Mechanics of Material
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Displacement due to uniaxial loading, temperature and bending
Lecture – 47
Stepped shaft subjected to raise in temperature

Now, having seen this; next let us move on to thermal stresses.

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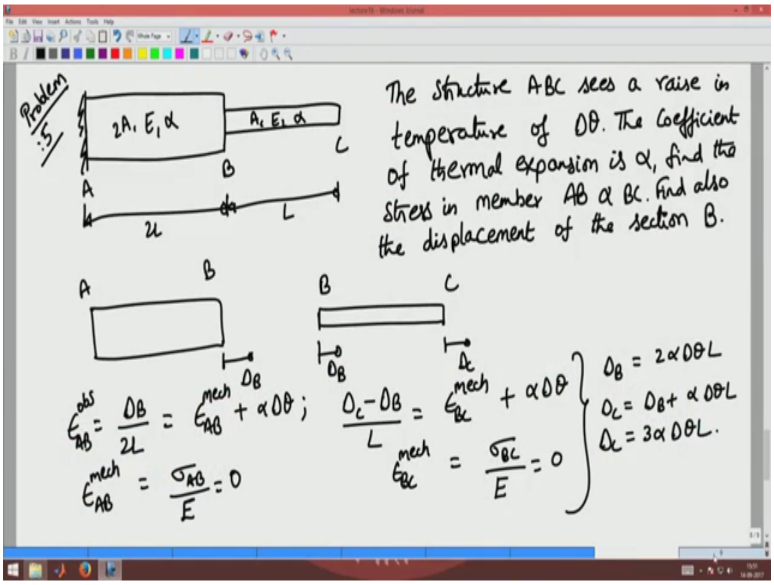

So, I have the same stepped shaft A, B, C, 2 A, E, A of cross section Young’s modulus E. This length is 2L and this length is L; this problem 5. These are subjecting to raise in temperature of delta theta. Structure ABC sees a raise in temperature of delta theta and the coefficient thermal expansion is alpha for both. The coefficient of thermal expansion is alpha, find the stress in member AB and BC.

Now, how will you solve this problem? There is no applied force here. Just the temperature of the structure increases by delta theta. Apart from the stress, I want to find also the displacement of the section B. I want to see by how much this section B moves and I want to find what will be the stresses in number AB and BC. Now, what can you say? When temperature raises, A is fixed? So, it cannot move. B will move by delta B. This AB, BC, B moves by delta B and C moves by delta C.

There is no force applied. So, what will be the observe strain? Observe strain would be delta B by 2L (Refer Time 03:52) of the member. This will be equal to epsilon mechanical strain plus alpha delta theta, which is the thermal strain along the direction, axial direction for AB is mechanical strain in member AB. Similarly, for BC the strain would be delta C minus delta B by L. That is coming from a strain displacement relationship difference B moves by delta B. C moves by delta C. So, the strain would be difference between displacement of C and B.

So, there is delta C minus delta B by the length of the member which is L. This will be equal to the mechanical strain in BC plus alpha delta theta which is again this strain in member BC to the thermal effects. Now, since there is no force applied in AB, what will be the mechanical strain AB? Mechanical strain AB from the constitutive relation would be the stress in AB by the Young's modulus E. Since, there is no force applied in member AB; this stress is going to be 0.

Similarly, the mechanical strain in member BC would be the stress in BC by the Young's modulus. I am applying no force in this member BC. So, the stress in member BC would be 0 and this will also be 0. From this I get delta B to be equal to 2 alpha delta theta into L and delta C would be delta B plus alpha delta theta to L which will be 3 alpha delta theta into L. That is delta C. That is the change in cross section does not matter. The total length of the member is 3L. The elongation at point C would be 3L times alpha delta theta. Now, let us solve the final problem of this lecture which will be problem number 6.

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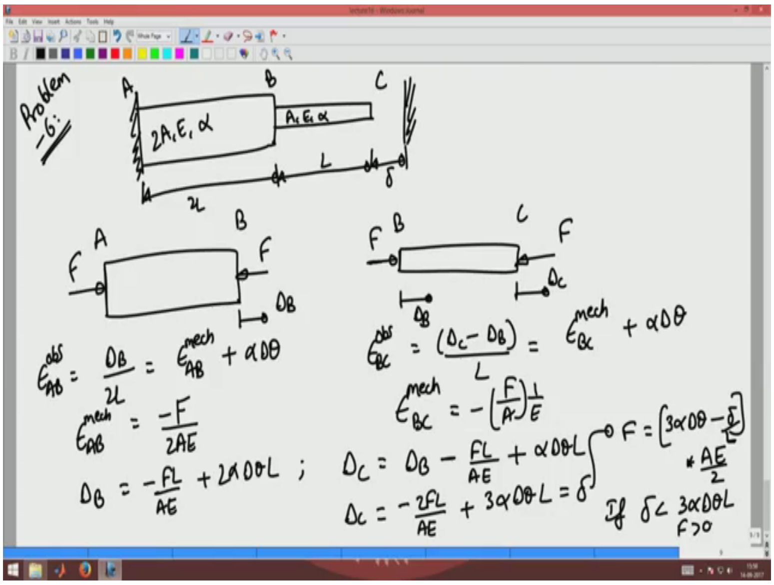

This is same as problem 5 except that I have restrained at C. Just like in problem 3, I have this distance given by delta; this is A, B, C; this as 2A as state of cross section E as Young's modulus; alpha is a question of thermal expansion A, E alpha for BC. This has a length 2L and this has a length of L. BC as length of L. Now, I heat it. Now, what happen? Until delta, the temperature change is such that the total elongation of the member is delta. There would not be any force coming in; this will be same as problem 5.

Now, after the gap closes there will be mechanical stress arising. Because there will be a compressive force coming at C which will propagate and it will change the entire dynamics of the problem. So, basically how do you solve this problem? You assume again as we did in problem 3. That the gap is closed to the change in temperature and then what you do is you draw a free body diagram AB, BC. There is a compressive force F applied here. There arises another compressive force F.

There will be by continuity at B, there should be a compressive force F. Here and F here too; so, since I assumed F4 here and I do not know what this F is. I have to find the F such that the displacement of point C is delta. They find F such that the displacement of point C is delta not more than delta. Now, again I write the same equations. I assume delta B this moves by delta B. These moves by delta B and these moves by delta C. Same things I write as before. This observed strain in member AB is delta B by 2L. That will be equal to the mechanical strain in AB plus alpha delta theta.

Similarly, the observed strain in member BC would be delta C minus delta B by L which will be equal to the mechanical strain in BC plus alpha delta theta. Now, the mechanical strain in member AB would be minus F into minus F by 2 AE. There will be a mechanical strain in member AB. F by A is if stress in member AB, the stress by strain F by 2A will be the strain in member AB. F by 2A will be the stress in member AB and that stress divided by the Young's modulus will give me the strain in member AB. Similarly the mechanical strain in BC is given by F by A into 1 by E. The stress in member BC divided by the Young's modulus of member BC.

Now, substituting this back in there, I get delta B to be minus F by AE into L plus 2L alpha delta theta and I get delta C to be delta B plus FL by A E minus FL by AE plus alpha delta theta into L. Substituting for delta B. This will be minus 2FL by AE plus 3 alpha delta theta into L would be delta C. We know from the comparability condition that the displacement of point C cannot exceed delta. So, this will be equated to delta and from this equation we obtain F as 3 alpha delta theta into L minus delta AE by L. From this you get F as 3 alpha delta theta minus delta into AE by 2 delta by L.

We will get F as that and from here is clear that if delta is less than 3 alpha delta theta into L, then F would be positive and that is a case that we want. If F is become negative then what happens is, it is not possible. F cannot be a tensile stress in member BC. When the temperature increases the members can be only in compression and it cannot be in tension.

So, you from the physical argument you see that delta has to be less than 3 alpha delta theta into L and that is a condition when F will be greater than 0. If F have to be less than 0, if delta where to be less than 3 alpha delta theta into L. Then the force F would be less than 0. Not possible physically. Possible physically hence limit F to be equal to 0.

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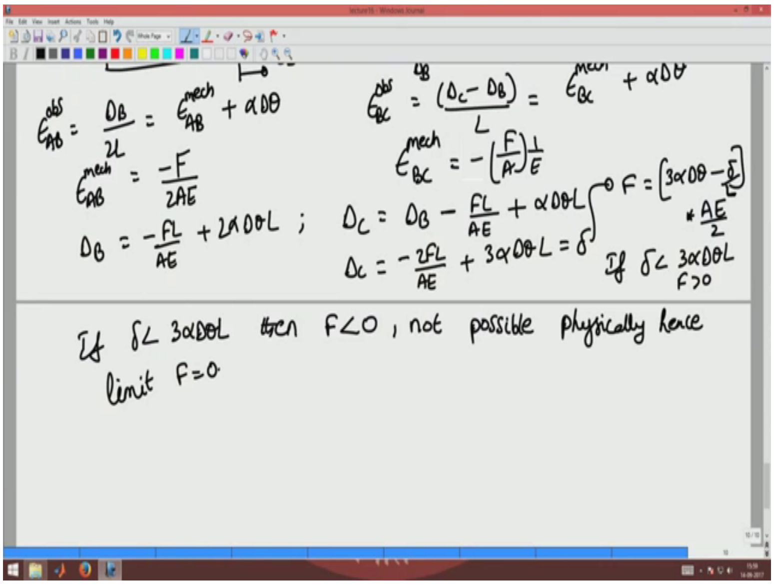

So, that is how you solve this problem. Just like what we did for problem 3; same arguments here to solve for problem 6. So, what we have seen in today's class is some 6 different (Refer Time 13:53) problems involving actual members. The first 3 where all the same type, where in you have only 4 separate on a homogenous bar (Refer Time 14:03) which are different cross sectional areas. In problem 4 we saw a bar, which is inhomogeneous made of two different materials and in problem 5 and 6; we looked at effect of thermal strains or what is the effect of change in temperature on these structures?

Thank you.