Mechanics of Material

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Lecture – 49
Governing equilibrium equations

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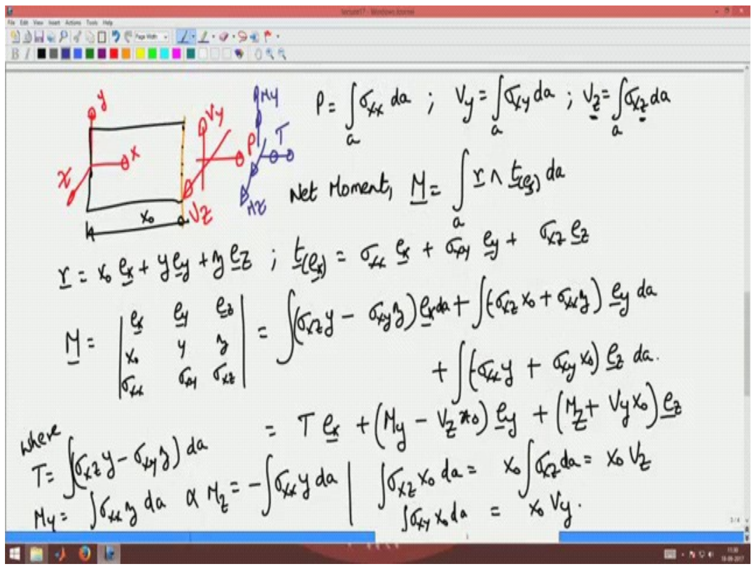

Now, there are other surfaces also the body right. So, I have to find; what is the net effect of the tracks reciting on the other surfaces too. So, let us do that.

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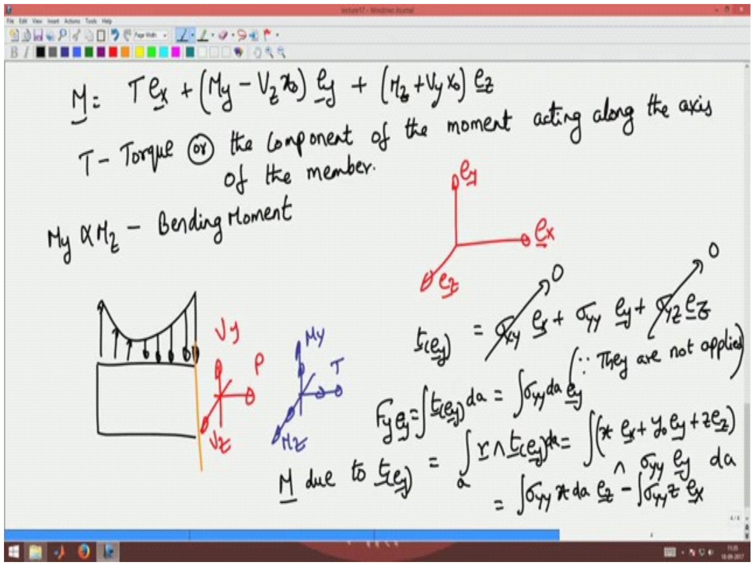

Let us take the cut surface cut body say I have I combine both the acting load on the reaction load to say that there is some net force acting on the body which is given by maybe something like this net force acting on a body something like that around the y direction a combined the reaction force with the force acting on the body. So, basically that has resulted in some forced distribution like this ok.

Now, in this cut surface there are 3 forces p, V y and V z. Now, there are 3 moments also acting on this cut surface which is the torsion moment the M y moment and M z moment. Now, what is the effect of this force on to this moments is what we are interested in finding. So, basically I have to find net force acting on this surface. I have find the net force acting on this surface and then I have to find the moment produced due to this force acting there. So, let us do that what is that face that face as a normal e y. So, I define first piece of e y is going to be in this case I am applying only are the top surface I am applying only sigma y y. So, in general will be sigma x y e x plus sigma y y e y plus sigma x sigma y z e z.

On the top surface I am applying only sigma y y, this 2 stresses are 0, since they are not applied and then the net force now acting along the y direction is integral t of e y da would be integral sigma y y da times e y this is the net force e y acting along that direction ok. Now, what is the moment produced due to this force, now I have to find the moment M let us say M e M due to t of e y that will be integral r cross t of e y or it is again same as before ok.

So, this is going to be integral x naught e x not x naught x e x plus y naught e y because it is at some this section is at some y naught plus z e z cross sigma y y e y da, I left the da there da. Now, what will this moment be? This moment would be integral sigma y y into x da e z plus integral sigma y y into z minus e x ok. So, there will be net moment produced due to this force sigma y y acting on that surface ok.

Now, there is no force or there is no externally applied load at the other 2 faces that is there is no external force applied on this face or on this face right. So, basically now I have to satisfy the equilibrium equations force and moment equilibrium for this section ok.

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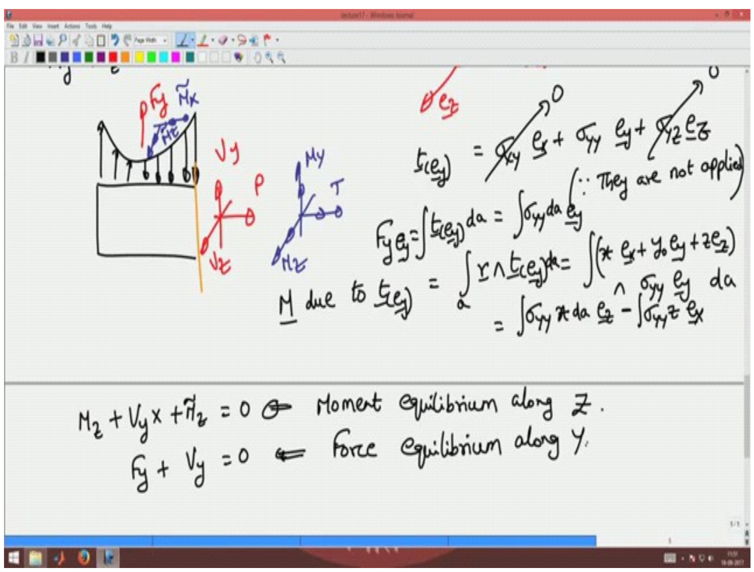

Now, here what I am getting? I am getting the F y I am getting a force F y. Yeah and I am getting a moment M z I had to use different symbols here. So, let us use M tilde z and I am getting a moment M x tilde at that face.

So, now equating the moments we saw that there are 2 components of moments due to the traction acting along the x direction that is M z moment plus V y into x naught this is the total moment component around the z direction for the traction expose on the x surface. So, you will have M z plus V y into x see this distance that distance x plus M z tilde has to be equal to 0 this is moment equilibrium along z along z direction ok.

Similarly, force equilibrium along y will tell you that F y plus V y has to be equal to 0 for force equilibrium along y. Now, force equilibrium along equilibrium along X direction would tell you that p has to be equal to zero and then force equilibrium along z direction will tell you that V z has to be equal to 0. We are not bother about other 2 moments in this course, but anyway for completeness let me write those also.

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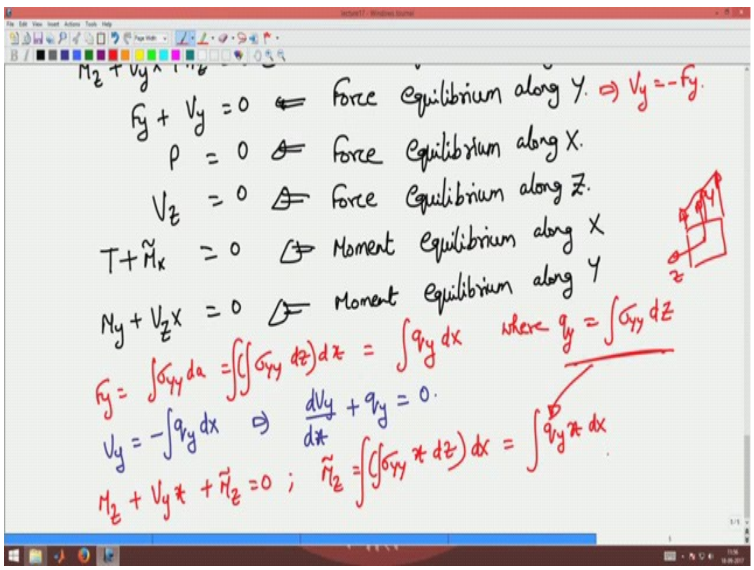

The moment equilibrium along X would imply that T plus M x tilde has to be equal to 0 and moment equilibrium along y would imply that M y tilde no m y will imply that M y plus V z into x must be equal to 0. Now from this force equilibrium I get the condition that V y is minus F of y recollecting what F of y was; F of y was integral sigma y y da and here da is sigma y y dz dx. I split there integral to what the are integral is and this we by definition was Q y dx ok.

Where Q y is define as integral sigma y y dz that is we have cross section this is y and this is z and I have a varying sigma y y along the cross section I integrate that over z and I get the effect lower that is recycling per unit length the (Refer Time: 10:09) this is q y dz. So, from here I get using this expression and using this expression and this expression I get that V y is minus integral q y da q y dx ok in other words this implies that d V y by dx plus q y is equal to zero ok.

Now, let us go to the first equation let us go to the first equation we had M z plus V y into x plus M z tilde equal to zero and here M z tilde was integral from here M z tilde is integral sigma y y here from here M z tilde is sigma y y into x da. So, will do that sigma y y into x dz dx, this will be integral Q y x dx again using the fact that Q y is different as this from this fact I will get this as Q y x dx ok.

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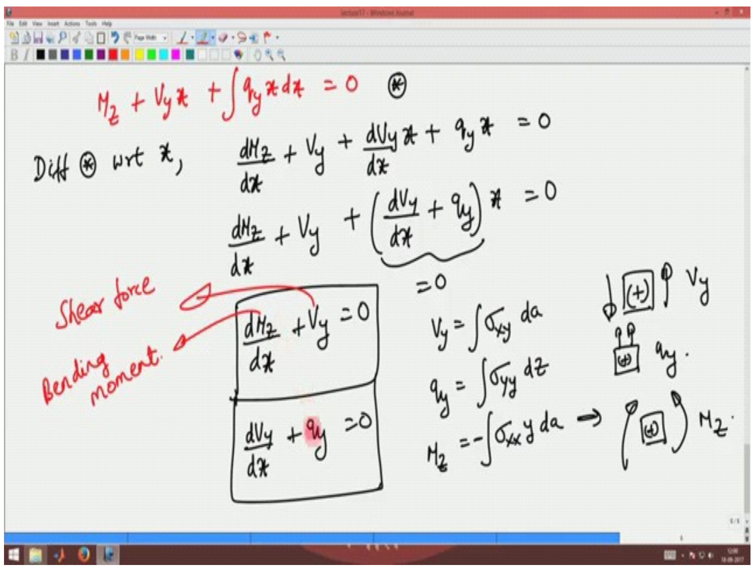

So, I get the equation from here M z plus V y x plus integral q y into x dx is equal to 0, .I differentiate this equation differentiating star with respect to x, what do I get I get d M z by dx plus V y plus d V y by dx into x plus q y into x is equal to 0 ok. From this equation this I can write rewrite it as d M z by dx plus V y plus d V y by dx plus q of y into x equal to 0 we saw that from the first equilibrium this has to be equal to 0 and hence we get the second governing equation that d M z by dx plus V y equal to 0. So, what you are seen today is seen to governing equations for beam bending, one is this and the second one is d V y by dx plus q y equal to 0 ok.

Here you have to note that V y is nothing, but integral sigma x y da and hence the positive V y is this is a positive V y and q y is given by integral sigma y y into dz and hence q acting upwards is positiveq y ok similarly for the M z moment; M z moment is integral sigma x into y da (Refer Time: 14:59) ok.

We will see next class that this imply that this is the positive moment this is the positive M z value essentially what we are done is we have taken beam cut a section expose the traction and then we wrote the global force and moment equilibrium for the acting forces on the beam and the expose traction from that we got the governing equations for how the bending moment and shear force will vary along the axis of the beam ok this V y it is called as the sheer force and M z is called as the bending moment.

So, in the next class we will take it forward and see how the definition of v will take place. From the definition we will write the strain displacement relationship, constellation, one dimension and then use the equipment equations again to solve for the unknowns. The things to take away from this lecture is these 2 governing equations and the fact that you have that 6 equations right all the 6 equations we simplified the first 2 the other 4 we have to carry over for the next lecture.

Thank you.