Mechanics of Material
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Displacement due to uniaxial loading, temperature and bending
Lecture – 48
Traction in member subjected to bending

Welcome to the seventeenth lecture in mechanics of materials, in the last lecture we saw how to solve a Bernoulli problem involving an axial member. In particular we saw that the observer strain can be written as delta by L, where delta is the deformation of one of the section points where the other section point is L fix.

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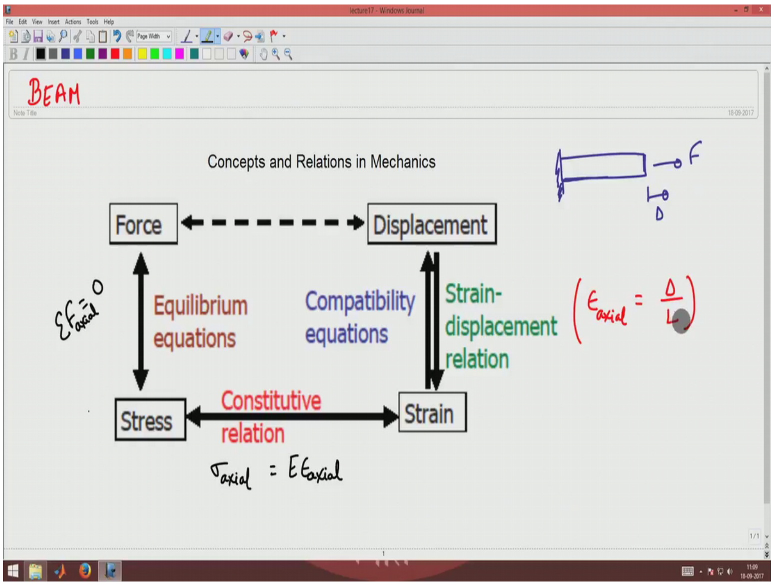

Here both the section points move, then it will be delta a minus delta b by L and we saw that the; we use a constitutive relation that sigma axial is equal to E time axial to solve the problem where in we use the equilibrium equation to find the unknown deltas. So, this how we proceeded to solve for axial members we saw axial members place in series with different area of cross section, with different young small s also you can use the same approach to solve it. We saw how to solve problems involving temperature changes, and then we saw how to solve axial members where in the property varies across the cross section. For example, we saw a steel casing enclosing a concrete cylinder how to solve Bernoulli problem involving that how to write the strain comparability condition and then how to proceed to solve the problems what we saw in the last lecture.

In today's lecture we will solve, another Bernoulli problems which is beams. It will take us two or three lectures to formulate, what is the displacement what is the how you go about solving a problems involving beams.

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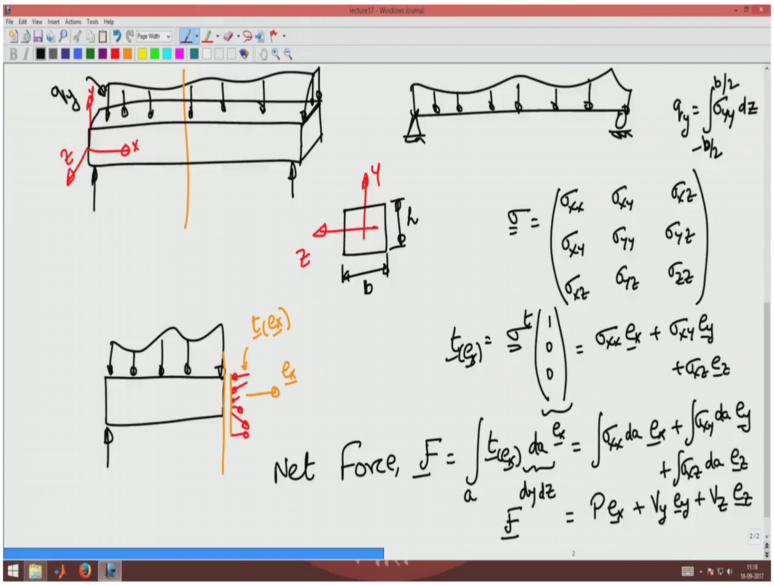

In particular we are interested in solving this problem, say I have some three dimensional body for example, I am taking a square cross section subjected to some loading arbitrary loading, and there are some support reactions also coming in which is which may be here I am assuming its simply supported ok.

It the idealization of this beam would be as a one dimensional member would be something like this. Where in this the arbitrary loading that the beam is subjected to with a simply support boundary condition like this. This load is distributed uniformly across the thickness of the cross section, across this dimension this load is uniformly distributed, where the load can vary across the length of the axial member along the axis of the member the load can vary, but it is uniform across the width of the cross section.

So, this loading is given as load per unit length this load denoted by q of y is given by given as load per unit length. In particular if you are looking at the stress components q y would be integral sigma y y into d z 0 to b, if I take my coordinate system as the following x y and z, and the rectangular beam is y z, now cross section is given by b, and this is given as h; this integral b minus b by 2 to b by 2 in this case, because I assume the origin to be as the central of the cross section. So, this will be minus b by 2 to b by 2 we will see all this in much more detail shortly. Now I want to do is when a structure is subjected to this arbitrary loading, I want to take a section at some point; say I want to take a section at this point and then expose the traction and write the global force equilibrium and moment equilibrium to find what are the result interaction this is acting at that cut surface ok.

So, basically what I want to do is cut there. So, let me take a plane section subjected to some arbitrary loading like that and here at this cut surface the traction will add, not necessarily parallel or perpendicular to the plane, but it can have an inclination. For example, here it might be like this and then it might have an angle like that with a magnitude much smaller and then it can change direction from there to something like this and finally, can have orientation like this ok.

So, the traction we will see changes like this across the deep y all the cross section. So, the traction will change across deep of the cross section both in magnitude and in direction in a general loading case. We want to find what is traction t of e x is, because normal to the surface of the assumed coordinate system is c x. So, now, let us assume a general stress state let us assume that the stress is given by sigma x x, sigma x y, sigma x z, sigma x y, sigma y y, sigma y z, sigma x z, sigma y z, and sigma z z ok.

That is a general state of stress; for this general state of stress what will be the traction t of e x is what I want to fine t of e x would be that stress acted upon by 1 0 0, that is the e x vector stress is semantic. So, I drop the transpose or else I will have a transpose there. So, now, this will result in me getting sigma x x e x, plus sigma x y e y plus sigma x z e z. So, what happens is in this cross section, the traction can vary from point to point it can earn three emission inclination like this ok.

It can have three emission inclination like that, where in there will be horizontal component which is sigma x x, the vertical component will be sigma x y and the component along this direction would be sigma x z. That is what we get this what we saw even when we were interested in drawing the stress cube and things like that. So, we got the traction to be this, now I am interested in what is a net force acting in that cross section. I am interested in net force F which is integral t of e x d a; where this a d is nothing, but d y d z, it is a area integral I have to integer this over d y and d z then I will get this as integral sigma x x d a e x plus sigma x y d a e y plus sigma x z d a e z. Now this we write it as the axial load P time e x plus the sheer force V y acting e y direction plus the sheer force V z acting along e z direction these the force vector, net force vector acting because of the expose traction.

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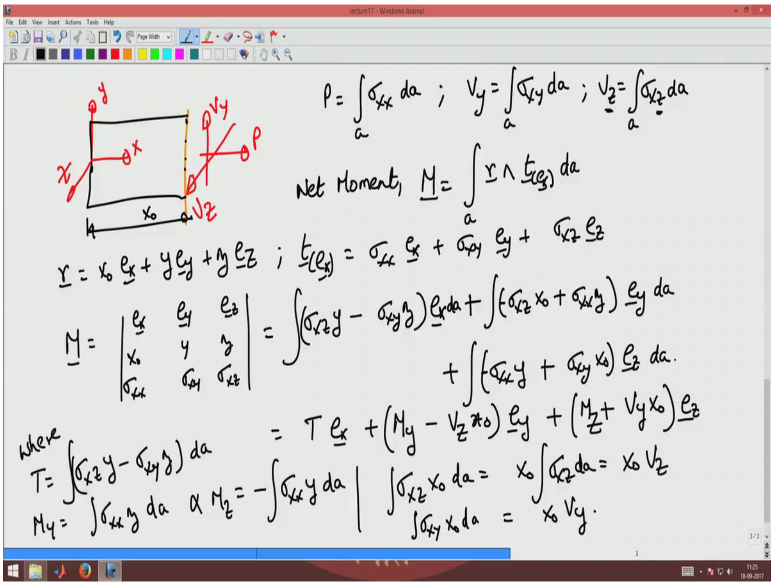

In other words again going back to the section add section the net force acting on the surfaces p V y and V z. These are the net forces where p is given by integral sigma x x d a and v y is given by integral sigma x y d a, and v z is given by integral sigma x z d a. You can note that the second index is what is the suffix for the shear force V x and V y and V z ok.

Now, the traction were to act uniform across that section, then you will would not have any moment in taking about the centre of the cross section. On the other hand traction is not having a uniform magnitude nor direction. So, what is going to happen is, there going to be three moment that are produced due this traction let us compute the moment due to this traction. Net moment M is given by integral r cross f, f is t of e x d a. This if you recollect it is a cross product. So, now, what is r? R is let us assume it as r would be for any point in this cross section what will be r; say this distance from the origin was x naught r would be x naught e x plus y, e y plus z, e z ok.

Recollecting that the origin was from here this x, this is y, and this is z I have r given by this expression near and t of e x is found that is given by t of e x is given by sigma x x, e x plus sigma x y, e y plus sigma x z, e z. So, now, r cross f moment would be you have to do the determinant of this e x, e y, e z x naught, y and z sigma x x, sigma x y, sigma x z its a shortcut, but that is. So, this will be sigma x z into y, minus sigma x y into z into e x, plus this integral over d a plus sigma x z into x naught minus plus sigma x x into z e y d a plus integral sigma x x into y, minus sigma x y into x naught e z d a ok.

Now, this I will rewrite it as the torsion moment T times e x there is a moment acting around the axis of the member, there is along e x direction plus M y moment minus V z into x naught I will explain shortly why this comes about, plus M x M z moment minus V y no I left a negative sign here this should be negative minus minus plus V y into X naught into e z, where T is integral sigma x z y minus sigma x y z, d a. M y is integral sigma x x into z d a and M z is I would not write with the negative sign. So, this will be minus integral sigma x x into y d a.

Now this x naught is the constant. So, since x naught is a constant integral sigma x z x naught d a would can be written as x naught into integral sigma x z d a which will be x naught into v z. From the definition that sigma x e d a is v z coming from this definition you write this as X naught into V z and similarly we write integral sigma x y into x naught d a as x naught into V y. So, we got that the moment is given by this expression let us proceed.

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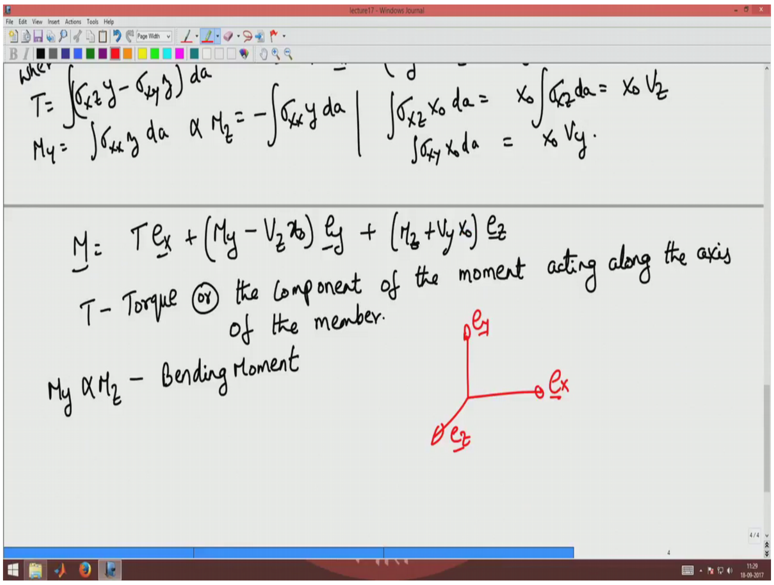

Moment M is given by T of e x plus M y minus V z x not into e y, plus M z moment plus V y into x naught e z, where this T is called as torque or the component of the moment acting along the axis of member. We will come to this after we finish the bending moment expression the other 2 M y and M z are called as bending moment ok.

That is acting not around the axis of the component of moment not along axis of the member is called as bending moments. In particular M z moment will produce a bending like this is M z right for a coordinate system we have the coordinate system as the following, we had e x, e y and e z oriented like this. So, the moment M z which is this moment the moment which is this moment will produce a deflection along the y direction similarly a M z mo M y moment is a moment acting like this a moment trying to bend a member like this ok.

That moment is M y moment and it will produce a diffraction along the z direction. When you bend it like this is going to curve like this, and that is the displacement along the z direction. Whereas the torque is not going to produce a displacement normal to any of the access, but what is going to do is, it is going to produce a angle change that torque is along the axis of the member. So, what happens is it produces shear stress or a shear stress causes a torque which produces a angle change in the member. So, that is why we are not interested in the torque right now, we are interested in the bending moments ok.

You can see the bending moments are dependent upon sigma x x with the corresponding your arm being either y or z. So, sigma x x the normal x sigma x x is the cause for both bending moment along y and bending moment along z directions ok.

So, now, then what is this term V z into x naught and V y into x naught doing here. Basically if I have to this is finding the moment only due to the cut surface here. I found the moment acting on this cut surface, wherein I have this forces plus now three moment acting on this cut surface, I will indicate moments by double arrows this is a torsion moment, this M z moment and this is the M y moment. Basically the traction of my term points to the direction of the moment. If a moment is bending like this my term points the z direction.

So, that is a M z moment is a moment is bending the beam my thumb is pointing the M y direction. So, that is M y moment. So, basically we understood that the cut surface produces some force and some moment in the body, because it is not uniformly distributed.