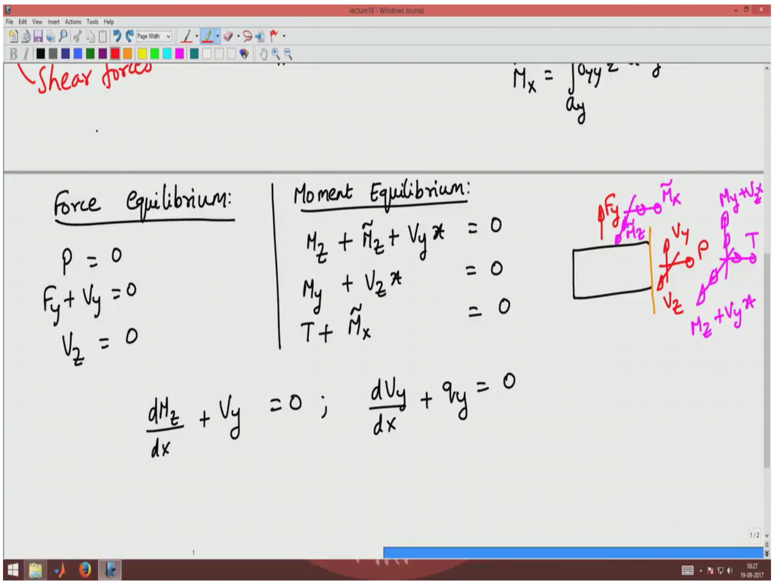

Now, we wrote so the net force and the net moment that is acting on a cut surface of the beam is given here, P acting along the axial direction, V y acting like this, V z acting like this F acting like this, F y acting like this and M z moment acting like this, M x moment acting like this and then M x tilde moment acting along on this phase causing it to twist like this and then M z moment which is again acting like to deform the beam like this M z tilde moment.

So, the net equilibrium equation for the force in a moment is given by here, which we manipulated to get these governing equations for the moment and the shear force and the applied loading. Now, if there is a loading on the z plane, then you will have order F z force and you will have other moments coming in M y tilde and M x tilde will come over there on this phase of the beam.

Basically on this phase of the beam, there are to be some loading acting on it, there will be other stresses that are coming in. So, there will be other stresses that are coming in, if there is some force acting on this surface of the beam, we consider only forces to act on this surface that too only transverse normal to the surface. We did not add any shear stress in this direction or in this direction acting on the surface, we had only normal stress acting on the surface.

That is a basic recommend for a beam to behave like a beam; for a member to behave like a beam. So, basically if it were to bend like this, I have to apply some force here, you can say I am applying some force in this direction for it to bend like that. Then, there will be some other forces and moments coming because of the boundary force, boundary traction is acting on this surface.

So, this is a special case that we are considering, we are considering only first start on the top surface of the beam. If there are force acting in a bottom surface, again you have to include that and you have to rework all the equation that we have derived till now. So, basically this is the governing equation where we stopped in last class, let us proceed further. I am interested in estimating this stresses sigma xx in this course, in this lecture.

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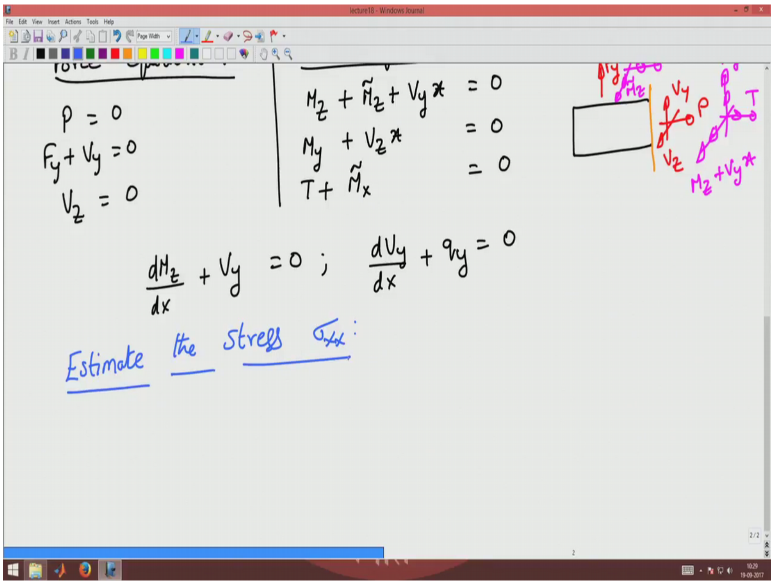

Estimate the stress sigma x x, this is the aim of this lecture today. So now, as I told you, you add a beam, you add a beam like this.

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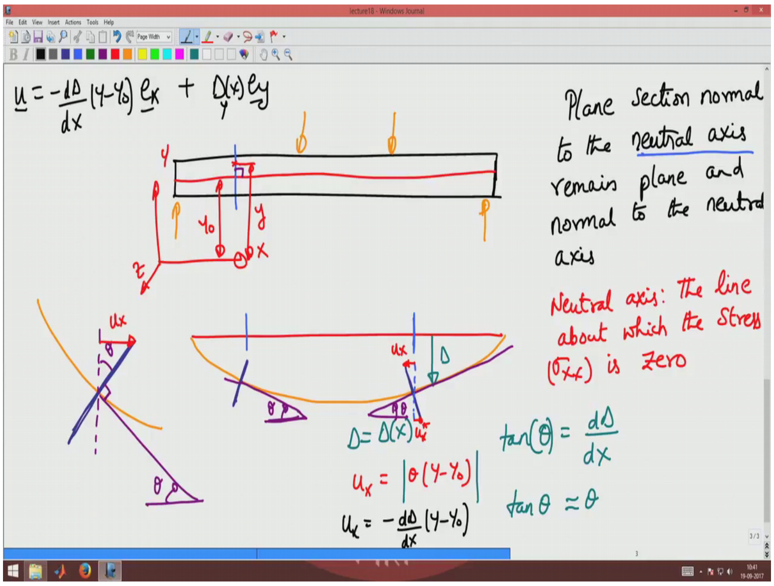

I am going to represent the beam using a 1 dimensional representation that is I am going to represent the central axis of the beam alone. That is, if this is a central axis of the beam, I am going to represent this beam using the central axis alone. The central axis due to the application of a load, some load acting like this, some load acting like this, would deform some load acting like this, would deform into some curved shape like that.

Now, let us look at a cut surface, this cut surface there that is this surface here, what we are going to assume is, we are going to assume this cut surface how it deforms, all this blue cut surface deforms. So, we are going to assume, how this plane sections will deform, how this blue line is going to deform. So, basically what we are saying is plane sections normal to the neutral axis remain plane and normal to the neutral axis.

Here I use the word neutral axis, what this word neutral axis means is, neutral axis means this red line is a neutral axis here, the line about which the stress is, the stress sigma x x is 0. We will see, once we derive we will see why such a axis will exist in a beam, but for now, let us assume that there exists such axis called a neutral axis, our stress goes to 0 and I am giving a cut perpendicular to that neutral axis in the initial configuration or the reference configuration.

After it deforms, what happens is, this blue line will become into some blue line here. So, the tangent to this line at that point, times the line at that point let us say makes an angle theta with the horizontal, measured in the clockwise direction, this is theta measured in the clockwise direction. Let us zoom in here and draw an exaggerated figure. I have the surface, I have tangent to that surface at that point, I have the blue line which is perpendicular to that let us say this makes an angle theta, then the angle made by this, this 90 degrees, the tangent because I assume plane sections remain plane a normal to the neutral axis.

So, initially it was 90 degrees here and then should be 90 degrees here too. So, this angle will become theta, then this displacement along the x direction, this u of x is given by theta into y minus y naught, where y naught is, say I add my coordinate system already like this y z, this location of the neutral axis is y naught and any other location here as a distance y from the x axis.

So, u of y is theta into y minus y naught. Now, if my vertical deflection here is, this vertical deflection down here is delta, delta is a function of x; delta is some function of x because you see that, as I bend the beam, the vertical deflection is not same at different points, the vertical deflection varies along the acts of the beam, hence this delta as a function of x.

So, delta is a function of x, then what will this theta be? This theta is going to be, tan of the theta is going to be d delta by dx, from you know that slope of a line is given by the tangent of, the curve at a point is given by the derivative of the function with respect to x. You know that, the tangent at a given point is given by the derivative of the function with respect to the x, so theta is this.

Now, that is a catch though here on what you substitute for theta, we are assuming small deformations. So, tan theta is approximated as theta and then here the deformation is in the clockwise, the rotation is in the clockwise direction. In other words, the theta that I will get will be negative in this case, but I want a positive displacement because it is deforming in the positive x direction. Since I want a positive direction, I have to put a absolute sign here, which means the theta that I get will be negative, to make it positive I have to write it as u of x as minus d delta by dx into y minus y naught, into y minus y naught this what I will get here.

Now, let us look at different section, let us look at a section at this side, now the tangent to that point here is something like that. Now, the cut surface deforms parallel remains plane and perpendicular the to the normal, so it will be like this, it will be like that. Now, what happens at the surface, that theta that you measure, this is a theta that you get from differentiating delta with respect to x that is a positive theta.

Now, for this positive theta what happens, the deformation is in the negative x direction you can see that right, the deformation is in negative x direction here, it is in this negative x direction this is u of x. So, when delta is positive you are automatically getting that then displacement is not negative (Refer Time: 17:03). So, you have the negative sign put in here, remember y is greater than y naught, since we are looking at a point above the neutral axis.

If a point is below the neutral axis y minus y naught would be negative and it will be positive in this case for positive d delta by d x which is right because at the bottom you have a displacement ux star which is the power along the positive x direction, so ux is given by this expression here. In general, now u the displacement vector is given by u x e x that is minus d delta by dx into y minus y naught e x plus delta is a function of x into e y, to be more specific let us write it as delta of y, being a y component or the displacement.