Mechanics of Material
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Lecture – 38
Constitutive relation, strain energy and potential

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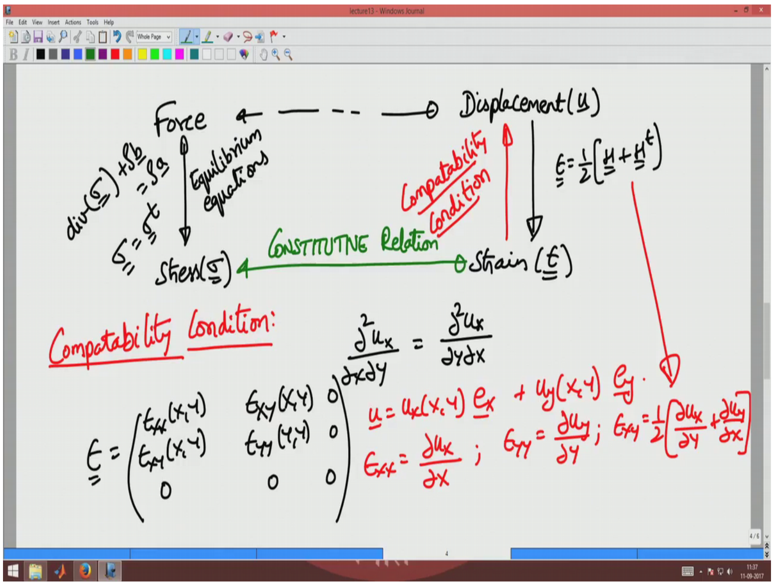

Now, the constitutive relation is developed.

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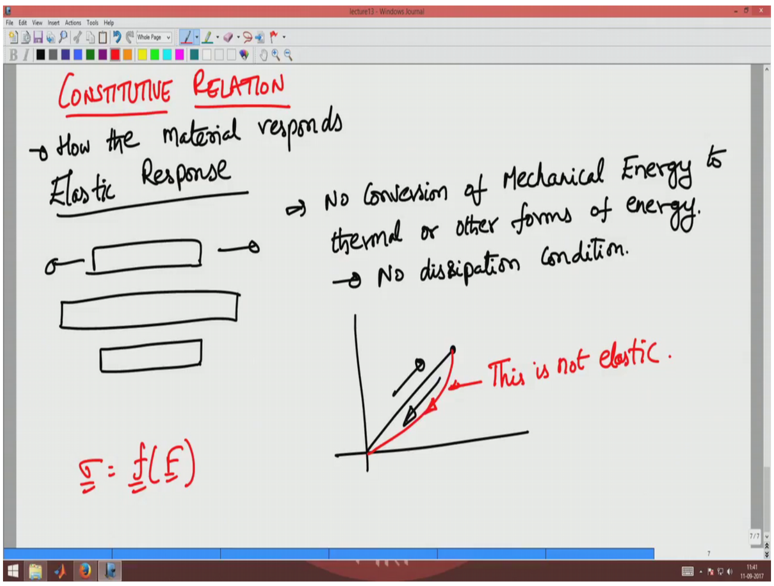

The constitutive relation tells us how the material responds; it tells us how the material responds. So, basically we saw that the stress and strain are concepts introduced to lose geometric dependence, and constitutive relation is a metal dependent relationship where in you know that the steel response differently from a rubber bar right.

So, that material dependencies brought in by constitutive relation. In the constitutive relation you have various class of constitutive relation called as elastic constitutive relation, plastic constitutive relation, visco elastic constitutive relation, and visco plastic constitutive relation depending upon how the stress a kinetic quantity, and a strain a kinematic quantity are related. The stress and strain can be related directly or stress, stress straight, strain and strain rate can be a related, or there are different ways where which the relationship can be proposed depending upon it will fall into the metal be elastic or plastic or viscos elastic or visco plastic. In this course we will be interested only in the elastic response of the material.

So, let us understand what you mean by elastic response. A text book definition of elastic response is if I load a bar in uniaxial tensor and let go the force it should be come back to its original state right. If I pull a bar and let the force go, it will come back to its original state that is a definition of a elastic response; that is if I pull a bar, it elongates to this shape may be and when I let it go it comes back to its original shape. So, that is a definition of elastic response. In other words what we are saying here is there is no dissipation, elastic response means there is no conversion of mechanical energy to thermal or other forms of energy ok.

So, essentially what we are saying is, the mechanical energy will be stored in the body as a mechanical energy and we release from the body as mechanical energy when you remove the load. This is called as no dissipation condition; that is if I load a bar say this is a loading part, if I unload it from here we will try as same path again it will would not do this. If the unloading is like this, even though it reaches a same state this is not elastic. Loading and unloading part are to be the same the shape should be preserved. So, all this are the attributes of a elastic response ok.

Now, we will be interested only in this elastic response of the body and hence we will relate the stress directly as a function of the definition gradient F. Since we are interested in only elastic response we will say that the stress is a function of the reformation gradient f. Now what happens now there are certain requirements of constitutive relation?

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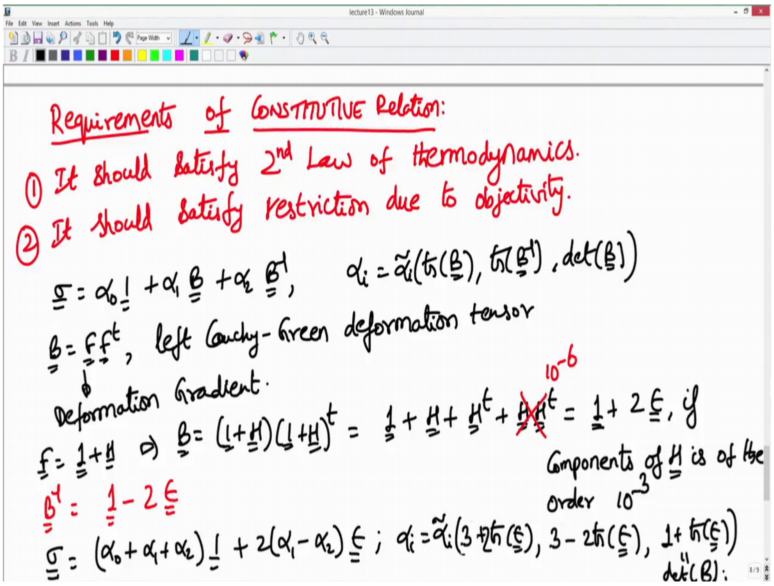

One is it should satisfy second law of thermodynamics, 2 is it should satisfy restriction due to objectivity. What does the second restriction mean is, it does not matter either you do an experiment in India or whether you do experiment in America or whether you do an experiment on surface of moon or you do experiment on mass, the constitutive relation that you infer for a given martial should be independent of where you do the experiment, there is a restriction due to the objectivity. Irrespective of where you do the experiment, the constitutive relation for a given material will not change ok.

In other words what it means is, when you place a body in the equitant space each of us can paste place a body in a different region. This is the body I can place it here I can have my corner system oriented like this oriented like this, oriented like this, it can be here it can be there any where can be, but this body's constitutive relation should not depend upon any of displacement of the potion vector or how I represent the body and its surroundings in equilibrium point space ok.

That is what a restriction due to objectivity means. The restriction due to the objectivity means anywhere you do the experiment a constitutive relation of the body should be a same. So, based on this we will find that to satisfy these 2 restrictions, you will find that the more general expression for the stress would be equation of this form alpha 2 B inverse or alpha i is some function of stress of B, stress of B inverse and determent of B ok.

Now, B is F F transpose, this is called as the left Cauchy green deformation tensor. B is call as a left Couchy green defamation tensor; B is called as a left Couchy green defamation tensor, this F times of transpose where F is the deformation gradient that we have seen before. Now just like we computed C in transpose epsilon the right Couchy green deformation tensor lets compute what is B in terms of the displacement gradient.

You know that F is related to the displacement gradient also 1 plus h this will be 1 plus H transpose, this is 1 plus H plus H transpose, plus H H transpose, again if this will be 1 plus 2 epsilon if components of H is of the order 10 power minus 3. What I have done I have neglected this term H is transpose saying this of all 10 power minus 6. So, I have neglected this and I have written this as 2 epsilon ok.

Similarly, I can find B inverse would be 1 minus 2 times epsilon, I would not go on to a detail derivation of this, B inverse would be this then substituting these 2 expression for B and B inverse this expression for stress what we will get is it will get sigma to be alpha naught plus alpha 1 minus alpha 2 times identity plus alpha one minus alpha 2, 2 times epsilon ok.

So, what you have done is this alpha i is would be function of stress of B, there will be 3 plus 2 times stress epsilon, this will be 3 minus 2 times stress epsilon, that is B inverse and determinant B can be shown the 1 plus stress of epsilon this is nothing, but determinant of B, I am not interest in deriving this I am just interested to show you that how you obtain one of the constitutive there will be using in this course or the only constitutive that will be using this course that is (Refer Time: 10:58) law ok.

So, basically what you find from here is, this alpha i this all functions of stress epsilon.

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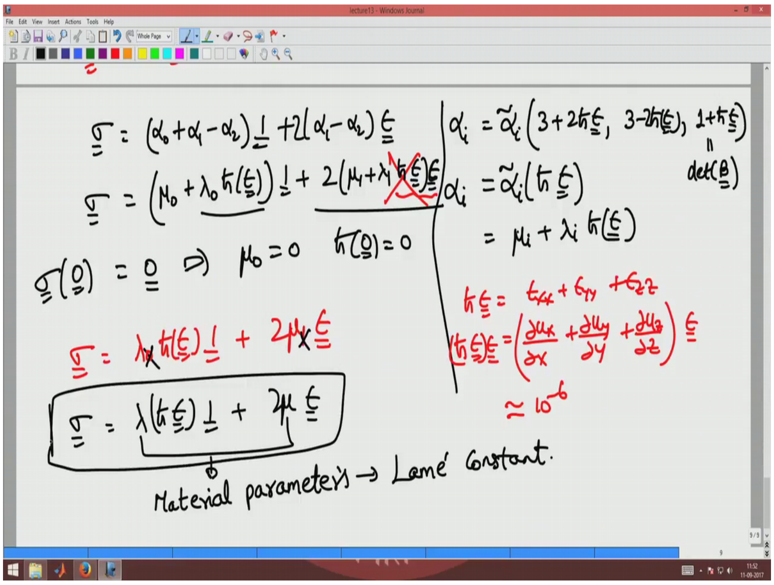

So, alpha i is the function of stress of epsilon. Since we neglected the higher order terms of the displacement gradient, this alpha r plus alpha 1 minus alpha 2, and alpha 1 minus alpha 2, cannot depend upon i and r terms of epsilon. So, you even though this is function stress epsilon this can be at most mu i plus lambda i stress of epsilon, it can be at most function of this linear functions let us substitute that in here. So, what will I get I will get some of linear function. So, it will be mu i plus lambda i stress epsilon times identity, plus 2 times let me say mu 0 lambda 0 mu 1 plus lambda 1 stress of epsilon, epsilon ok.

Now, what happens when this is a stress, when strain is 0 what should be the stress? Stress has to be 0 right when there is no strain the stress has to be 0, which will imply that mu 0 has to be 0 because stress of a 0 is 0 these terms will vanish this, this and that term will vanish (Refer Time: 12:41) mu identity which are which means that mu 0 as to be 0 ok.

Now, then I have here stress epsilon times epsilon, that is a non-linear terms it will have gradient of h component which are squared right because stress of epsilon is epsilon x x plus epsilon y y, plus epsilon zz, which is dou u x by dou x plus dou y by dou y plus dou u z by dou z. This multiplied by epsilon stress epsilon multiplied by epsilon would be of order 10 power minus 6. Since I ignore this order before I have to ignore this terms also.

So, you get finally, the stress to be given by lambda naught stress epsilon identity plus 2 mu 1 epsilon ok.

I will drop this suffix and simply write sigma as lambda stress epsilon identity plus 2 mu epsilon where this lambda and mu are material parameters; in particular it is called as lame constant. So, this is a constitutive realtion that we will be using in this course. In the next class we will see how to estimate this metal parameters and then we will dwell more into this constitutive realtion.

So, in this lecture we looked at the 2 remaining equations that we on looked at till now, the cast in this lecture I looked at 2 equation that we all look till now the capability condition constitutive relation. In the previous lecture we looked at all the four concepts force displacement stress and strain, and we looked at equilibrium equation or strain displacement relationship, in this lecture we have looked at what compability conditions are and what the constitutive relation is. So, this complete basically the four concepts and four equation that connect them in mechanics used in mechanics ok.

Thank you.