Mechanics of Material
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Displacement due to uniaxial loading, temperature and bending
Lecture – 45
Stepped shaft subjected to axial force

Welcome to the 16 lecture in Mechanics of Materials. From this lecture on, we are going to look at how to solve different bond value problem that is of engineering interest. Till now we have seen about the four concepts in mechanics namely force, displacement, stress and strain; the four equations that connect this concepts.

Basically the equilibrium equations, the strain displacement relationship, the comparability conditions, constitutive relations, we also saw what energy is, and how to compute the strain energy per unit volume, your load potential and the total potential in the last lecture.

From today's lecture on, we will try to apply this concept to solve some bond (Refer Time: 01:00) that are of engineering interest. The first point (Refer Time: 01:03) we are going to look up is these are often axial members.

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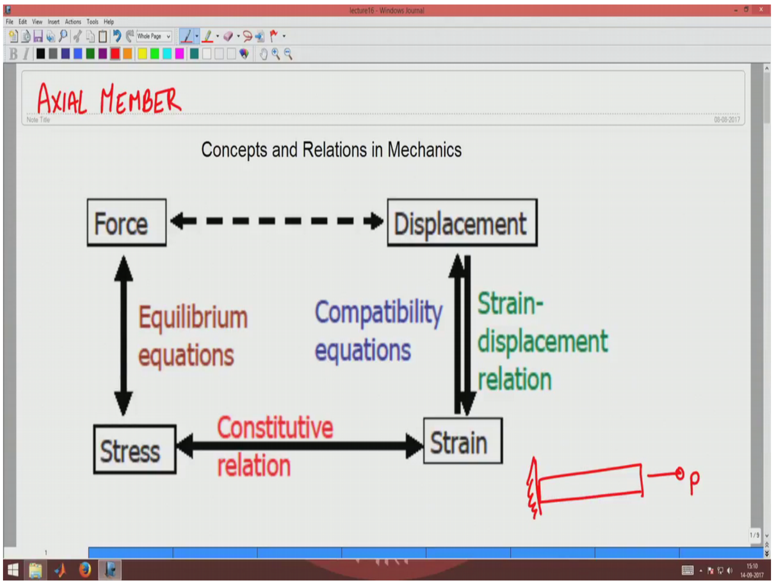

The axial member is one wherein I have this member subjected to some axial force and one end is fixed. These are axial member that we are interested in analyzing first. Let us go ahead.

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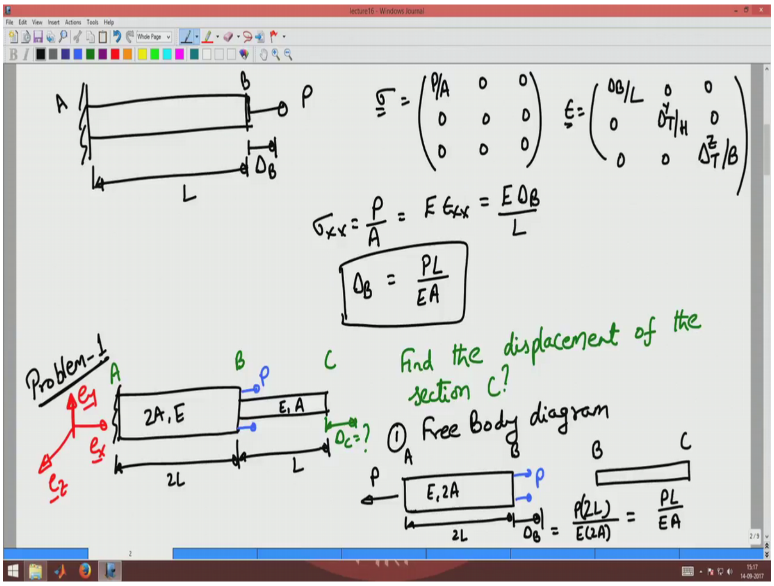

So, I have this member which I am applying a force P, all of us know that ones I apply a force P it is like a uniaxial state of stress; uniaxial loading. So, we saw when you looked at uniaxial state of stress we saw that stress is given by this tensor P by A 0 0 0 0 0 0 0 0. And in the uniaxial loading case, we understood that the strain would be that is A and B.

And if this displacement amount delta B, this delta B by L, where L is length of this member 0 0 0 and transfers displacement in y direction by the height of the member 0 0 0 transfers displacement along z divided by breath of this member. We saw this when you looked at the uniaxial loading case.

To solve Bernoulli problems what we are interested is in this relationship of the sigma x x stress to this strain epsilon x x, which was this which was E times delta B by L. So, you want to relate delta B now using this relation delta B is P L by E A. This is a take away equation that will be using to solve different Bernoulli problems involving axial members. What we have used this, we have used the equilibrium equations to understand that the stress is beyond by P by A, and then we have used the strain displacement relationship to write the strain as delta B by L. And here we are use the constitutive relation to write this stresses E time epsilon x x.

In other words, what we have done is we have used the equilibrium equations relate the applied force to the stress, we have used the strain displacement relationship to relate the displacement to the strain. Here used a constitutive relationship to relate the stress to the strain, so that is what you have done in this problem. And we have got the relationship the delta B is P L by A E or E A.

Now, let us apply this to some problems that is of interest. The first problem that I am going to solve is the following. I have a stepped cylinder, this is of length 2 L, this of length L. It has 2 A as area, and E is young's modulus. This member has E as young's modulus, and A as area. Let me assume as usual this to be E x, E y, and this to be E z. And let us assume that I am applying a force P along this rim along that rim I am applying a force P.

Let us also assume that this is A, B and C. And I am interested in finding what this displacement delta C is, find the displacement of the section C for this loading.

Now, how will you go about doing this, first I have to draw the free body diagram of A B. First step is to draw the free body diagram of the respective axial members, the two axial members are A B and B C. Since there no force acting at C, there would not be any force acting at B either; whereas here there a force P acting, there is a fist P acting there.

So, what will happen is if I were to remove this support, if I were to remove this support that support is going to prevent from A from displacing, it will offer reaction force P there. It is similar to the case where in we saw just now AB subjected to axial load P. So, this edge would displace by delta B which is given by P 2 times L by E 2 times the area because this as the young's modulus C 2 A as area of cross section and length is 2 L. In other words, delta B is P L by E A.

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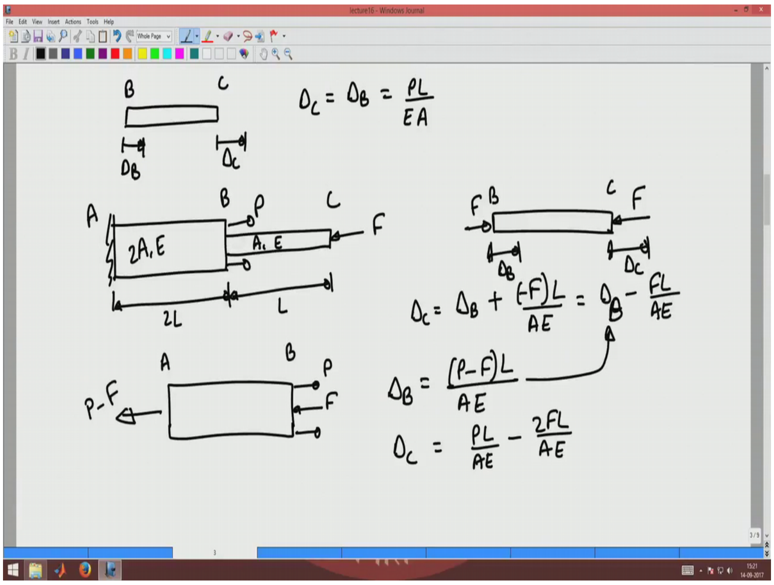

Since there is no force at B, if I were to analyze the structure if I were to analyze a structured BC, B C there is no force here this point moves by delta B and so this will move by delta C. Since, this edges moves, this surface moves, this surface also will be moving by some amount delta C. This case in there is no force applied delta C is going to be equal to delta B, which is P L by E A. On the other hand, for applied a force F at C n say, so in so solving that problem I was looking at this problem say as a force F applied here.

And there is a force P applied over there then delta C A B C again this is 2 A, E, A, E, this is 2 L, and this is L. Then the free body diagram of B C would be something like this F. So, this delta B, and if this displacement is delta C, delta C would be delta B plus in this case it is a compressive force so minus F into the definition cause by this 4 C F which will be again given by P L by A E. This members of length basis of length L, it has a cross sectional area of A, and Young's modulus E. So, from our previous discussion the delta B is P L by E A for applied tensile load.

Here I am applying a compressive load, so that is why it is negative minus F into L by A E. So, now, delta C would be minus F L by A E. Now, delta B is not same as what we had before because the force acting on member AB also will differ now. So, the force acting on member AB would be the following. There is a force P acting here, and there will be a force F acting here. So, net force from equilibrium is P minus F, because they are in opposite direction the net force acting on member A B would be P minus F.

So, delta B would be this force, this P minus F into L by A E from our expression for the displacement at a point P to a member for which applied actual force is a P which is in tension. So, this will be the displacement of B. So, now, delta C becomes P L by A E minus 2 F L by A E substituting for delta B in there, so that is the displacement of C, the point C. Now, let us assume that the C is not free to displace, but I have boundary at a distance delta from C. Now, let us assume that C is not free to displace, but its limited to displace only by a certain amount delta that is this problem three that I am looking at.

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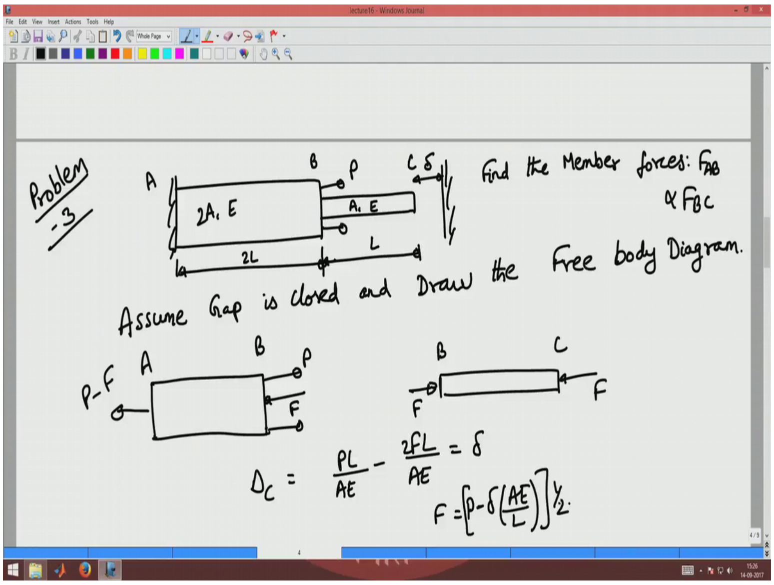

This as area 2 A, A, A E, and let say this is distance delta and this length is as before 2 L and this distance is L and this A, B and C. Now, I am applying a force P at the rim of A B at the intersection remembers A B and B C, in this member applying a force P. Now, I want to find, I want to find the member forces forcers F AB and F BC. Now, there are various ways by which we can solve this problem; clearly, there are two cases the first cases before the gap process and after the gap process. The simplest way of solving this problem is the following procedure.

Again you draw a free body diagram, assuming the gap is closed. First you assume gap is closed and draw the free body diagram. So, for member AB, BC, gap is closed, what does that mean C there will be some reaction force coming here which is F say, there will be that reaction force F, there will be F here, P there, and there will be a force P minus F similar to what we had in the problem 2.

But in problem 2; the F was a prescribed value, but now f is not a prescribed value; problem 2, F was given to us; now we do not know what f is and we have to find what that f there will satisfy this recommend that C cannot displace by more than a amount delta.

So, now, delta C would be this similar to what we have in the problem 2. So, delta C now would be P L by A E minus 2 F L by A E, and this displacement cannot be more than delta, so that is the displacement that you can have delta C can be delta. So, you use this equation, now to find what F is F is delta A E by L P minus delta A E by L into one half. So, F is one half P into P minus delta A E by L. Now, let us understand what this means?

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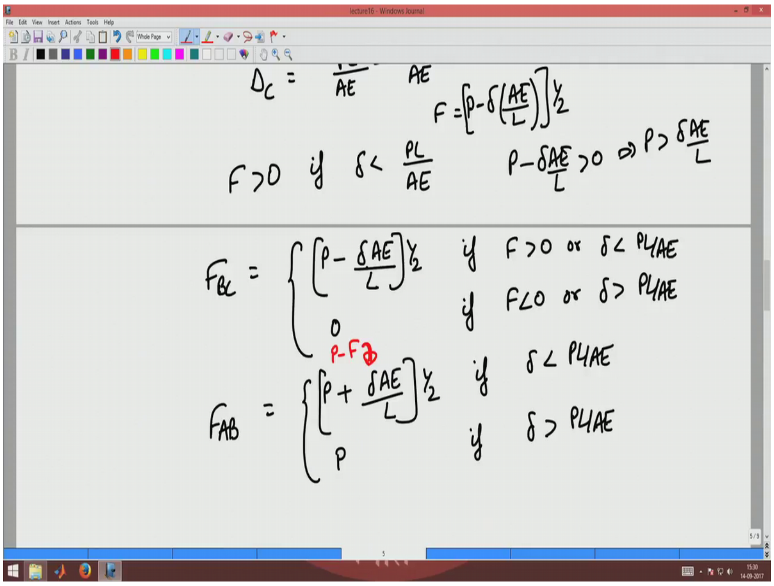

F would be greater than 0 if delta is less than P L by A E; F will be greater than 0, if delta is less than P L by A E. How do I get that I get from the fact that P minus delta A E by L has to be get than 0 which means P must be greater than delta A E by L. This implies delta as to be lesser than P L by A E, P L by A E; else what happens F becomes negative. It is a negative force possibly here at C, which means is a tensile can B C B in tension B C cannot be in tension, because there is no force applied in C, and if the gap is there it can be only in compression not in tension.

So, if you solve this problem and get your force B tensile force rather than being a compressive force; in other words, if F happens be negative where we assume the positive (Refer Time: 18:36) F to be the compressive force then what happens is you have to set that to 0. So, the solution is F BC is equal to two cases P minus delta A E by L into half if F is greater than 0 or delta is less than P L by A E; else it is 0, if f less than 0 or delta is greater than P L by A E.

Now, in problem 1, we saw that the expansion that C will have to do the separate load P is P L by A E right. In problem 1, we found that delta B is P L by A E and that is equal to delta C, delta C and delta B were P L by A E. So, the gap is more than P L by A E, and the C will have free expansion and there would not be when the delta is greater than P L by A E, C will expand freely as there would not be any force reaction force that comes about at C, that is what we are saying here.

That is what we are saying when I say F BC is 0, that is what we are saying when we say F BC is 0. Similarly, F AB would be P minus F, F, so it will be P plus delta A E by L into one half if delta less than P L by A E. And it will be P, if delta is greater than P L by A E, because F is 0 this is essentially P minus F this is what this is. So, P minus F will give me that that is comes from the observation that at AB P minus F is a axial force that this member A, B, C, this P minus F.