Mechanics of Material
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Constitutive relation, strain energy and potential
Lecture – 44
Strain energy, load potential and total potential

Next, what I have going to look up is some concepts in mechanical energy.

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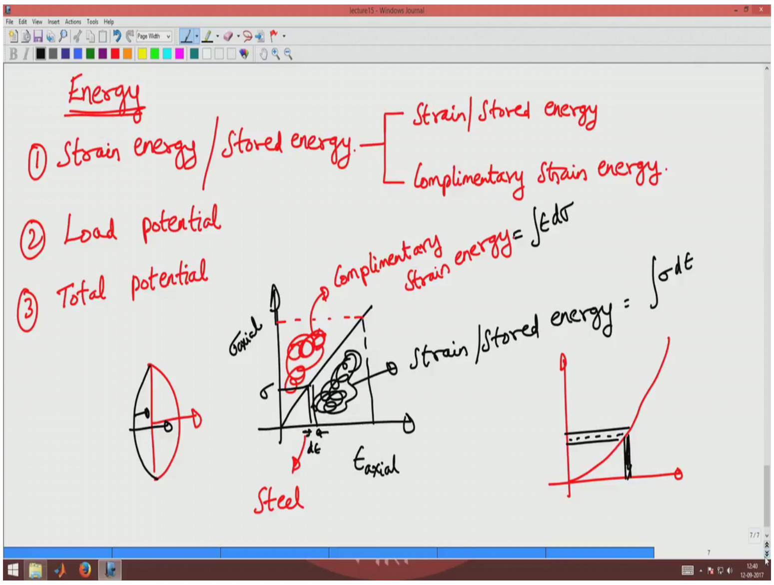

Just like force is the mathematical construct to explain displacement of bodies, energy is also a mathematical construct for an alternate formulation to explain displacement of bodies. So, basically here we are interested in looking at three concepts; one what is called as strain energy or stored energy.

The second concept is load potential and the third concept is total potential. There are two kinds of stored energies; one which is just called as plain, strain or stored energy. And the other thing is complementary strain energy; let us understand first what is strain energy and stored energy is; say I have a bow and arrow.

Now, what happens when I pull this back? This strain gets deform like this and this thing comes in here like this. Now, if I let go after deformation the spring comes back here and the arrow flies off right. So, basically what is the energy that is stored in this string because of you pulling back? It is the release, when I let the arrow go and the strain energy is converted into a motion of the arrow.

So, that is what strain energy is; it is in a sense the capacity of the body to do useful work. Now how do you compute strain energy? Let us look at the uniaxial case, to compute a strain energy for uniaxial case; I have sigma versus epsilon axial stress versus axial string. Say I had a curve; I have a straight line like this, what is typically of metallosis linearly elastic.

Now I have two options, I can compute the area under this curve; that is called as strain energy or stored energy or and compute the complementary of it, which is this area; this is called as complementary strain energy. Now for a straight line, there is no difference between complementary strain energy and strain energy because the state line; this half times delta the area remains the same.

So, on the other hand; if I add a material like rubber this is for material like steel and if I add a metal like rubber, it will show some sustained response like this. And then as a difference between whether I compute this part of the energy or whether I compute this part of the energy. Now, how do you compute this area that area? To compute a strain energy, I have to do integral sigma d epsilon because I am varying epsilon here; this is d epsilon.

And that is the value of stress at that particular point, I sum that up to get the strain energy. Similarly to find the complementary strain energy, I have to integrate epsilon with respect to the stress; they integrated epsilon with respect to the stress.

Now, let us do this for a uniaxial case; uniaxial case of a linear elastic material, we know that for linear elastic material sigma is given by lambda stress epsilon, identity plus 2 mu epsilon.

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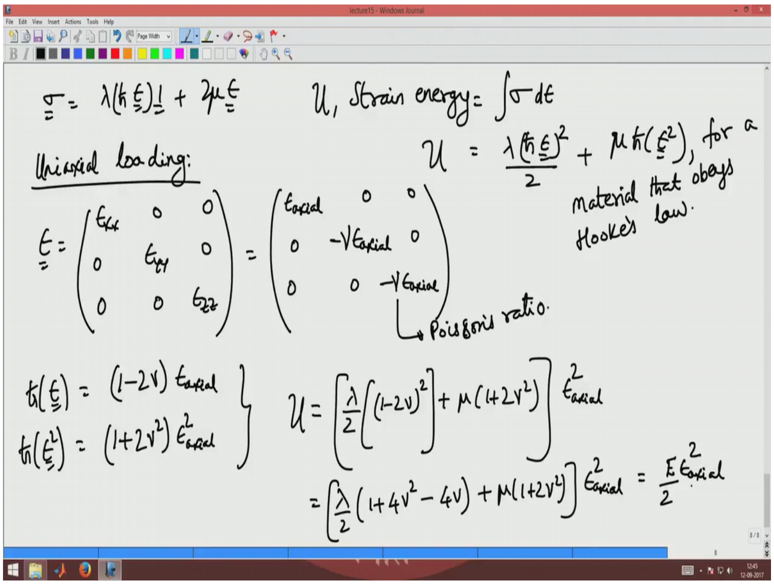

So, when I want to find strain energy; I have to integrate sigma with respect to d epsilon, which means I have to find sigma as a function of epsilon which internal boil down to 2 plus mu stress of epsilon square. This is the general expression for the strain energy, in a linear elastic material; for a material that obeys Hooke's law. Now let us find out what the strain energy would be for a uniaxial state of stress?

Let us assume for the uniaxial experiment; loading you add strain as epsilon XX; 0, 0, 0, epsilon YY; 0, 0, 0 epsilon ZZ; from Hooke's law, from what we did define Young's modulus as, this will be epsilon axial 0, 0, 0 minus nu times epsilon axial 0, 0, 0 is mu times epsilon axial from our definition of the Poisson's ratio; this is Poisson's ratio. Now what will be U then? What is stress of epsilon?

Stress of epsilon is 1 minus 2 mu epsilon axial and stress of epsilon squared would be 1 plus 2 mu square epsilon axial square. So, substituting this in the expression above; we get the strain energy as lambda by 2; 1 minus 2 mu squared plus mu times 1 plus 2 mu square into epsilon axial squared.

So, this will be nothing but lambda by 2 into 1 plus 4 nu squared minus 4 nu plus mu times 1 plus 2 nu square; epsilon axial square. Now, this I can rewrite it as E by 2 epsilon axial square where I substituted for lambda and mu; the expression from involving Young's modulus and Poisson's ratio and I will simplify this equation to get this as Young's modulus times epsilon axial square by 2.

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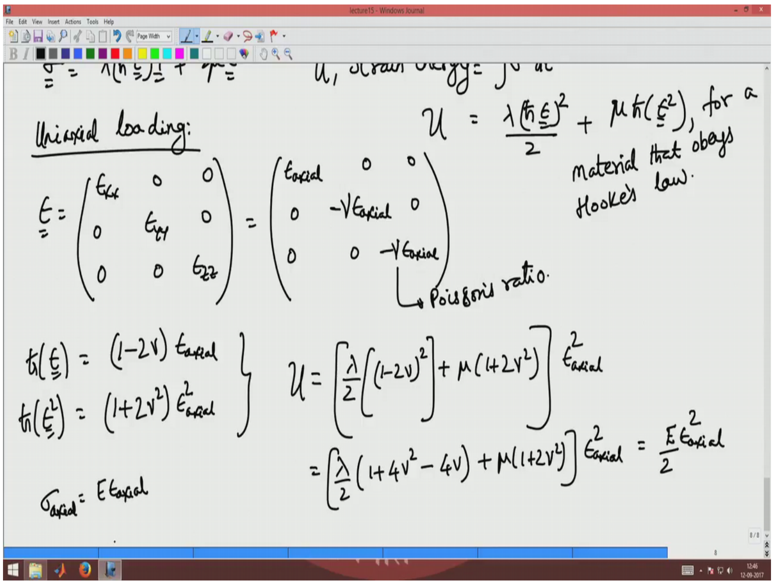

In other words, what you have is you have to integrate you know that sigma axial is E times epsilon axial, the other components of stresses are 0.

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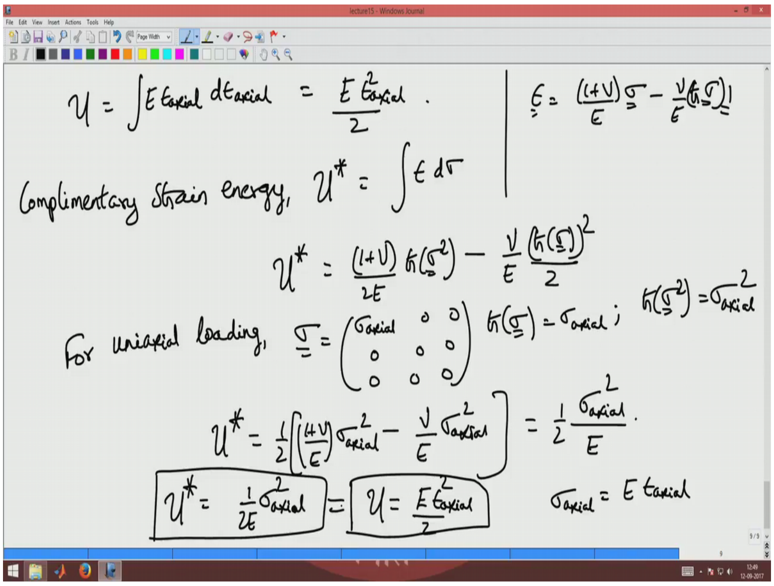

So, what will happen is this U is integral E times epsilon axial, the other components of stresses are 0.

So, it will be d epsilon axial; so, this is nothing, but E epsilon axial squared by 2. So, the simpler way of doing that is that the correct or the straightforward way of doing this;

what we did before; what we did here. Now let us find the complementary strain energy, which denoted by U star this is nothing, but epsilon d sigma.

For s epsilon was 1 plus nu by E; sigma minus nu by E, stress of sigma identity. So, this will boil down to 1 plus mu by 2; E stress of sigma squared, minus mu by E stress of sigma squared by 2; that is a complementary strain energy. Now for the uniaxial state of stress; for uniaxial loading, you know that sigma is sigma axial 0, 0, 0, 0, 0, 0, 0, 0. So, stress of sigma would be sigma axial and stress of sigma squared would be sigma axial squared.

So, U star would be 1 by 2 into 1 plus nu by E; sigma axial squared minus mu by E, sigma axial squared which will be 1 by 2 sigma axial squared by E. So, U star is 1 by 2 E sigma axial square; so, basically now you have found that what the complement strain energy; the strain energy are, now how do you know that these are the same? Because you have the relationship between sigma axial and epsilon axial as sigma axial is E times epsilon axial, so if I substitute in there; we find that these two are same.

That happens because the material is linearly elastic, for a non-linear elastic material in general, the complementary strain energy and the strain energy would be different.

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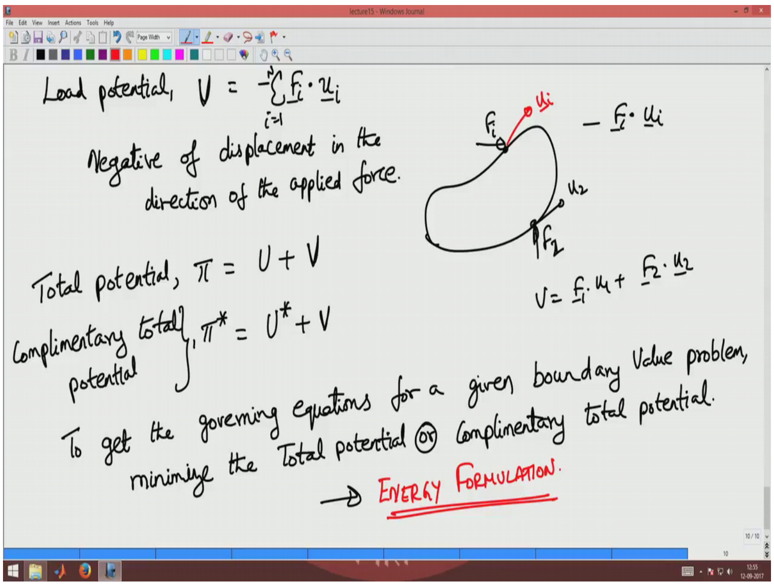

Let us look at load potential, this is denoted by V; this is defined as negative of summation of applied force into the displacement of the dot product with the applied force; dot product of the displacement at the point of the applied force.

So, for example, if I have body; if I have force applied F at this point and this point displaces by U of phi. Then it is the work done by this force for it to displace by that distance U of i. So, the load potential is negative of F i dotted with U of i; that is a scalar that is you are projecting F 1 to U i that is the displacement in the direction of the applied force.

Load potential is negative of displacement; in the direction of the applied force, that is a load potential. Finally, the total potential is you have to sum this from i equal to 1 to N; for all the forces that you have applying. If you have F there, another F here and this point displaces by U 2; V would be F 1 dotted with U 1 plus F 2 dotted with U 2. Finally, the total potential is defined as pi is defined as U plus V. If I did not have negative sign here, I would have introduced a negative sign here.

So, basically that is the tradeoff; I introduced negative sign there because I did not want to introduce a negative sign the total potential. Similarly there is a complimentary total potential position to the pi star which nothing, but compliment disorder energy plus the load potential V.

So, basically we have seen what a strain energy is? What complementary strain energy is? What dot potential is? And what total potential is? And what complimentary total potential is? Now to solve problems just like we had four concepts, what we are do is; you have to minimize its potential in terms of some unknowns that heals the solution to the boundary problem and energy formulation.

We will see more offered in the coming lectures, but just like there are some concept that we saw in the force base formulation; the energy based formulation the basic concepts are the strain energy or complementary strain energy, the load potential, the total potential or the complimented ordered potential. To solve a problem; we will minimize the potential; to get the governing equations for a given boundary value problem, we have to minimize the total potential or complimentary total potential. This is in energy formulation, this is essentially energy formulation; with this we will conclude today's lecture.

Thank you.