Mechanics of Material
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Displacement due to uniaxial loading, temperature and bending
Lecture – 46
Inhomogeneous bar subjected to axial force

Let us, look at a different kind of problem you have seen stepped shaft with some condition on the comparability condition, this condition that delta c is delta small delta is a comparability condition ok.

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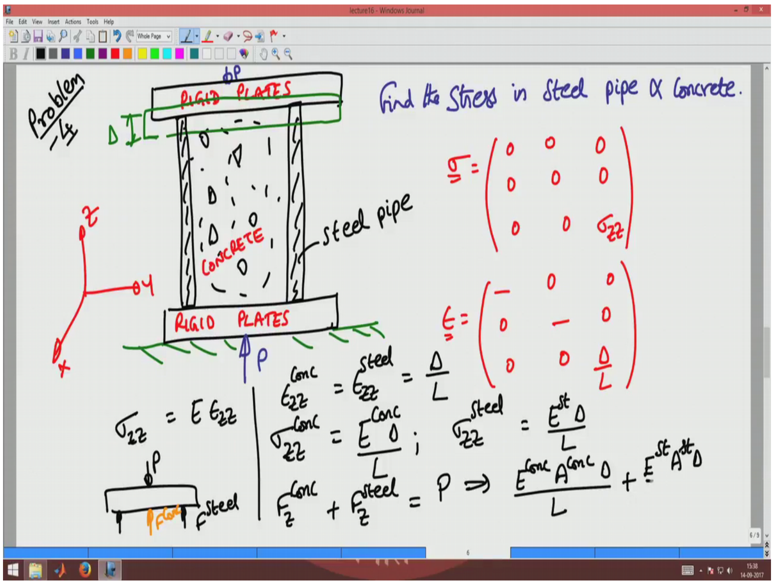

Next let us, look at a composite cylinder; say I have a steel pipe filled with concrete. I have the steel pipe filled with concrete, this is concrete, this is a rigid plate. I have two rigid plates on top and bottom, and I am subjecting this to a compressive load P. So, I have steel pipe filled with concrete and I am submitting it to compressive force P. Now I want to find the stress in steel pipe and concrete ok.

Now, do you go for solving this problem, a systematic way is to assume some distress and field to compute based on distress and field stain, from the strain you compute the stresses in the constellation then use the equilibrium equations to related to the applied force P. So, what is the possible deformation for this composite system, the plate can move down like this is it plate can moved on like this by an amount delta say, and I am assuming that this is fixed on the ground. So, this does not move I am assuming that top plane alone moves down by an amount delta.

So, what will be the and let us assume that the corner system is given by this the corner system is given by this orientation, x and y in the plane and z along the axis of the cylinder ok. Then what will be the stress state, what will be the sigma and what will be this epsilon for this definition that is a question. Since, I am applying a force along the z direction, there will be stress sigma z z alone and let us assume for the moment that the stresses in other directions and other share normal stresses are 0 ok.

This would not be true in the poissons ratio of steel and concrete are different. In fact, persons ratio of steel and concrete are different and hence, the stresses in the other direction will would not be the same will would not be 0, but we assuming that they are 0. Now what will be the strain corresponding to this it has displace the top plate has displace by amount delta.

So, stay in roughly is the change in length by the original length are displacement delta divided by L, as we saw in the case of uniaxial member, here it is compressing along the z directions and hence, the starriness along the z direction and then there will be poissons strain in the other two directions which for now we will ignore.

We will ignore this poissons strain there has no shear strains. So, we will ignore those strains. Now what happens is the stress and the strains are related through the constitutive relation, now sigma z z is E times epsilon Z Z, right this is broadly the member to we made of the same material, but I have now the body made of two materials. So, what will happen allow epsilon z z of concrete will be equal to epsilon z z in steel, which will be equal to delta by L, because our the plates rigid there is no rotation there for a steel and concrete two are different strengths, the rigid plate moves as a rigid plate and the hence the strain in the steel and concrete would be the same ok.

Now, since the strain I have competed the strain, so next I compute the stress in the concrete would be E of concrete times delta by L, just change the concrete and sigma z z of steel would be the Young's modulus of steel in to delta by L.

Now what should I do next after the equilibrium equations, equilibrium equations is the force in the concrete plus force in the steel the actual force F z in the steel and F z in the concrete started to give me the force P, because my rigid plate as this force P coming in there and there is this force F steel and there is the force F concrete ok.

So, this force should adapt to applied force P, now what is force and concrete force and concrete would be the force and concrete would be the stress in the concrete times the area of the concrete. So, basically from the I get that this implies E concrete into area of concrete into delta by L, this is the force concrete plus the force in the steel which will be E of steel to area of steel into delta by L, must be equal to the applied force P. Now from here, I get what is delta from here I get delta to be P L by area of concrete.

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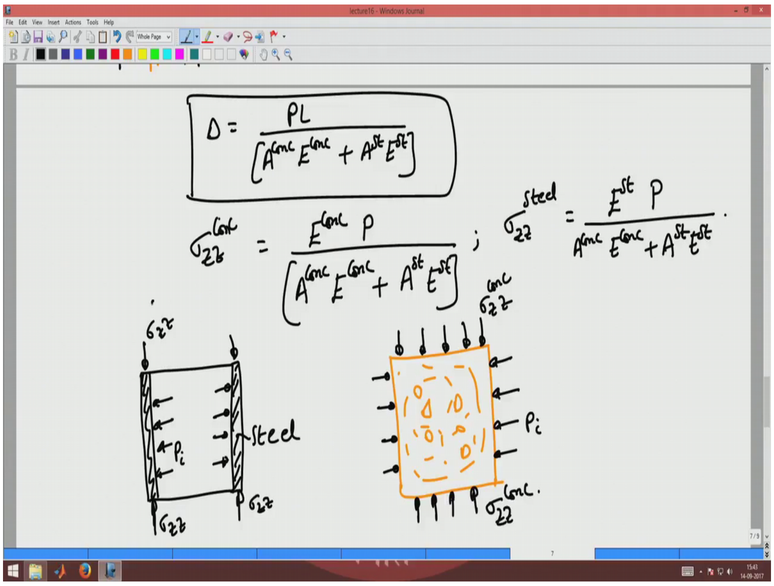

The Young's modulus of concrete plus area of steel, and the Young's modulus of steel so; I have found for delta is and related to the applied force P ok.

So, what are we interested in mechanics so relating the force to the displacement. So, given a force P and now what will be the displacement delta L know, but what I am interested in this problem, I am interested in finding the stress in steel pipe and stress in the concrete pipe.

So, coming back to the expression for the stress in the concrete, so sigma z z of concrete would be E of concrete into delta by L. So, there will be P divided by area of concrete to Young's modulus of concrete, plus area of steel into Young's modulus of steel. Similarly sigma z z of steel would be Young's modulus of steel into P divided by area of concrete Young's modulus of concrete plus area of steel the Young's modulus of steel ok.

So, this will be the set of stress in the concrete and the steel pipe, on the other hand if I want to consider difference in the poissons ratio what will happen there is a question next that we want to consider, if I want to consider the poissons effect what will happen, the strain delta by L, here will result in a strain of minus mu delta by L and here it will be minus mu delta by L right.

Now, since the possion ratio is different, but delta by L is same what will happen is the concrete will expand differently from how much this steel expands. So, there will be an interfacial stress that develops within the concrete and the steel.

So, in that case the free body diagram for this member will become, there is a steel pipe they will be some inner pressure P, I acting here the steel added to that there will be sigma z z acting along the circumference of the steel pipe, there will be a sigma z z circumferential stress acting along the circum action stress along the circumference of the steel pipe to sigma z z and in the concrete, the concrete pipe there will be a counter part of this P i, newton's third law coming to effect this concrete cylinder will be subjected to a external pressure P i ok.

And it will also be subjected to an axial compression along this circular cross section to sigma z z of concrete sigma z z of concrete; this is sigma z z of steel. Now the state of this is no longer uniaxial it is biaxial to say the least if not triaxial. So, basically you have to know then how to solve the problem of un (Refer Time: 12:47) cylinder being inflated or a solid cylinder being depleted to be able to solve this problem.

You come to this full solution of a three dimensional inhomogeneous axial member later in this course, already I want to say is this solution that sigma z z of concrete is of concrete times uploaded divided by E of concrete in the area of concrete plus E of steel in to the area of steel and.

So, on applies only when you make the assumption that the steel and concrete are the same poissons ratio for ignore the second dimension or third dimension effects of the body you are interested only primarily in the axial deformation of the body then only this expressions will hold.