Mechanics of Material
Dr. U. Saravanan
Department of Civil Engineering
Indian Institute of Technology, Madras
Introduction and Mathematical Preliminaries

Lecture – 02
Part 2
Mathematical Preliminaries
Vector Algebra

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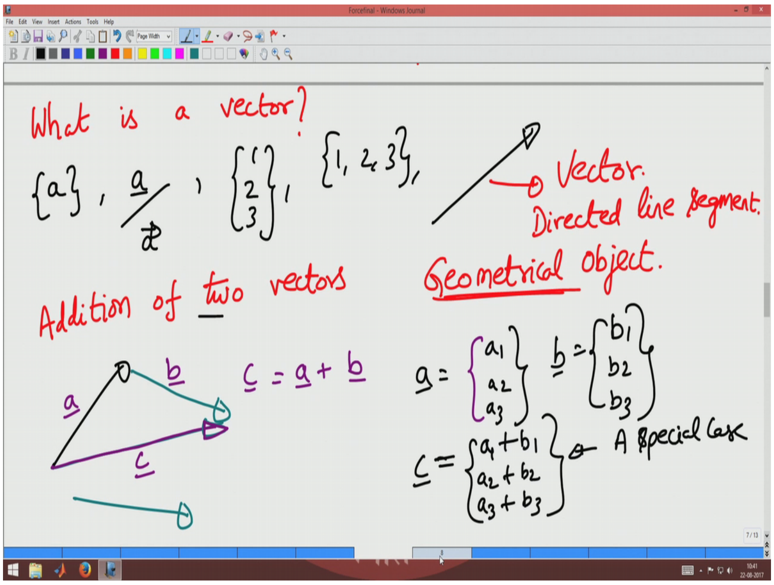

We are going to look at what a vector is. A vector, is this a vector or is this a vector or some of you would have been using a notation with the arrow on top, is this a vector or is this a vector, is this a vector or is this a vector which of these is a vector. I have given five choices which of these five choices is a vector.

For mathematicians any of these five choices represents a vector, but for mechanics purposes when you say a vector we mean a directed line segment. This is what you mean by a vector this what you mean by a vector this is for is a vector that is directed line segment. It is a geometrical object, this is a geometrical object and that is a vector.

In mechanics we consist this has a vector, now let us understand what you mean by addition of 2 vectors say I add dotted line segment like this and another dotted line segment like that. And I say I am going to add these 2 I bring this dotted line segments such that I move this parallelly on to this, assume these 2 are parallel then this is the resultant vector this is the resultant vector c if you denote this by a b a, write it as a plus b to be equal to c.

For us if a were to be, if a were to be written as a 1 a 2 a 3 and b were to be written as b 1 b 2 b 3 then c is a 1 plus b 1, a 2 plus b 2, a 3 plus b 3 is this is a special case. We will see when we can write a as a 1 a 2 a 3, b as b 1 b 2 b 3, but what we mean by addition is the diagonal form by the parallelogram with a and b as a sides of a parallelogram is what you mean by addition of 2 vectors. There are cases where you can write the components of vector a and b as this and the components of vector c will become this and does special circumstances what they are and how to when it becomes like this we will see subsequently.

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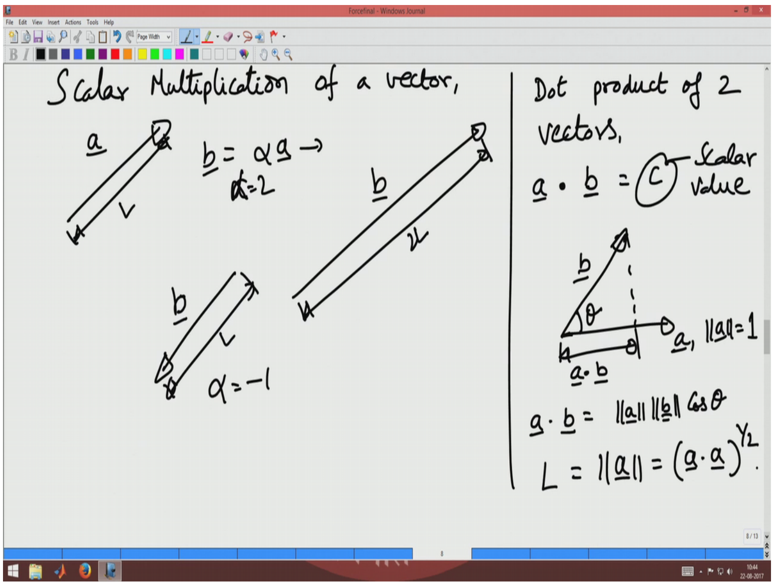

Next we are interested in seeing scalar multiplication. Multiplication of a vector that is if I have vector a which is this dark line segment if I want alpha times a what it means is and if alpha is if alpha is 2 then this vector this a parallel lines where the length has become 2. This length was L this length has now become 2 L and direction remain the same and other and if alpha where to be minus 1 the direction reverses and the length remains as same this is remains L, but direction has reverse and this would be if this b vector this is the b vector here that is a b vector there. That is what scalar multiplication of a vector mean.

On the other hand a dot product means of 2 vectors is denoted by a dotted with b is equal to sum scalar value c. This is not a vector, scalar value. What it means is if I have a vector a I have a vector b that line segment I have project this vector on to this and this length is denoted by a dotted with b that length is denoted by a dotted by with b. In other words in terms of length of a and b this would be mounted of a mounted of b times cosine of theta a theta this is a angle between a and b that is what a dotted product means. Here I am assuming that mounted of a here I have assume mounted of a to b equal to 1.

So, that is why its production of b on to a. The length is given by mounted of a which is a dotted with a power half. I am sure you know all this things already this is just you a recap on certain basic operations of vector algebra and how do we represent those operations in this course.

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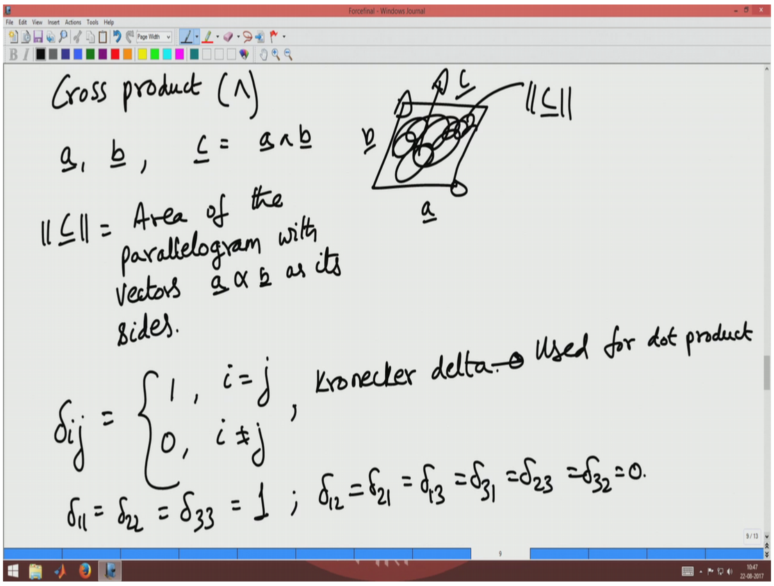

Next is cross product, we will denote this by this symbol in this course. If I add 2 vectors a b then c is defined as a cross b c is defined as a cross b then c would be perpendicular to this plane containing a and b which is a b c is a vector which is perpendicular to this plane mounted of c would be nothing, but area of the parallelogram with vectors a and b as its sides there is I complete this parallelogram. This area denotes the mounted of c. So, that is what a cross product does.

Now, we have to introduce 2 symbols to be able to mathematically represent the cross product and the dot product delta ij is equal to 1, i equal to j and 0, i is not equal to j this is called as Kronecker delta. That is if delta 11 would be equal to delta 22 b equal to delta 33 would be equal to 1, but delta 12 would be equal to delta 21 equal to delta 13 would be equal to delta 31 would be equal to delta 23 equal to delta 32 all this would be 0. So, this is called as a Kronecker delta. This is used to mathematically used for dot product.

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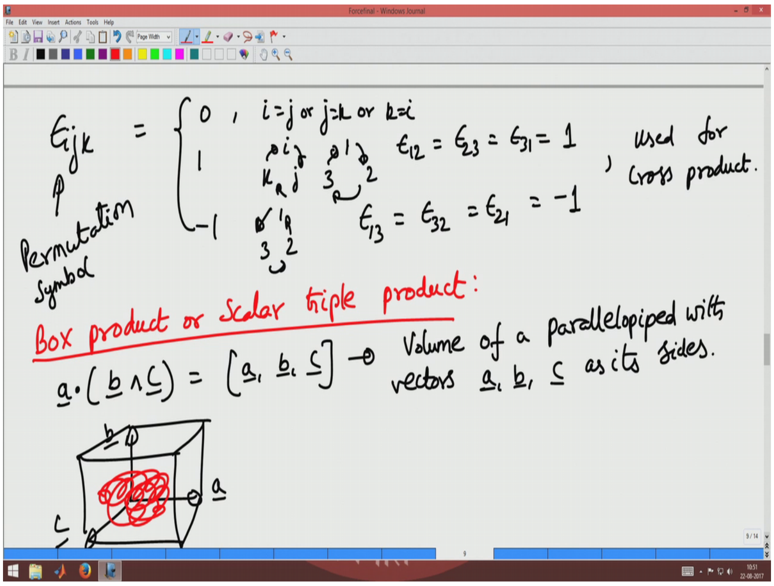

For cross product what we do is we use the symbol epsilon i j k which is called as the permutation symbol which takes values 0 if i equal to j or j equal to k or k equal to i and 1 if i j k go in a clockwise direction that is 1 2 and 3 go in a clockwise direction. That is epsilon 12 is equal to epsilon 23 is equal to epsilon 31 all this would be equal 21 and it will be minus 1 if 1 2 and 3 are going in a anticlockwise direction there is epsilon 13 is equal to epsilon 32 is equal to epsilon 21 would be minus 1. To note the order of index 1 2 1 2 1 are different signs here. It is called as a permutation symbol, is used for cross product; where it is used how it is used will understand shortly.

Now, there is one more provided we have to see that is called as the box product or scalar triple product. What is tells us is gives this is if you have a 3 vectors a dot b cross c will be denoted by a comma b comma c this is a combination of a cross product and a dot product and what this represents is volume of parallelepiped with vectors a b c as its sides. So, basically if I have a vector, b vector and the c vector a form of parallelepiped which is this and the volume contain in this parallelepiped is what this scalar triple product gives us.

So, we have seen 4 products now at dot product, a cross product, a scalar triple product, scalar multiplication and vector addition. So, basically these are some vector operation and we have seen you will see that we will use this to scalar triple product to find the volume changes when the body undergoes deformation will use the cross product to determine the area changes when a body undergoes a deformation and so on when you look at the corresponding deformation patents of the body.

So, we will stop here for today's lecture. Basically in today's class we have called an introduction to linear algebra how do you multiply 2 matrices and then we got to know about initial notation what is called as a dummy index and a free index. And the in the dummy index we will repeat only twice on one side of the equal to sign and a free index will appear on a as the set of the equal to sign a dummy index has to be summed from 1 2 3 whereas, the free index you not be summed from 1 2 3.

And then we got an introduction to vector algebra is what a vector is for as a vector is a dotted line segment, it is a geometrical object. And then we saw addition of what addition of 2 vectors mean what scalar multiplication of 2 vectors of a vector means and then we saw what a scalar multiplication or dot product of a vector means 2 vectors means, and then we saw what the scalar triple product means, and then what a vector product of 2 vectors means.

So, with this we will conclude today's lecture.

Thank you.