Mechanics of Material
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Introduction and Mathematical Preliminaries

Lecture – 02
Part 1
Mathematical Preliminaries
Linear Algebra

Welcome to the second lecture of Mechanics of Materials course. In this lecture we will get an introduction to linear algebra and then we will see how to use initial notations.

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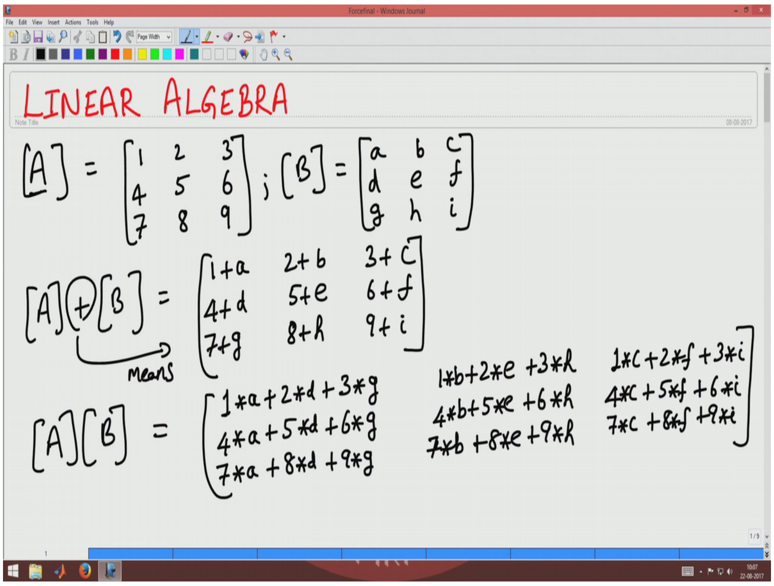

First let assume that A is a matrix which we will denoted by the symbol, let us say it has entries 1 2 3 4 5 6 7 8 9 and B is another matrix which has entries a b c d e f g h i. I am going to define some basic operations in linear algebra many of you might be knowing this already, but for completeness say let me go out and define those operations when I say a matrix A plus matrix B what I mean is this resulting matrix 1 plus a, 2 plus b, 3 plus c; 4 plus d, 5 plus e, 6 plus f; 7 plus g, 8 plus h, 9 plus i. This is what this operator a plus means.

Similarly if I want to multiply this matrix A times B then what I mean is multiplying these columns with these rows A multiplied by B means multiplying these columns with these rows. So, now, what is this A into B means? A into B means 1 into a plus 2 into d plus 3 into g will be the first row first column entry out this matrix A into B. The second entry of this would be 1 into b plus 2 into e plus 3 into h. The third entry of this row would be its going to be 1 into c plus 2 into f plus 3 into i.

Similarly, for the second row of this A into B matrix I have to multiply second row of A with each of these columns of B vector, B matrix. So, it will be 4 into a plus 5 into d plus 6 into g. Similarly the second entry is going to be 4 into b plus 5 into e plus 6 into h. Third entry is going to be 4 into c plus 5 into f plus 6 into i. The last row of this I am sure you would have recognize a pattern by now it would be 7 into a plus 8 into d plus 9 into g and here it is going to be 7 into b plus 8 into e plus 9 into h and here is going to be 7 into c plus 8 into f plus 9 into i. This is what matrix multiplication means.

Now, in A into B is not going to be equal to B into A.

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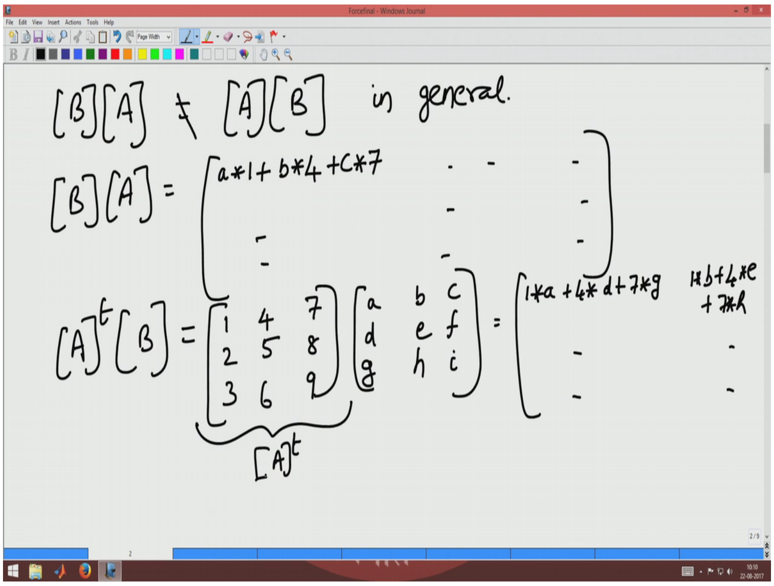

Let us see that now I am interested in multiplying finding the result for B times A, this is not equal to A times B in general. To see what it means let us write first row of B into A, B into A what should I do I have to multiply the rows of b with the columns of a. So, the first entry there is going to be B times A is going to be a times 1 plus b times 4 plus c times 7 right when I multiply the rows of b the columns of a there is what I will have. In contrast you what you had for a times b consider a difference between a times b and b times a here. So, A multiplied by B is now same as B multiplied by A. I leave this has an exercise for you to complete what the entries of these are going to be.

Now, I am also interested in finding what A transpose B would be for (Refer Time: 05:46) become evident to us end of this lecture. A transpose B would be 1 to a this a transpose 6 7 8 9, this is a transpose that is have to interchange the rows and the columns A was initially 1 2 3 4 5 6 7 8 9 arrange along the rows now it becomes arrange along the column because I am interested in transposing this matrix A.

So, B remain same B is a b c d e f g h i. So, this gives me 1 times a plus 4 times d plus 7 times g. So, first entry second entry would be 1 times b plus 4 times e plus 7 times h and so o. I leave it as an exercise for you to complete the remaining components of A transpose B. So, this is what this multiplication means.

So, we have seen what addition of 2 matrices mean and what multiplication of 2 matrices mean. Till now you have been doing it by writing all the elements of the matrix and then you have been multiplying it and writing what the resulting matrices now say I am lazy or I do not want to explicitly state what the components of the matrix is that is 1 2 3 4 5 6, but I want to delete with some symbols like a b c that I have use. I am going to use an alternate symbol to represent the matrix a as matrix a can be represented as A ij where we will follow the convention that i means rows and j denotes the column of this matrix A.

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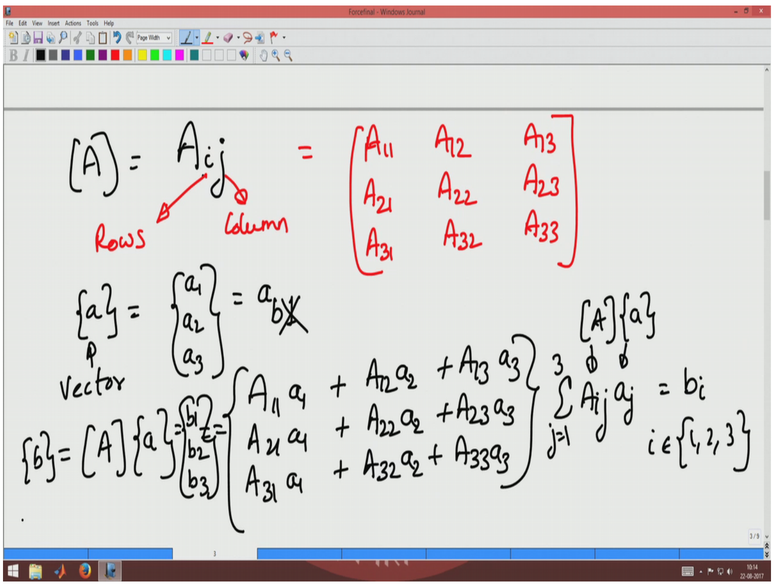

So, this should have been A 11 because I am in the first row first column and A 12 because I am the first row second column and A 13 because I am in the first row third column here. Similarly it will be A 21 second row first element first column A 22 A 23, A

31 A 32 and A 33, would this has in a short form written as A ij this matrix in short form has been written as A ij. Now similarly if I have a vector which I will denoted by the symbol a this denotes a vector it can I will write it as a column vector a 1 a 2 a 3 which I will denote it as a b, a b in general, I will not have this 1 to indicate the column. So, vectors will be represented using just 1 index by default vectors for us should be a column vector.

Now if you want to multiply this A matrix with this vector a I know that it will be A 11 a 1 plus A 12 a 2 plus A 13 a 3, A 21 a 1 plus A 22 a 2 plus A 23 a 3, A 31 a 1 plus A 32 a 2 plus A 33 a 3. This equation I want write it in short form by writing this a matrix as A ij and this vector A as a j and I am going to indicate that I have to sum this j from 1 2 3 this will be another vector say this vector was equal to b this will be the b ith component of the vector B.

So, basically what we have is you have here b 1 b 2 b 3 represented as A 11 a 1 A 12 a 2. So, that is a same equation here where I can take elements from 1 2 or 3. So, basically this matrix vector multiplication is can be represented as this.

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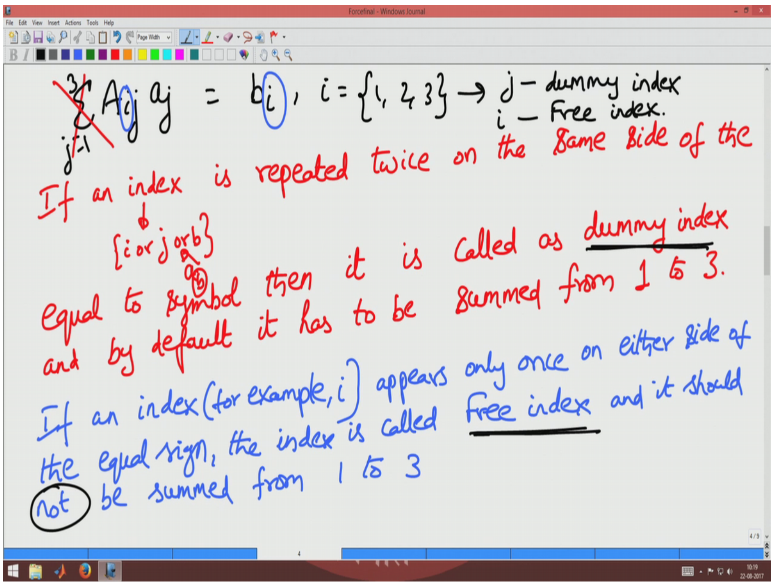

Now, I am going to become a little bit more lazy and say that the simplify the equation that I wrote there I wrote A ij a j equal to b i, j equal to 1 2 3 and I takes elements from the set 1 2 3. I am going to rewrite the same equation dropping off the summations symbol saying that if a index is repeated twice if an index indexes index means i or j or b or b what we use for a b what I write in a suffix is called as the index.

If an index is repeated twice on the same side of the equal to symbol then it is called as dummy index and by default it has to be summed from 1 to 3. In contrast this index I which appears on either side of the equal to sign is called as a free index. If an index for example, i, in the above equation appears only once on either side of the equal sign the index is called free index in contrast to dummy index this is called as free index here. So, called as free index and it may not it should not be summed from 1 to 3 important word is it should not be summed dummy index has to be summed from 1 to 3. Whereas, a free index need not be summed from it should not be summed from 1 to 3.

In this example j is a dummy index and i is a free index. Now why is this dummy and free has to be distinguished this? A dummy index can be given any alphabet a it can change into any other symbol just like a variable can be denoted by x y or z or t or a, this dummy index can be changed consistently throughout into any other symbol j can become b j can become k, but j cannot become i because i appears on either side of the equal to sign.

So, x ray for the indexes that appear on either side of the equal to sign which are free indexes. The dummy index can take any other symbol also because by default you have to summing you have going to sum that from 1 2 3. So, this dropping off of the summation sign is a conventional we are going to adopt. So, what that means is an index should not appear more than twice on side of the equal to sign, if the index appears more than twice on one side of equal to send there is something wrong with your initial notation this is called as a initial notation of this matrix equation, this called as a initial notation of the matrix equation matrix. What is multiplication is initially denoted by in this particular form here in this particular form here.

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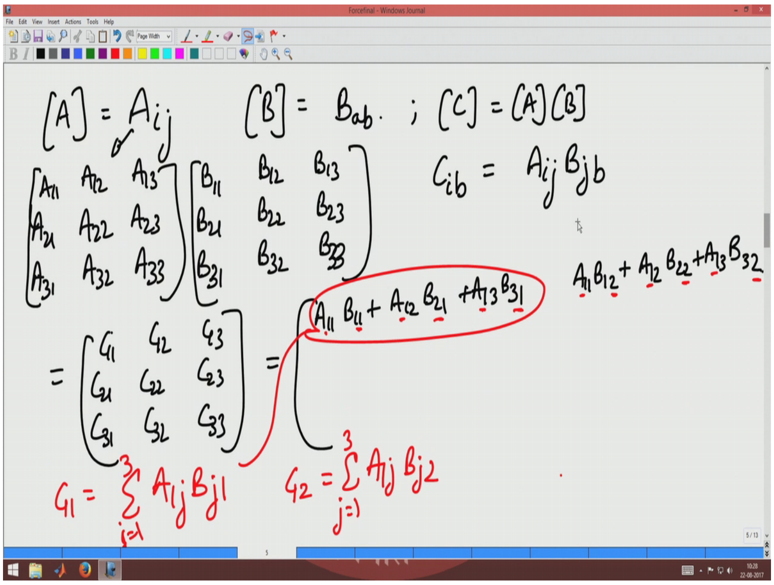

So, now let us understand how to write matrix matrix multiplication say I have A which I write it as A ij and another matrix B I write it as B ab, where i j denotes the components of that matrix 11 12 13 14 like that. Now I want to define a matrix C which is A times B, then what should I do? I have to sum the columns of A with the rows of B, so this j and a have to be of the same symbol. So, this means by a notation this has to be A ij, B jb would be C ib. So, going back to a equation that we are here this is C ab. So, C 11 corresponds to this term right that is we are multiplying the column indexes of A with the row index of B.

So, let us go back and check whether we are getting the same thing for this matrix. So, let us do it the long way. So, A would be written as A 11 A 12 A 13, A 21 A 22 A 23, A 31 A 32 A 33 right this is what this means. Similarly B would be B 11 B 12 B 13, B 21 B 22 B 23, B 31 B 32 B 33 now this is going to be equal to C 11 C 12 C 13, C 21 C 22 C 23, c 31 c 32 c 33 which in terms of A and B would be A 11 B 11 plus A 12 B 21 plus A 13 B 31.

The second entry would be A 11 B 12 plus A 12 B 22 plus A 13 B 32 plus they will be additional terms. These 2 terms has of suffix is to see that what you are summing up is in a given this thing this if I write C 11 here C 11 would be summation A 1j B j1, j equal to 1 2 3 let us see whether this is equal to this right. So, I have 11 here, so its 11 here, 11 here, 11 here and j has been summed from 1 2 and 3. So, this corresponds to C 11.

Similarly, C 12 would be A 1j B j2 summation j equal to 1 2 3 which is 1 and 2 here the 1 and 2 here corresponds to 1 and 2 their, 1 and 2 there, 1 and 2 here and then j has to be summed from 1 2 3 which it has been 1 2 1 3 j has means sum. So, you see that this is the short form notation of doing a matrix matrix multiplication, I can write it is a subsequently as A ij B jb. So, let us now continue to see what will, how will I write this following multiplication.

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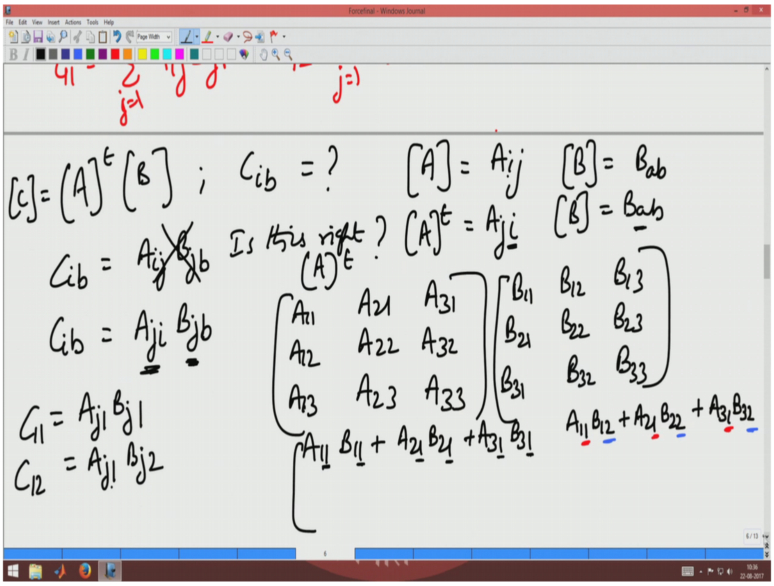

I want to do A transpose B is C matrix. So, how will I write C ib as is the question. I know that A is written as A ij and B is written as this B matrix is written as B ab ok.

Now, since is transpose I have to sum the A transpose should be A ji and this I have to multiply with B ab. So, now, multiplication means I have to make these 2 indexes the same. So, it will become C ib would be A ij B jb this is right. Now if I write would like this and use the fact that I have interchange the indexes i and j here. So, this is not right the correct expression for this is A ji B jb would be C ib because I have to interchange this indexes.

So, this is the right way of writing A transpose B. Let us examine that a transpose would be A 11 A 12 A 13, A 21 A 22 A 23, A 31 A 32 A 33. Here the meanings of the indexes are change. So, that is why the first index is the row index, first index has come like this, this multiplied by b this will be B 11 B 12 B 13, B 21 B 22 B 23, B 31 B 32 B 33 now multiply this what do I get the first term would be. So, if you multiply these 2 matrices what will get is A 11 B 11 plus A 21 B 21 plus A 31 B 31, the second entry would be A 11 B 12 plus A 21 B 22 plus A 31 B 32 and so on.

So, if you see now C 11 would be A j1 B j1, this 1 1 has to repeat at the end the column index of B and A should be the same you can see that it is the same here. The column and the indexes of A and B are same whereas, the j means you have to sum 1 2 3 which you have doing on the row index of A and B. Similarly C 12 would be A j1 B j2. So, the column index of A has to be 1 and the column index of B has to be 2 in the second term, you can see here the column index of A is 1 and the row index of B is 2 as suggested by this expression here and you have to sum j from 1 2 3 which you are doing by summing it from 1 2 3.

So, basically now this what A transpose B means basically you change where you are the repeated index where a dummy index comes changes from A multiplied by B to A transpose multiplied by B. This is important because in many places you will be using this concept of initial notation in this course. Now, let us move along and understand what a difference between vector is, now we have see matrix multiplication.