Mechanics of Material
Dr. U. Saravanan
Department of Civil Engineering
Indian Institute of Technology, Madras

Constitutive relation, strain energy and potential
Lecture – 42
Restriction on material parameters

Welcome to the 15 lecture in mechanics of materials, in the last lecture we saw six different metal parameters Young's modulus, Poisson's ratio, bulk modulus, share modulus and 2 lame constants. The relationship that we obtained between this various constants is tableted here again.

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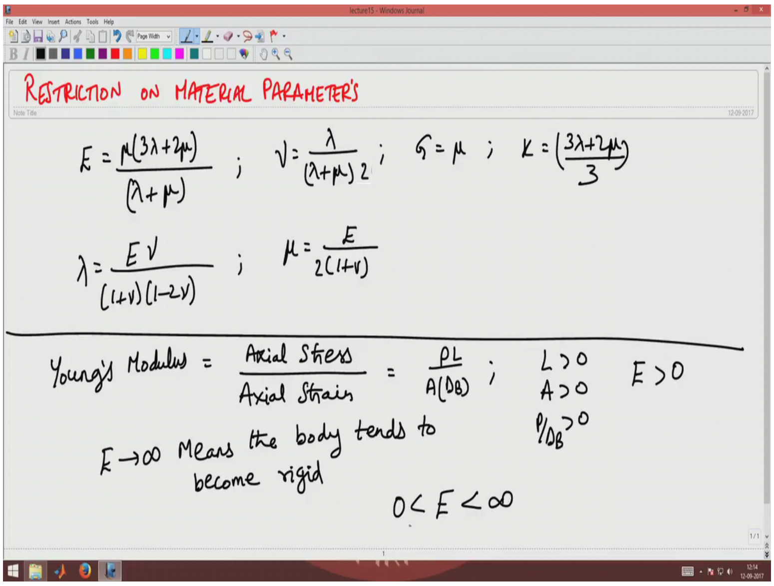

Basically, you have Young's modulus related to a lame constants, Poisson's ratio related to the lame constant through this expressions and lame constant related to Young's modulus and Poisson's ratio. So, these equations now what you are going to see is what are the restrictions are what are the possible values that this metal parameters can take. Now what we define for the Young's modulus is, we define the Young's modulus as axial stress by actual strain, which was P by A into delta B into L right.

So, now what happens, if I apply L and A are positive, L is positive, A is positive, if I reply a tensile force, I should get a displacement in the action of the force right. So, P by delta B must be positive because a displacement should be in the direction of force and apply compressive force the length will reduce delta B will be negative and apply tensile force P will increase and delta V will increase will be positive and hence, P by delta B has to be positive, from this registration you find that Young's modulus as to be a positive number.

Now, the question is kind Young's modulus 10 to infinity, if Young's modulus 10 to infinity what they are saying is, even though I am applying axial stress tensile are compressive there is no strain develop that is a meaning of E been 10 infinity. Which is a rigid body, E 10 into infinity means the body tends to become rigid, that is just what defamation even though I am applying a force that is the displacement between two particles, is the same irrespective of or two particles I constant the body.

So, since in this course we not interested the rigid bodies what we say is the Young's modulus has to live between 0 and infinity, it cannot be 0, it cannot be infinity, but it has to live between 0 and infinity or it can be a positive number.

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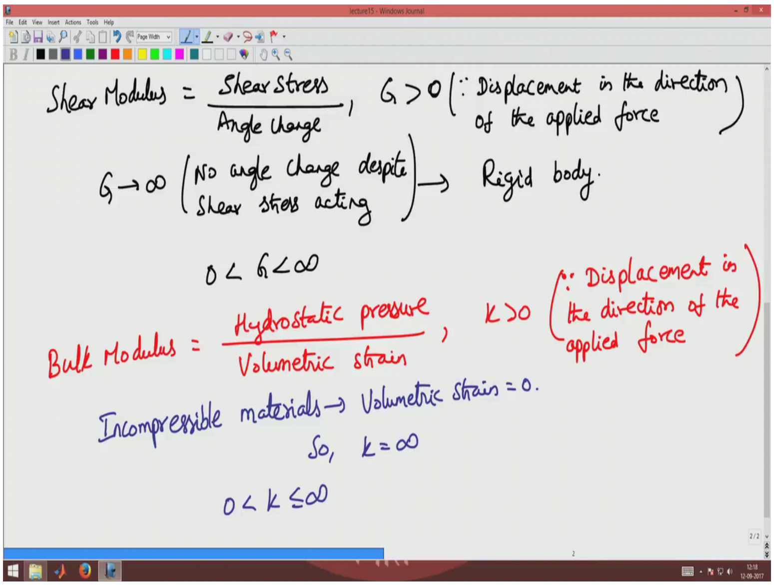

A similar argument we make for the shear modulus was defined as shear stress by angle change cause to shear stressed. So, a share starting like this on a body should produce angle changed in the direction of the stress right shear stress stating like this should produce angle change in the direction of the stress that is acting.

So, G has to be positive for same reasons that displacement has to be in the direction of applied force, displacement has to be, since displacement in the direction of the applied force. Again shear modulus been infinity means, shear modulus 10 into infinity means there is no angry change the despite been a shear stress applied infinity means no angle change despite shear stress acting this means the body is a rigid.

Body which we are not interested in this course because we are interested only in deformable bodies. So, the range of G is this, it has be a positive number. On the other hand bulk modulus is defined as the ratio of hydrostatic pressure divided by volumetric change.

Now, again bulk modulus as to be positive because if apply a compulsive force the volume should decrease, if I apply tensile hydrostatic pressure volume as to increase for the same reason as above K has to be greater than 0, since displacement in the direction of the applied force. Now there are some materials which will not change its volume despite what hydrostatic pressure you apply or what are the stress state, certain martial that will won't change its volume, such materials are called as incompressible materials real's the volumetric strain is equal to 0 for volumetric strain is 0.

So, K can be equal to infinity, so domain for K is 0 less than K less than or equal to infinity. So, that is a domain for K, now with this restrictions you want to find; what is the restriction of K G and E, on the Poisson's ratio and the lame constants lambda and mu.

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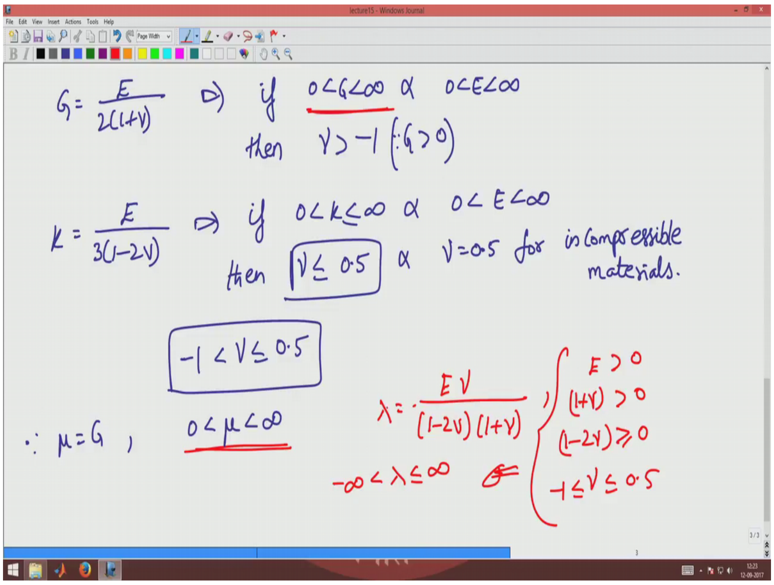

So, now we have G given by E by is given by 2 times 1 plus mu the shear modulus is given in terms of Young's modulus and Poisson's ratio as that right. If G this implies if 0 less than G less than infinity and 0 less than E less than infinity, then nu as to be greater than minus 1 right mu equal to the Poisson's ratio equal to minus means, G is infinity which is not allowed and if mu is greater than minus 1 is positive denominator is negative.

So, G becomes negative this is because G as to be greater than 0, when E is always greater than 0. So, you get the restriction that the Poisson's ration has to be greater than minus 1.

Now, the bulk modulus K is given as E by 3 to 1 minus 2 mu, since bulk modulus E by 3 into 1 minus 2 mu and implies that if 0 less than kappa less than k less than or equal to infinity and 0 less than E less than infinity then nu ask to be greater than equal to 0.5 and nu is 0.5, for incompressible materials.

So, from here you get that then nu should be less than or equal to 0.5 and mu as to equal to 0.5 for incompressible materials, from this 2 restrictions from this and that restriction you get nu to vary between minus 1, minus 1 and 0.5 nu as to vary between minus 1 0.5, from this 2 restrictions. Now since, nu is G the restriction 1 mu is also from 0 less than mu less than infinity it comes from this restriction G is that, and hence you get this restriction on mu.

Now, the other lame constant lambda is E times nu divided by 1 minus 2 mu into 1 plus mu for a given range you know that E is positive 1 plus mu is positive 1 minus 2 mu is greater than equal to 0 mu as range between minus 1 and point five all this together would imply that lambda varies from minus infinity to plus infinite.

The equal to sign here is because the mu is 0.5 that is what is allowed it will be positive mu is 0.5 that is when this will be positive whereas, this mu is cannot be minus 1, and hence it can't be negative infinity. So, to summarize you have seen that the following are the restrictions on metal parameters.

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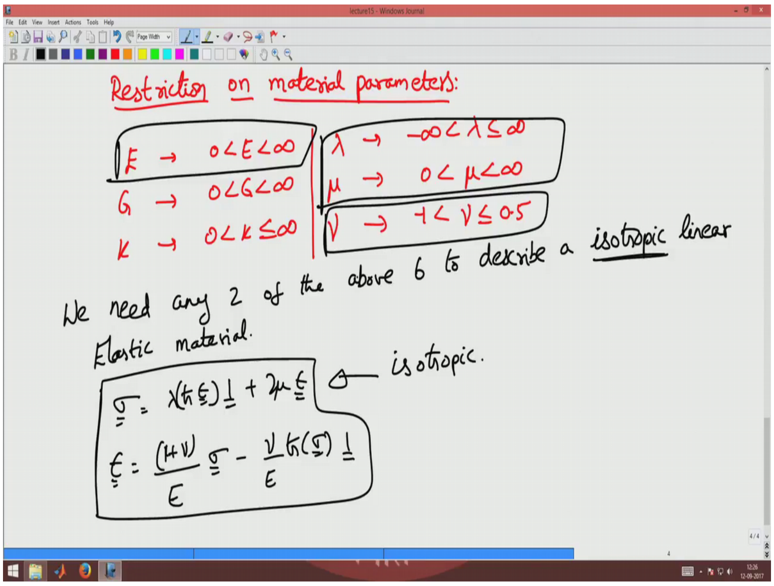

For E varies between 0 less than E less than infinity G varies between 0 less than G less than infinity kappa varies between 0 less than bulk modulus less than infinity.

Similarly, lambda varies between minus infinity and infinity the other lame constants varies between 0 and infinity and the Poisson's ratio nu varies between minus 1 less than nu less than or equal to 2.5. So, this are restriction on the metal parameters, now we have this 6 metal parameters do you need all the 6 metal parameters describe a constitutive relation no you need only any two of this subset.

So, you have 6 metal parameters, we need any two of the above 6 to describe a isotropic linear elastic material. What is this isotropic means, we have seen it means that the response of the body in different directions would be the same. So, that is what we mean by isotropic and when we wrote the sigma is lambda stress Epsilon identity plus 2 mu Epsilon, we assumes this isotropic that is the response in different directions are the same.

So, here we wrote in terms of lambda and mu we write in terms of Young's modulus and posissions ratio, we can write in terms of Young's modulus and Poisson's ratio are bulk modulus and shear modulus any two we can pick and we can write this constitutive relation in terms of those 2 parameters.

So, to go forward and remember that G is given by the stress is given by this expression and your strain is given by a expression 1 plus mu by E sigma as mu by E stress of sigma identity. These two expressions you have to remember and the range of values that the Poisson's ratio can take, the Young's modulus can take and this lame constant can take, that is what I have to remember to move forward.